International Mathematical Forum, 1, 2006, no. 40, 2003 – 2017

**A Multi-Entry Simulated and Inversed Function Approach
for Alternative Solutions**

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Abstract

Usually, a simulated and inversed function associated with its generated random variables is used as a simulation input supporting alternative solutions. Single-entry simulated and inversed function is popularly used in industry management for many years. But in most of cases, multi-entry components problems existed such as the multi-median traffic (voice, data & fax) in telecommunication network and the multi-sized container (container sized with 20, 30 & 40 feet) in marine transportation management. The purpose of this compound inversed function approach is to solve the multi-entry alternative problems. Furthermore, a case study of network multi-service path traffic event by MATLAB genetic module is demonstrated to verify the model we propose.

Keywords: Multi-entry inversed function; simulated; ARIMA; K-S test; genetic

1. Introduction

Usually, a simulated and inversed function associated with its generated random variables is used as a simulation input supporting alternative solutions. Single-entry simulated and inversed function is popularly used in industry management for many years. But in most of cases, multi-entry components problems existed such as the multi-median traffic (voice, data & fax) in telecommunication network and the multi-sized container (container sized with 20, 30 & 40 feet) in marine transportation management. The purpose of this compound inversed function approach is to solve the multi-entry alternative problems [1] [2]. Furthermore, a case study of network multi-service path traffic event by MATLAB genetic module is demonstrated to verify the model we propose.

This approach mainly includes several processes: (1) Building a multi-entry inversed model; (2) Verify the selected distribution; (3) Alternative index calculation & decision making.

2. Mathematical Modeling

The applied mathematical model definition and formulation of this study are explained as follows.

2.1 Formula A: A single-entry inversed function

Let x be a variable with cumulative distribution of $F_j(x)$, j denotes each data sampling event. For $F_j(x)$ is a non-decreasing function, the inverse simulation function of $F_j^{-1}(y)$ could be defined as

$$F_j^{-1}(y) = \inf\{x; F_j(x) \geq y\} \quad 0 \leq y \leq 1 \quad (1)$$

- Assume that sampling data can be grouped into n adjacent intervals $[b_0, b_1]$, $[b_1,$

$b_2), \dots, [b_{n-1}, b_n]$, and define the piecewise-linear function:

$$F_j(x) = \begin{cases} 0 & , x < b_0 \\ F_j(b_{i-1}) + \frac{x - b_{i-1}}{b_i - b_{i-1}} [F_j(b_i) - F_j(b_{i-1})] & , b_{i-1} \leq x < b_i \text{ for some } i < n \\ 1 & , x > b_n \end{cases} \quad (2)$$

Where $F_j(b_0) = 0$, $F_j(b_i) = \sum_{k=1}^i r_k / r$, $i = 1, 2, \dots, n$

r_i is the number of observations in the i th interval (b_{i-1}, b_i) and $r = \sum_{i=1}^n r_i$

• Find the minimum positive integer k ($0 \leq k \leq n-1$), such that $Y \leq F(X_k)$ and return

$$X = F_j^{-1}(Y) = b_k + [Y - F_j(b_k)] \times (b_{k+1} - b_k) / [F_j(b_{k+1}) - F_j(b_k)] \quad (3)$$

2.2 Formula B: A multi-entry inversed function

Define a compound function as an input of multi-entry inversed formula (3)

$$\hat{F}(x) = \sum_{j=1}^n F_j(x) P_j(x) \quad (4)$$

where $0 < P_j \leq 1$, $\sum_{j=1}^n P_j = 1$ and P_j denotes the Percentage Weight (PW) value of

service entry j . The summation of the percentage parameters of voice, data and fax is equal to value 1. Usually, there are two cases for the calculation of PW values:

a. Case 1: If the multi-service traffic is in a stable stage, then the constant PW values of each service will be derived by sampling data statistics directly

b. Case 2: Otherwise, the multi-service traffic is in a big variation stage, a statistical ARIMA (Auto-Regression Integrated Moving Average) time-series estimation formula for each telecommunication service (voice, data, fax) PW value will be used. Construct a statistical ARIMA (Auto-Regression Integrated Moving Average) time-series estimation formula for each telecommunication service (voice, data, fax) as **Formula C** [3] [4].

2.3 Formula C: An ARIMA formula of order (p, d, q)

- Auto-regression model

$$Y_t = \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + \dots + \phi_p Y_{t-p} + e_t \quad (5)$$

where ϕ : Index of auto-regression ; $\phi_1 Y_{t-1}$: Service observation of time

series t-1 term ; $\phi_p Y_{t-p}$: Service observation of time series t-p term;

e_t : Partial error

- Moving average model :

$$Y_t = e_t - \theta_1 e_{t-1} - \theta_2 e_{t-2} - \dots - \theta_q e_{t-q} \quad (6)$$

where θ : Service observation of time series t term ; $\theta_1 e_{t-1}$: Partial error of

time series t-1 term; $\theta_q e_{t-q}$: Partial error of time series t-p term

- Integrated model :

$$Y_t = \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + \dots + \phi_p Y_{t-p} + e_t - \theta_1 e_{t-1} - \dots - \theta_q e_{t-q} \quad (7)$$

In ARIMA (p, d, q) model, p denotes the order of Auto-Regression, d denotes the order of difference, q denotes the order of Moving Average.

2.4 Formula D: K-S model verification test

The authors use K-S testing to verify the inverse function model [5] [6]. It is defined as the follows :

$$F(x) = x, \quad 0 \leq x \leq 1 \quad (8)$$

$$S_N(x) = \frac{R_1, R_2, \dots, R_N \leq x}{N} \quad (9)$$

Where N is the largest observation number and $S_N(x)$ is the optimal

approximation value of F(x).

Step 1: Rank the data from smallest to largest, let denote the ith smallest observation, so that $R_{(1)} \leq R_{(2)} \leq \dots \leq R_{(N)}$

$$\text{Step 2: Compute } D^+ = \max_{1 \leq i \leq N} \left\{ \frac{i}{N} - R_{(i)} \right\} \quad (10); \quad D^- = \max_{1 \leq i \leq N} \left\{ R_{(i)} - \frac{i-1}{N} \right\} \quad (11)$$

$$\text{Step 3 : Calculate } D = \max(D^+, D^-) \quad (12)$$

Step 4: Determine the critical value D_α , for the specified significance level α and the given sample size N.

Step 5: If the sample statistic D is greater than the critical value D_α , the null hypothesis that the data a sample from a uniform distribution is rejected. If $D \leq D_\alpha$, conclude that no difference has been detected between the true distribution of $\{R_1, R_2, \dots, R_N\}$ and the uniform distribution.

2.5 Formula F: A queue and related score function

The queue modeling M/M/K can be used as a basis to calculate the related performance parameter values. Further, the related explanations are provided in the following part.

$$\rho = \frac{\lambda}{k \mu} \quad (13) \quad L = k\rho + \frac{(k\rho)^{k+1} p_0}{k(k!)(1-\rho)^2} \quad (14) \quad \omega = \frac{L}{\lambda} \quad (15)$$

where λ denotes inter-arrival rate; μ denotes service rate; ρ denotes system utilization ; L denotes service number in system; ω denotes delay time (service time); and P_0 denotes the probability in idle status. For simplicity of computing processes, a linear transformation function of score function raging from 1 to 10 was employed supporting the alternative indexes calculation.

2.6 Formula E: An object function

By using the genetic analysis [7] [8] of MATLAB software package, the authors define a general object function $g_i(X_i, Y_i)$ supporting alternative solution:

$$\text{MAX} \sum_{i=1}^n g_i(X_i, Y_i) \quad (16)$$

$$\text{ST} \sum_{s=1}^n \alpha_{k,s} X_s \leq c_k, \quad k=1, 2, \dots, l \quad (17)$$

$$\sum_{s=1}^n \beta_{k,s} Y_s \leq d_k, \quad k=1, 2, \dots, l \quad (18)$$

where $g_i(X_i, Y_i)$ denotes the score function path performance parameters; X_s, Y_s denote the path performance parameters; $\alpha_{k,s}, \beta_{k,s}$ denote the coefficients to achieve the studied path ; c_k, d_k denote the constrained upper bound values ; l denotes the number of constrained conditions.

Inter-Arrival Time (Milli Sec.)	Frequency	Relative Frequency	Cumulative Frequency	Inverse reference point
$0.5 \leq x < 2.0$	10	0.10	0.10	(0.10,2.00)
$2.0 \leq x < 4.0$	15	0.15	0.25	(0.35,4.00)
$4.0 \leq x < 6.0$	20	0.20	0.45	(0.75,6.00)
$6.0 \leq x < 8.0$	40	0.40	0.85	(0.85,8.00)
$8.0 \leq x \leq 10.0$	15	0.15	1.00	(1.00,10.00)

Table 1. Voice pack service sampling frequency

3. Applied Example

3.1. Sampling & Model Verification

Firstly, this study takes a group of network sampling data as above and below:

Table 2. Data pack service sampling frequency

Inter-Arrival Time (Milli Sec.)	Frequency	Relative Frequency	Cumulative Frequency	Inverse reference point
$0.5 \leq x < 2.0$	25	0.25	0.25	(0.25,2.00)
$2.0 \leq x < 4.0$	40	0.40	0.65	(0.65,4.00)
$4.0 \leq x < 6.0$	20	0.20	0.85	(0.85,6.00)
$6.0 \leq x < 8.0$	10	0.10	0.95	(0.95,8.00)
$8.0 \leq x \leq 10.0$	5	0.05	1.00	(1.00,10.00)

Fig 1 Voice pack inverse function

Fig 2 Compound inverse function

Fig 3 ARIMA ($p=0, d=1, q=1$) voice service PW values estimation

Table 3 An example of K-S test model verification

K-S Calculations \ Terms	1	2		99	100
$R_{(i)}$	0.021	0.051		0.969	1.000
i / N	0.025	0.05		0.975	1.000
$i / N - R_{(i)}$	0.003	-		0.005	0.000
$R_{(i)} - (i-1)/N$	0.021	0.026		0.019	0.025

An example of Table 3 is the Calculation parameters of each term for K-S test, here K-S Calculation denotes the parameters. Terms denote the indexes we make for this statistics. From this table, the authors can find $D^+ = \max_{1 \leq i \leq N} \left\{ \frac{i}{N} - R_{(i)} \right\}$ and $D^- = \max_{1 \leq i \leq N} \left\{ R_{(i)} - \frac{i-1}{N} \right\}$ values, and finally, we can get $D = \max(D^+, D^-)$ for the decision model fitting. H_0 : the random variable of failure inter-arrival time is exponentially distributed; H_1 : not exponentially distributed. An example of $D=0.063, \alpha = 0.05, D_\alpha=0.136$; where D value derived from empirical test of Table 3 and D_α value derived from statistical reference table; If $D \leq D_\alpha$, accept the H_0 : the random variable exponentially distributed. Once the hypothesis

model of K-S test is accepted, the authors can use the selected distribution parameters to calculate the queue performance indexes; otherwise, while model test rejected, the classical statistical minimum variance method will be then, used to estimate the performance index .

3.2 Alternative Study

Secondly, for simplicity, a MATLAB software genetic program associated with an object function is used to support the following research [9][10]. (Using **path 1-2-4-7-9** as an example in Fig 4)

Fig 4 Network architecture of this study

Table 4 Factor scores of each link

Link	1→2	2→4	4→7	7→9
X	2.6500	1.8838	3.1046	4.3100
Y	6.6000	7.2615	6.8616	7.7000

Table 4 has factor scores for each link. For each link, there are two factor scores X and Y, X is a linear transformation of delay time, and Y is a linear transformation of utilization. There are some of genetic parameter definitions: (1) Objective

Function: $X^2 + Y^2 - XY$; (2) No. of population: 500; (3) Crossover rate: 0.9; (4) Mutation probability: 0.05; (5) Generation maximum evolution: 30

Table 5. A genetic example for alternative (3: $p=0$, $d=1$, $q=1$)

No. of generations	Optimal values of fitness	Average values of fitness	Total values of fitness	No. of population
01	763.29	370.26	185131.33	500
02	763.29	435.40	217702.53	500
03	771.38	478.05	239028.68	500
04	771.38	525.05	262525.93	500
05	771.38	543.91	271954.89	500
06	771.38	565.50	282753.60	500
07	777.84	581.19	290595.83	500
08	778.35	599.84	299920.11	500
::	::	::	::	::
19	778.35	675.41	337705.15	500
20	778.35	670.36	335183.42	500
<u>21</u>	<u>779.63</u>	<u>676.73</u>	<u>338365.87</u>	<u>500</u>
22	779.63	676.58	338293.90	500
23	779.63	682.49	341247.16	500
::	::	::	::	::
29	779.63	686.62	343313.84	500
30	779.63	696.66	348334.08	500

In Table 5, the genetic experiment result of alternative is derived by numeric trend analysis. This analysis shows that if the iterative of generation over 21, the optimal fitness value will be rather smoothing. Therefore, in general, this fitness value of generation 21 can considered as optimal fitness value for alternative.

Table 6 Managed alternatives definition

Alternatives of Strategies	Traffic Status	Attribute Intensity	Order (p, d, q) of ARIMA
1	$A < 75\% \& B+C > 25\%$	L	$p=0, d=0, q=1$
2	$A < 85\% \& B+C > 15\%$	M	$p=0, d=1, q=0$
3	$A < 85\% \& B+C > 15\%$	M	<u>$p=0, d=1, q=1$</u>
4	$A < 85\% \& B+C > 15\%$	M	$p=1, d=0, q=0$
5	$A > 85\% \& B+C < 15\%$	H	$p=1, d=0, q=1$
6	$A > 85\% \& B+C < 15\%$	H	$p=1, d=1, q=0$
7	$A > 85\% \& B+C < 15\%$	H	$p=1, d=1, q=1$

(Here, A denotes the PW value of voice service, B denotes the PW value of data service, C denotes the PW value of voice service, H denotes Higher performance requirement, M denotes Middle performance requirement, L denotes Lower performance requirement)

Table 7 Genetic fitness indexes of alternative experiments

Alternatives of Different ARIMA Order (p, d, q) for Multi-entry Inversed Functions	Sum of Averaged Square Error	PW Values of Multi-entry Inversed Function for (Voice, Data, Fax)	Simulation Data Numbers			
			n			
			500	1000	2000	Averaged Genetic Fitness Indexes (AGFI)
1: p=0,d=0,q=1 A	0.01260	(0.72,0.15,0.13)	532.63	552.39	577.59	<u>554.20</u>
2: p=0,d=1,q=0 B	0.02352	(0.77,0.13,0.10)	750.51	802.43	843.27	<u>798.74</u>
<u>3: p=0,d=1,q=1 B</u>	<u>0.03570</u>	<u>(0.79,0.12,0.09)</u>	<u>779.63</u>	<u>830.30</u>	<u>887.59</u>	<u>832.51</u>
4: p=1,d=0,q=0 B	0.03610	(0.81,0.11,0.08)	1052.87	1127.82	1195.52	<u>1125.40</u>
5: p=1,d=0,q=1 C	0.03792	(0.86,0.09,0.05)	1285.98	1369.56	1464.06	<u>1373.20</u>
6: p=1,d=1,q=0 C	0.03876	(0.88,0.08,0.04)	1375.63	1463.67	1563.19	<u>1467.50</u>
7: p=1,d=1,q=1 C	0.04329	(0.91,0.06,0.03)	1498.65	1595.56	1703.79	<u>1599.33</u>

Finally, in Table 7, the authors also show the possible decisions/actions of Network Service Provider (NSP). Usually, the network users have different performance requirements according to how much transmission fee they want to pay, therefore, NSP defines service strategies with the constrained condition of Sum of Averaged Square Error less than 0.05 as:

- A.** Supporting lower performance requirement of path design (AGFI less than 600)
- B.** Supporting middle performance requirement of path design (AGFI greater than 600 and less than 1200)
- C.** Supporting higher performance requirement of path design (AGFI greater than 1200)

Fig 5. AGFI list of different alternatives

4. CONCLUSION

. In this paper, the authors presented a simulated analysis pattern to support multi-service network path alternatives. In specific, a series of simulated procedures are provided for the readership to follow up by this article. At the same time, this evaluation model can also be used to meet the requirement of the management level's decision making activities in many other industries.

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Received: May 30, 2006