k –Perfect Geodomination is Affected by Adding a Pendant Vertex

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Abstract

A k –geodominating set is a geodominating set S such that any vertex $v \in V(G) \setminus S$ is geodominated by a pair x, y of vertices of S with $d(x, y) = k$. A k –perfect geodominating set is a geodominating set S such that any vertex $v \in V(G) \setminus S$ is geodominated by exactly one pair x, y of the vertices of S with $d(x, y) = k$. The cardinality of a minimum perfect geodominating set in G is its perfect geodomination number $g_p(G)$ and the cardinality of a minimum k –perfect geodominating set in G is its k –perfect geodomination number $g_{kp}(G)$. We investigate the affection of k –perfect geodomination numbers of a graph G when a pendant vertex is added.

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1 Introduction

Let G be a non-trivial connected graph. A $x - y$ path of length $d(x, y)$ is called a $x - y$ geodesic. A vertex v is said to be in a $x - y$ geodesic P if v is an internal vertex of P . The closed interval $I[x, y]$ consists of x, y and all vertices lying in some $x - y$ geodesic of G , while for $S \subseteq V(G)$,

$$I[S] = \cup_{x, y \in S} I[x, y].$$

A set S of vertices is a *geodetic set* if $I[S] = V(G)$, and the minimum cardinality of a geodetic set is the *geodetic number* $g(G)$. A geodetic set of cardinality $g(G)$ is called a g –set, (see [1,2,3,4,5,6]).

Geodetic concepts were studied from the point of view of domination. Geodetic sets and the geodetic number were referred to as geodominating sets and geodomination number [2] that we adopt in this paper.

A pair x, y of vertices in a nontrivial connected graph G is said to *geodominate* a vertex v of G if either $v \in \{x, y\}$ or v lies in an $x - y$ geodesic of G . A set S of vertices of G is a geodominating set if every vertex of G is geodominated by some pair of vertices of S . A vertex of G is *link-complete* if the subgraph induced by its neighborhood is complete. It is easily seen that any link-complete vertex belongs to any geodominating set. For a graph G and an integer $k \geq 1$, a vertex v of G is *k-geodominated* by a pair x, y of distinct vertices in G if v is geodominated by x, y and $d(x, y) = k$. A set S of vertices of G is a *k-geodominating set* of G if each vertex v in $V(G) \setminus S$ is *k-geodominated* by some pair of distinct vertices of S . The minimum cardinality of a *k-geodominating set* of G is its *k-geodomination number* $g_k(G)$. A *k-geodomination set* of cardinality $g_k(G)$ is called a *g_k -set* of G . All graphs are connected and for an edge $e = \{u, v\}$ of a graph G with $\deg(u) = 1$ and $\deg(v) > 1$, we call e a pendant edge and u a pendant vertex.

2 How k-perfect geodomination is affected by adding a pendant vertex

How *k-geodomination numbers* are affected by adding a vertex, was studied in [5]. Here we study this concept for *k-perfect geodominating sets*.

A perfect geodominating set is a geodominating set S such that any vertex $v \in V(G) \setminus S$ is geodominated by exactly one pair x, y of the vertices of S

Definition 2.1 *Let $k \geq 1$ be an integer. A k-perfect geodominating set is a geodominating set S such that any vertex $v \in V(G) \setminus S$ is geodominated by exactly one pair x, y of vertices of S with $d(x, y) = k$.*

The cardinality of a minimum *k-perfect geodominating set* in G is its *k-perfect geodomination number* $g_{kp}(G)$.

Theorem 2.2 *Let G be a graph with no pendant vertices and G' obtained from G by adding a pendant edge uv with $v \notin G, u \in G$, and also the vertex u belongs to some $g_p(G)$ -set. Then $g_p(G') = g_p(G)$.*

Proof Let S be a $g_p(G)$ -set containing the vertex u , then it is easily seen that $(S \setminus \{u\}) \cup \{v\}$ is a perfect geodominating set for G' . So $g_p(G') \leq g_p(G)$. Now if S' is a $g_p(G')$ -set with size less than $g_p(G)$ then $v \in S'$, so $(S' \setminus \{v\}) \cup \{u\}$ is a $g_p(G)$ -set with size less than $g_p(G)$, which is a contradiction. Hence $g_p(G') \geq g_p(G)$ and the Proof is complete.

Note that the condition u belongs to some $g_p(G)$ -set in the above proposition is necessary. For this order consider the following:

Example 2.3 Let H be the graph C_{2n} with vertices $v_1, v_2, \dots, v_n$. Add ears $v_1, w_2, w_3, \dots, w_n, v_{n+1}$ and $v_1, w'_2, w'_3, \dots, w_n, v_{n+1}$ to obtain a graph G with $g_p(G) = 2$. Now if G' obtained from G by adding one of the pendant edges $xw_i, 2 \leq i \leq n, x \notin V(G)$, then $g_p(G') > g_p(G)$.

To adding of a pendant edge to a connected graph G can produce a graph H with $g_{kp}(H) > g_{kp}(G)$.

Example 2.4 Consider the cycle $C_6 : v_1, v_2, \dots, v_6, v_1$ and add a pendant edge v_6u with $u \notin C_6$, and also add $m \geq 4$ ears v_1, w_{i2}, w_{i3}, v_4 for $i = 1, 2, \dots, m$ to obtain a graph G . Now let G' obtained from G by adding the pendant edge v_4w with $w \notin V(G)$, then $g_{3p}(G') = 2 + g_{3p}(G)$. So the addition of a pendant edge to a connected graph G can strictly increase its k -perfect geodomination number.

Also the addition of a pendant edge to a connected graph G can produce a graph whose k -perfect geodomination number is strictly smaller than that of G .

Example 2.5 Consider the cycle $C_{2m} : v_1, v_2, \dots, v_{2m}, v_1$ for some $m \geq 2$. Since $\text{diam}(C_{2m}) = m$, then $g_{(m+1)p}(C_{2m}) = 2m$. Let G be a graph obtained from C_{2m} by adding the pendant edge uv_{m+1} . Since $\{u, v_1\}$ is a $(m+1)$ -perfect geodominating set, then $g_{(m+1)p}(G) = 2$.

Theorem 2.6 For every pair k, n of positive integers there exists connected graphs G and G' where G' is obtained from G by adding a vertex v with more than one edge incident with v and $g_{kp}(G) - g_{kp}(G') = n$.

Proof Let $P_i : v_{i1}, v_{i2}, \dots, v_{i(k-1)}$ be a path of order $k-1$ for $i = 1, 2, \dots, n$ and let G be obtained from the paths P_i by adding a new vertex v and joining v to each of the vertices v_{i1} for $i = 1, 2, \dots, n$. It is easily seen that $\{v_{i(k-1)} : i = 1, 2, \dots, n\} \cup \{v_{11}, v_{21}\}$ is a k -perfect geodominating set of G . Moreover $g_{kp}(G) = n + 2$. Now let G' obtained from G by adding a new vertex u and joining it to each of the vertices $v_{i(k-1)}$ for $i = 1, 2, \dots, n$. Thus $d(u, v) = k$ and $g(G') = 2$, so $g_{kp}(G') = 2$. Hence $g_{kp}(G) - g_{kp}(G') = n$.

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