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A Maple program for computing Adomian polynomials

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Abstract

Adomian Decomposition method is a well known device for solving many functional equations such as differential equations, integral equations, integro-differential equations and this method is extended for solving systems of such equations by the firs author and some of his collaborators [4-7]. Adomian Decomposition method yields an analytical solution in terms of a rapidly convergent infinite power series with easily computable terms. During the computations one encounters especial Polynomials so called Adomian polynomials. Many researchers have suggested different methods and algorithm for computing these polynomials. To end these attempts we are presetting a Maple program to compute easily these polynomials for functions with one, two or several variables. This Program also computes Adomian polynomials for a system of functional equations as well.

Keywords: Adomian decomposition method, Adomian Polynomials, Maple Package

1 Introduction

ADM well addressed in [1-3] is a powerful tool for solving many functional equations in the following canonical form,

$$\frac{1}{3}u = f + G(u) \tag{1}$$

Adomian considers the solution as the summation of a series say,

$$u = \sum_{n=0}^{\infty} u_n \tag{2}$$

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And $G(u)$ as a series say

$$G(u) = \sum_{n=0}^{\infty} A_n(u_0, u_1, \ldots, u_n) \tag{3}$$

Adomian introduces these polynomials, for functions with one variable, by the following formula [2]

$$A_n(u_0, u_1, \ldots, u_n) = \frac{1}{n!}[G(\sum_{n=0}^{\infty} u_n \lambda_n]_{\lambda=0} \tag{4}$$

Many researches have been working to derive a simple procedure for computing Adomian polynomials. Wazwaz [9] suggestion is to substitute (2) into (1) and then by manipulation terms, based on algebraic operations , trigonometric identities and Taylor series as appropriate, all terms can be collected such that the subscripts of the components of in each terms is the same. With this step preformed, the calculation of the Adomian polynomials is thus completed.

Javadi has sujected another method [] these two methods are very difficult for functions with more than one variable.

Biazar and some of his collaborators have used "an alternate Algorithm for computing Adomian polynomials..."[10], which is a straightforward method and can be easely extended for functions with multivariable. The short coming of this algorithm is modified in an improvement to an alternate algorithm for computing Adomian polynomials in special cases [11].

For functional equations with several variables the following extension of (1) can be used.

$$A_n(u_{10}, \ldots, u_{1n}, u_{20}, \ldots, u_{2n}, u_{m0}, \ldots, u_{mn}) = \frac{1}{n!}[G(\sum_{n=0}^{\infty} u_{1n}\lambda_n, \ldots, \sum_{n=0}^{\infty} u_{mn}\lambda_n)]_{\lambda=0} \tag{5}$$

Where $G(u_1, \ldots, u_n)$ is a functional depending on$n$ variables, each of them is an unknown function which are considered as the summation of series say, $u_j = \sum_{n=0}^{\infty} u_{jn}\lambda_n \; j = 0, 1, 2, \ldots$

2 Maple program for computing Adomian Polynomials

>restart;
>with(student):
INPUT
>**G[1]:=G(u[1],u[2],...,u[N])**;
G[2]:=G(u[1],u[2],...,u[N]);

```
⋮
G[k]:=G(u[1],u[2],...,u[N]);
>m:=M:
>n:=N:
>k:=K:
PROGRAM
> for i from 1 by 1 while i <= n do
    u[i,lambda]:=sum(u[i,b]*(lambda)∧b,b=0..m):
end do:
>for i from 1 by 1 while i <= n
   do for j from 1 by 1 while j <= n
       do G[i,lambda]:=subs(u[j](t)=u[j,lambda],G[i]):
         G[i]:=G[i,lambda]
   end do:
end do;
>for i from 1 by 1 while i <= n do
   s[i]:=expand(G[i,lambda],lambda):
   ft[i]:= unapply(s[i],lambda):
end do:
>for j from 1 by 1 while j <= k
   do for i from 0 by 1 while i <= m
      do A[j][i]:=((D@@i)(ft[j])(0)/i!):
          print( A[j,i],"=", A[j][i])
   end do;
end do;
```

3 Examples

To illustrate the Program that some examples for computing Adomian polynomials are presented here

Example 1: This examples computes Adomian polynomial for a function of one variable.

$$G_1 = 1 + xu_1(t)^2$$

where m=6 and k=1 and n=1

Adomian polynomials:

$$A_{1,0} = 1 + xu_{1,0}^2$$

$$A_{1,1} = 2xu_{1,0}u_{1,1}$$

$$A_{1,2} = xu_{1,1}^2 + 2xu_{1,0}u_{1,2}$$
$$A_{1,3} = 2xu_{1,0}u_{1,3} + 2xu_{1,1}u_{1,2}$$
$$A_{1,4} = 2xu_{1,0}u_{1,4} + 2xu_{1,1}u_{1,3} + xu_{1,2}^2$$
$$A_{1,5} = 2xu_{1,2}u_{1,3} + 2xu_{1,0}u_{1,5} + 2xu_{1,1}u_{1,4}$$
$$A_{1,6} = 2xu_{1,1}u_{1,5} + 2xu_{1,2}u_{1,4} + 2xu_{1,0}u_{1,6} + xu_{1,3}^2$$

Example 2: Adomian polynomials for a function with two variables are computed in this example.

$$G_1 = \frac{u_2(t)}{2 + u_1(t)}$$

where m=6 and k=1 and n=2
Adomian polynomials:

$$A_{1,0} = \frac{u_{2,0}}{2 + u_{1,0}}$$

$$A_{1,1} = \frac{-u_{2,0}u_{1,0}}{(2 + u_{1,0})^2} + \frac{u_{2,1}}{2 + u_{1,0}}$$

$$A_{1,2} = \frac{u_{2,0}u_{1,1}^2}{(2 + u_{1,0})^3} - \frac{u_{2,0}u_{1,2}}{(2 + u_{1,0})^2} - \frac{u_{2,1}u_{1,1}}{(2 + u_{1,0})^2} + \frac{u_{2,2}}{2 + u_{1,0}}$$

$$A_{1,3} = \frac{-u_{2,2}u_{1,1}}{(2 + u_{1,0})^2} + \frac{u_{2,3}}{(2 + u_{1,0})} - \frac{u_{2,1}u_{1,2}}{(2 + u_{1,0})^2} - \frac{u_{2,0}u_{1,3}}{(2 + u_{1,0})^2} + \frac{u_{2,1}u_{1,1}^2}{(2 + u_{1,0})^3}$$
$$- \frac{u_{2,0}u_{1,1}^3}{(2 + u_{1,0})^4} + \frac{2u_{2,0}u_{1,2}u_{1,1}}{(2 + u_{1,0})^3}$$

$$A_{1,4} = \frac{-u_{2,1}u_{1,3}}{(2 + u_{1,0})^2} - \frac{u_{2,3}u_{1,1}}{(2 + u_{1,0})^2} + \frac{u_{2,4}}{2 + u_{1,0}} - \frac{u_{2,1}u_{1,1}^3}{(2 + u_{1,0})^4} + \frac{u_{2,0}u_{1,1}^4}{(2 + u_{1,0})^5} - \frac{3u_{2,0}u_{1,1}^2u_{1,2}}{(2 + u_{1,0})^4}$$
$$+ \frac{2u_{2,0}u_{1,3}u_{1,1}}{(2 + u_{1,0})^3} + \frac{2u_{2,1}u_{1,1}u_{1,2}}{(2 + u_{1,0})^3} + \frac{u_{2,2}u_{1,1}^2}{(2 + u_{1,0})^3} - \frac{u_{2,0}u_{1,4}}{(2 + u_{1,0})^2} - \frac{u_{2,2}u_{1,2}}{(2 + u_{1,0})^2}$$
$$+ \frac{u_{2,0}u_{1,2}^2}{(2 + u_{1,0})^3}$$

$$A_{1,5} = \frac{-u_{2,2}u_{1,1}^3}{(2 + u_{1,0})^4} - \frac{u_{2,0}u_{1,1}^5}{(2 + u_{1,0})^6} - \frac{u_{2,4}u_{1,1}}{(2 + u_{1,0})^2} + \frac{u_{2,3}u_{1,1}^2}{(2 + u_{1,0})^3} - \frac{u_{2,0}u_{1,5}}{(2 + u_{1,0})^2} + \frac{u_{2,1}u_{1,2}^2}{(2 + u_{1,0})^3}$$
$$- \frac{u_{2,2}u_{1,3}}{(2 + u_{1,0})^2} - \frac{u_{2,1}u_{1,4}}{(2 + u_{1,0})^2} + \frac{4u_{2,0}u_{1,1}^3u_{1,2}}{(2 + u_{1,0})^5} - \frac{3u_{2,0}u_{1,1}u_{1,2}^2}{(2 + u_{1,0})^4} - \frac{3u_{2,1}u_{1,2}u_{1,1}^2}{(2 + u_{1,0})^4} + \frac{2u_{2,0}u_{1,4}u_{1,1}}{(2 + u_{1,0})^3}$$
$$+ \frac{2u_{2,0}u_{1,3}u_{1,2}}{(2 + u_{1,0})^3} + \frac{2u_{2,2}u_{1,1}u_{1,2}}{(2 + u_{1,0})^3} - \frac{3u_{2,0}u_{1,1}^2u_{1,3}}{(2 + u_{1,0})^4} + \frac{2u_{2,1}u_{1,3}u_{1,1}}{(2 + u_{1,0})^3} - \frac{u_{2,3}u_{1,2}}{(2 + u_{1,0})^2} + \frac{u_{2,5}}{(2 + u_{1,0})}$$

$$+\frac{u_{2,1}u_{1,1}^4}{(2+u_{1,0})^5}$$

$$A_{1,6}=\frac{-u_{2,4}u_{1,2}}{(2+u_{1,0})^2}+\frac{u_{2,0}u_{1,3}^2}{(2+u_{1,0})^3}-\frac{u_{2,0}u_{1,2}^3}{(2+u_{1,0})^4}-\frac{u_{2,3}u_{1,3}}{(2+u_{1,0})^2}+\frac{2u_{2,0}u_{1,5}u_{1,1}}{(2+u_{1,0})^3}+\frac{2u_{2,0}u_{1,4}u_{1,2}}{(2+u_{1,0})^3}$$
$$+\frac{6u_{2,0}u_{1,1}^2u_{1,2}^2}{(2+u_{1,0})^5}-\frac{3u_{2,2}u_{1,2}u_{1,1}^2}{(2+u_{1,0})^4}+\frac{u_{2,6}}{(2+u_{1,0})}-\frac{u_{2,1}u_{1,5}}{(2+u_{1,0})^2}-\frac{u_{2,2}u_{1,4}}{(2+u_{1,0})^2}+\frac{u_{2,2}u_{1,2}^2}{(2+u_{1,0})^3}$$
$$-\frac{3u_{2,0}u_{1,4}u_{1,1}^2}{(2+u_{1,0})^4}+\frac{2u_{2,3}u_{1,1}u_{1,2}}{(2+u_{1,0})^3}+\frac{4u_{2,0}u_{1,1}^3u_{1,3}}{(2+u_{1,0})^5}+\frac{2u_{2,1}u_{1,2}u_{1,3}}{(2+u_{1,0})^3}-\frac{5u_{2,0}u_{1,1}^4u_{1,2}}{(2+u_{1,0})^6}+\frac{4u_{2,1}u_{1,1}^3u_{1,2}}{(2+u_{1,0})^5}$$
$$-\frac{3u_{2,1}u_{1,1}u_{1,2}^2}{(2+u_{1,0})^4}+\frac{2u_{2,2}u_{1,3}u_{1,1}}{(2+u_{1,0})^3}+\frac{u_{2,0}u_{1,1}^6}{(2+u_{1,0})^7}-\frac{u_{2,0}u_{1,6}}{(2+u_{1,0})^2}+\frac{2u_{2,1}u_{1,4}u_{1,1}}{(2+u_{1,0})^3}-\frac{3u_{2,1}u_{1,1}^2u_{1,3}}{(2+u_{1,0})^4}$$
$$-\frac{u_{2,5}u_{1,1}}{(2+u_{1,0})^2}-\frac{u_{2,1}u_{1,1}^5}{(2+u_{1,0})^6}+\frac{u_{2,2}u_{1,1}^4}{(2+u_{1,0})^5}+\frac{u_{2,4}u_{1,1}^2}{(2+u_{1,0})^3}$$
$$-\frac{u_{2,3}u_{1,1}^3}{(2+u_{1,0})^4}-\frac{6u_{2,0}u_{1,3}u_{1,2}u_{1,1}}{(2+u_{1,0})^4}$$

Example 3: A function with four variables and computing it's polynomials is considered here.

$$G_1=u_1(t)u_2(t)+u_3(t)^2+u_4(t)u_1(t)$$

where $m=6$ and $k=1$ and $n=4$
Adomian polynomials:

$$A_{1,0}=u_{1,0}u_{2,0}+u_{3,0}^2+u_{4,0}u_{1,0}$$
$$A_{1,1}=u_{1,0}u_{2,1}+u_{4,0}u_{1,1}+u_{1,1}u_{2,0}+u_{4,1}u_{1,0}+2u_{3,0}u_{3,1}$$
$$A_{1,2}=u_{3,1}^2+u_{1,2}u_{2,0}+u_{4,0}u_{1,2}+u_{1,0}u_{2,2}+u_{4,1}u_{1,1}+u_{4,2}u_{1,0}+u_{1,1}u_{2,1}+2u_{3,0}u_{3,2}$$
$$A_{1,3}=u_{1,0}u_{2,3}+u_{1,2}u_{2,1}+u_{4,1}u_{1,2}+u_{1,1}u_{2,2}+u_{1,3}u_{2,0}+u_{4,2}u_{1,1}+2u_{3,1}u_{3,2}+u_{4,3}u_{1,0}$$
$$+2u_{3,0}u_{3,3}+u_{4,0}u_{1,3}$$
$$A_{1,4}=u_{3,2}+u_{4,0}u_{1,4}+2u_{3,1}u_{3,3}+u_{4,1}u_{1,3}+u_{1,2}u_{2,2}+u_{1,4}u_{2,0}+u_{1,0}u_{2,4}+u_{4,2}u_{1,2}$$
$$+u_{1,3}u_{2,1}+u_{4,3}u_{1,1}+u_{1,1}u_{2,3}+u_{4,4}u_{1,0}+2u_{3,0}u_{3,4}A_{1,5}=u_{4,5}u_{1,0}+u_{1,0}u_{2,5}+u_{4,1}u_{1,4}$$
$$+2u_{3,1}u_{3,4}+u_{1,2}u_{2,3}+u_{1,5}u_{2,0}+u_{4,0}u_{1,5}+2u_{3,0}u_{3,5}+u_{4,2}u_{1,3}+u_{4,3}u_{1,2}+2u_{3,2}u_{3,3}$$
$$+u_{1,1}u_{2,4}+u_{1,3}u_{2,2}+u_{1,4}u_{2,1}+u_{4,4}u_{1,1}$$
$$A_{1,6}=u_{4,3}u_{1,3}+2u_{3,0}u_{3,6}+u_{4,5}u_{1,1}+2u_{3,2}u_{3,4}+u_{4,2}u_{1,4}+u_{4,0}u_{1,6}+u_{1,3}u_{2,3}+u_{4,4}u_{1,2}$$
$$+u_{4,1}u_{1,5}+2u_{3,1}u_{3,5}+u_{1,2}u_{2,4}+u_{4,6}u_{1,0}$$
$$+u_{1,6}u_{2,0}+u_{1,5}u_{2,1}+u_{1,1}u_{2,5}+u_{1,0}u_{2,6}+u_{1,4}u_{2,2}+u_{3,3}^2$$

4 Discussion

Simplicity and efficiency of the algorithm presented in this article, are illustrated briefly in the examples. Data's of the algorithm,n the number of unknowns,m the number of Adomian polynomials and k , the number of functionals are three advantages of the Algorithm which enable us to apply this algorithm to any functional , or any system of functional equations. Authors are working on programs for solving systems of functional equations, such as systems of ordinary differential equations, systems of partial differential equations, systems of Volterra integral equations of the first and second kind, and so on.

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