A buyer - seller game model for selection and negotiation of purchasing bids on interval data

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Abstract

Selection and negotiation of purchasing bids is a complex decision making process that requires consideration of a variety of vendor attributes such as price, delivery performance, and quality. Several decision models have been utilized for vendor evaluation and selection. Talluri [EUR. J. Operat. Res. 143(1) (2002) 171] and Zhu [EUR. J. Operat. Res. 154(2004) 150] propose a buyer - seller game model that evaluate the efficiency of alternative bids with respect to the ideal target set by the buyer. We prove some theorem in the current paper about this model. The bids cannot be easily evaluated and selected, while inputs and outputs each vary in interval. In this paper, presenting a new idea for evaluating the bids with interval data, an interval will be defined for the efficiency score of each bid. And finally, a method for selecting bids by the obtained efficiency interval is presented. And, the new technique will be applied to a set of real data.

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1 Introduction

As pointed out by Wise and Morrison (2000), one of the major flaws in the current business - to - business (B2B) model is that it focuses on price - driven transactions between buyers and sellers. In fact, a number of efficiency - based on negotiation models have been developed to deal with multiple attributes - inputs and outputs. For example, data envelopment analysis (DEA) is used

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by Weber and Desai (1996) and Weber et al. (1998) to develop models for vendor evaluation and negotiation. Vijayan (2000) discusses the importance of considering multiple vendor related attributes. It is suggested by him that companies are developing software that is considering several factors beyond price for effective B2B transactions.

Based on a survey of 170 purchasing managers, Dickson (1966) suggested that cost, quality, and delivery performance are the three most important criteria in vendor evaluation.

Willis et al. (1993) proposed a classification of vendor performance evaluation models that included categorical, weighted points, and cost ratio approaches.

Weber and Desai (1996) and Weber et al. (1998) identified two articles about decision models for vendor evaluation and negotiation bids.

In this paper, we review the models of Talluri (2002) and Zhu (2004) and prove some theorem.

In the recent years, models are observed to have inputs and outputs as interval. In this paper, an interval will be defined for each bid's efficiency score, and a method for selecting bids by the obtained efficiency's interval is presented.

This paper consists of the following sections: In section 2, the models in Talluri (2002) and Zhu (2004) are discussed and some theorems are proved. In section 3, this model with interval data is presented. One example with real interval data is provided. Finally, the conclusion is given.

2 Models introduce

In this section, we review the models of Talluri (2002) and Zhu (2004). Since the evaluations in these models are conducted from a buyer's perspective, inputs are defined as the resources spent and outputs as the benefits derived by the buyer. The scenario include single input - single output, single input - multiple output and multiple input - multiple output cases.

Case1 (single input - single output) Expression (1) depicts the model.

$$c_k = \frac{\frac{y_k}{x_k}}{\max\{\frac{y_j}{x_j}: j=1, \dots, n\}} \quad (1)$$

Then, when we have single input - single output, there is no require of the weights.

Case2 (single input - multiple output with simultaneous consideration

of n vendor bids) The model of Zhu (2004) is shown in (2).

$$\begin{aligned}
 \max \quad & \sum_{j=1}^n \left(\frac{\sum_{r=1}^s a_r y_{rj}}{b_1 x_j} \right) \\
 \text{s.t.} \quad & \frac{\sum_{r=1}^s a_r y_{rI}}{b_1 x_I} = 1, \\
 & \frac{\sum_{r=1}^s a_r y_{rj}}{b_1 x_j} \leq 1, \quad j = 1, \dots, n, \\
 & a_r, b_1 \geq 0 \quad r = 1, \dots, s.
 \end{aligned} \tag{2}$$

Where n represents the number of bids. a_r, b_1 represent unknown output and input weights, respectively. Each bid j has s outputs $y_{rj} (r = 1, \dots, s)$, $y_{rI} (r = 1, \dots, s)$ and x_I represent the best value for the r -th output and the best value for the input.

Because there is one input, so we can consider $b_1 = 1$. Since y_{rj} and x_j are scalar, this form is linear. Model (2) identifies the vendor who has the maximum efficiency with respect to the ideal target. If we regard this model with minimization form [Talluri (2002)], the buyer can determine the largest minimum efficiency, when the efficiency of target is assumed to be one. Let $\underline{h}_p^*$ and $\overline{h}_p^*$ be the optimal values to this models, and h_p be the true efficiency of vendor p with respect to the fact that the efficiencies of the targets are sets equal to one, then $\underline{h}_p^* \leq h_p \leq \overline{h}_p^*$. When both models are calculated, the buyer can have an efficiency range which the true efficiency lying within. This helps the vendor better in evaluating and selecting the vendors.

Case3 (single input - multiple output with individual consideration of n vendor bids) Expression (3) depicts the model.

$$\begin{aligned}
 \max \quad & \frac{\sum_{r=1}^s a_r y_{rp}}{x_p} \\
 \text{s.t.} \quad & \frac{\sum_{r=1}^s a_r y_{rI}}{x_I} = 1, \\
 & \frac{\sum_{r=1}^s a_r y_{rj}}{x_j} \leq 1, \quad j = 1, \dots, n, \\
 & a_r \geq 0 \quad r = 1, \dots, s.
 \end{aligned} \tag{3}$$

Where p represents the vendor being evaluated. This model is a linear programming.

Case 4 (multiple input - multiple output with simultaneous consideration

of n vendor bids) Expression (4) depicts the model.

$$\begin{aligned}
 \max \quad & \sum_{j=1}^n \left(\frac{\sum_{r=1}^s a_r y_{rj}}{\sum_{i=1}^m b_i x_{ij}} \right) \\
 \text{s.t.} \quad & \frac{\sum_{r=1}^s a_r y_{rI}}{\sum_{i=1}^m b_i x_{iI}} = 1, \\
 & \frac{\sum_{r=1}^s a_r y_{rj}}{\sum_{i=1}^m b_i x_{ij}} \leq 1, \quad j = 1, \dots, n, \\
 & a_r, b_i \geq 0 \quad r = 1, \dots, s, i = 1, \dots, m.
 \end{aligned} \tag{4}$$

Where m represents the number of bid's inputs and b_i represent unknown input weights. This model is non - linear programming. Talluri (2002) states that "the objective function can be replaced by $\frac{\sum_{r=1}^s a_r (\sum_{j=1}^n \frac{y_{rj}}{n})}{\sum_{i=1}^m b_i (\sum_{j=1}^n \frac{x_{ij}}{n})}$ ", but the objective function isn't equal to this expression, and this model isn't converted to a linear program.

Case 5 (multiple input - multiple output with individual consideration of n vendor bids)

Expression (5) depicts the model.

$$\begin{aligned}
 \max \quad & \frac{\sum_{r=1}^s a_r y_{rp}}{\sum_{i=1}^m b_i x_{ip}} \\
 \text{s.t.} \quad & \frac{\sum_{r=1}^s a_r y_{rI}}{\sum_{i=1}^m b_i x_{iI}} = 1, \\
 & \frac{\sum_{r=1}^s a_r y_{rj}}{\sum_{i=1}^m b_i x_{ij}} \leq 1, \quad j = 1, \dots, n, \\
 & a_r, b_i \geq 0 \quad r = 1, \dots, s, i = 1, \dots, m.
 \end{aligned} \tag{5}$$

This model by replacing the objective function's denominator with $\frac{1}{t}$ is transformed to a linear form.

Theorem 2.1 *Optimal value of objective function in (4) is less than or equal to objective function's optimal value in (5).*

Proof: Assume $(\bar{a}, \bar{b})$ to be optimal solution for objective function in (4), thus: $e_p = \frac{\bar{a} y_p}{\bar{b} x_p}$. obviously $(\bar{a}, \bar{b})$ is a feasible solution for (5). Then the optimal solution of (4) is less than or equal to optimal solution of (5).

2.1 Integer programming model for bid selection

The vendor ratings are evaluated in a 0 - 1 integer programming model, shown in expression (5).

$$\begin{aligned}
 \min \quad & z_0 = \sum_{j=1}^n z_j \\
 \text{s.t.} \quad & \sum_{j=1}^n \theta_j z_j - \theta_{avg} \sum_{j=1}^n z_j \geq 0, \\
 & \sum_{j=1}^n q_j = D, \\
 & q_j - c_{jmax} z_j \leq 0, \quad j = 1, \dots, n, \\
 & q_j - o_{jmin} z_j \geq 0, \quad j = 1, \dots, n, \\
 & q_j \geq 0 \quad j = 1, \dots, n, \\
 & z_j \in \{0, 1\} \quad j = 1, \dots, n.
 \end{aligned} \tag{6}$$

Where z_j is the binary variable that represents the selection status of vendor bid j , θ_j is the efficiency of vendor bid j , θ_{avg} is average efficiency target, q_j is the amount ordered from vendor j , D is the buyer's demand requirement, c_{jmax} is the capacity of vendor j , o_{jmin} is the minimum order quantity requirement of vendor j .

Theorem 2.2 $z_0 > 0$ for all feasible solution in (6).

Proof: Since $z_j \in \{0, 1\}$ for every j , thus $z_0 > 0$ or $z_0 = 0$. Let $z_0 = 0$, then $z_j = 0$ for every j . From $q_j - o_{jmin} z_j \geq 0$ we have $q_j \geq 0$, and from $q_j - c_{jmax} z_j \leq 0$, we have $q_j \leq 0$, therefore $q_j = 0$, that contradicts $\sum_{j=1}^n q_j = D$, because D is become zero, therefore $z_0 > 0$.

Theorem 2.3 Let in bids we have at least one nonideal vendor, then in (5) the vendor that has efficiency score 1, in (6) related z_j is 1.

Proof: Since there exist DMU_t subject to $\theta_t < 1$, so $\sum_{j=1, j \neq p}^n (1 - \theta_j) > 0$, therefore

$n - 1 = \sum_{j=1, j \neq p}^n 1 > \sum_{j=1, j \neq p}^n \theta_j$, we have
 $n - 1 > (n - n + 1) \sum_{j=1, j \neq p}^n \theta_j = n \sum_{j=1, j \neq p}^n \theta_j - (n - 1) \sum_{j=1, j \neq p}^n \theta_j$, we conclude
 $\frac{\sum_{j=1, j \neq p}^n \theta_j}{n-1} < \frac{\sum_{j=1, j \neq p}^n \theta_j + 1}{n} = \frac{\sum_{j=1, j \neq p}^n \theta_j + \theta_p}{n} = \frac{\sum_{j=1}^n \theta_j}{n} = \theta_{avg}$. Now assume to the contrary that $\theta_p = 1$ and $z_p = 0$, then:

$$\begin{aligned}
 0 & \leq \sum_{j=1}^n \theta_j z_j - \theta_{avg} \sum_{j=1}^n z_j = \sum_{j=1, j \neq p}^n \theta_j z_j - \theta_{avg} \sum_{j=1, j \neq p}^n z_j \\
 & \leq \sum_{j=1, j \neq p}^n \theta_j - (n - 1) \theta_{avg} = (n - 1) \left(\frac{\sum_{j=1, j \neq p}^n \theta_j}{n-1} - \theta_{avg} \right) < 0.
 \end{aligned}$$

This contradictions complete the proof, and if $\theta_p = 1$ then certainly $z_p = 1$.

3 Buyer - seller game model with interval data

Let input and output values of any bid be located in a certain interval, that is:

$$\forall i, \forall j \quad x_{ij} \in [x_{ij}^l, x_{ij}^u] \quad \text{and} \quad \forall r, \forall j \quad y_{rj} \in [y_{rj}^l, y_{rj}^u]$$

Therefore relative efficiency of DMU_p is also located in an interval. Now we evaluate the identified models for interval data.

3.1 Case (3) with interval data

The following pair of LP models has been developed to generate the upper and lower bounds of interval efficiency for each vendor.

$$\begin{aligned} e_p^l = \max \quad & \sum_{r=1}^s a_r \frac{y_{rp}^l}{x_p^l} & e_p^u = \max \quad & \sum_{r=1}^s a_r \frac{y_{rp}^u}{x_p^u} \\ \text{s.t.} \quad & \sum_{r=1}^s a_r \frac{y_{rI}^u}{x_I^u} = 1, & \text{s.t.} \quad & \sum_{r=1}^s a_r \frac{y_{rI}^l}{x_I^l} = 1, \\ & \sum_{r=1}^s a_r \frac{y_{rj}^u}{x_j^u} \leq 1, \quad j \neq p, & & \sum_{r=1}^s a_r \frac{y_{rj}^l}{x_j^l} \leq 1, \quad j \neq p, \\ & \sum_{r=1}^s a_r \frac{y_{rp}^l}{x_p^l} \leq 1, & & \sum_{r=1}^s a_r \frac{y_{rp}^u}{x_p^u} \leq 1, \\ & a_r \geq 0 \quad r = 1, \dots, s. & & a_r \geq 0 \quad r = 1, \dots, s. \end{aligned} \quad (7) \quad (8)$$

The dual of models (3) and (7) and (8) are as follows, respectively:

$$\begin{aligned} e_p = \min \quad & \theta + \sum_{j=1, j \neq p}^n \lambda_j + \lambda_p \\ \text{s.t.} \quad & \sum_{j=1, j \neq p}^n \lambda_j \left(\frac{y_{rj}}{x_j} \right) + \lambda_p \left(\frac{y_{rp}}{x_p} \right) + \theta \left(\frac{y_{rI}}{x_I} \right) \geq \frac{y_{rp}}{x_p}, \quad r = 1, \dots, s, \\ & \lambda_j \geq 0 \quad j = 1, \dots, n, \\ & \theta \quad \text{free.} \end{aligned} \quad (9)$$

$$\begin{aligned} e_p^l = \min \quad & \theta' + \sum_{j=1, j \neq p}^n \lambda'_j + \lambda'_p \\ \text{s.t.} \quad & \sum_{j=1, j \neq p}^n \lambda'_j \left(\frac{y_{rj}^u}{x_j^u} \right) + \lambda'_p \left(\frac{y_{rp}^l}{x_p^l} \right) + \theta' \left(\frac{y_{rI}^u}{x_I^u} \right) \geq \frac{y_{rp}^l}{x_p^l}, \quad r = 1, \dots, s, \\ & \lambda'_j \geq 0 \quad j = 1, \dots, n, \\ & \theta' \quad \text{free.} \end{aligned} \quad (10)$$

$$\begin{aligned} e_p^u = \min \quad & \theta'' + \sum_{j=1, j \neq p}^n \lambda''_j + \lambda''_p \\ \text{s.t.} \quad & \sum_{j=1, j \neq p}^n \lambda''_j \left(\frac{y_{rj}^l}{x_j^l} \right) + \lambda''_p \left(\frac{y_{rp}^u}{x_p^u} \right) + \theta'' \left(\frac{y_{rI}^l}{x_I^l} \right) \geq \frac{y_{rp}^u}{x_p^u}, \quad r = 1, \dots, s, \\ & \lambda''_j \geq 0 \quad j = 1, \dots, n, \\ & \theta'' \quad \text{free.} \end{aligned} \quad (11)$$

Theorem 3.1 Let $(\theta, \lambda_1, \dots, \lambda_n)$ be an optimal solution to model (9), then $\theta \geq 0$.

Proof: In model (3) the ideal constraint can be expressed as $\frac{\sum_{r=1}^s a_r y_{rI}}{x_I} \leq 1$, because in optimal solution we always have: $\frac{\sum_{r=1}^s a_r y_{rI}}{x_I} = 1$. Since, let a^* be an optimal solution, but we have $\sum_{r=1}^s \frac{a_r^* y_{rI}}{x_I} < 1$. I know there exists t such that $\sum_{r=1}^s \frac{a_r^* y_{rt}}{x_t} = 1$, if $t = i$ the theorem is proved, other wise since $x_I \leq x_t$ and $y_{rI} \geq y_{rt}$, for all r then: $x_I \leq x_t$ and $a_r^* y_{rI} \geq a_r^* y_{rt}$, for all r . Hence $1 > \sum_{r=1}^s \frac{a_r^* y_{rI}}{x_I} \geq \sum_{r=1}^s \frac{a_r^* y_{rt}}{x_t} = 1$, where it is a contradiction. Thus in optimal solution, we can write the ideal constraint with equality form. Therefore in optimal solution θ is non-negative.

Theorem 3.2 Let (θ^*, λ^*) be an optimal solution to model (9), then $(1 - \lambda_p^*) \geq 0$.

Proof: Consider the following model

$$\begin{aligned} \min \quad & \sum_{j=1}^n \lambda_j \\ \text{s.t.} \quad & \sum_{j=1}^n \lambda_j \left(\frac{y_{rj}}{x_j} \right) \geq \frac{y_{rp}}{x_p}, \quad r = 1, \dots, s, \\ & \lambda \geq 0 \end{aligned}$$

$\lambda = e_p$, is a feasible solution. Then the optimal objective function's value is always less than or equal to unity, thus $\lambda_j^* \leq 1$, for every j . Since if there exists j such that $\lambda_j^* > 1$, from $\sum_{j=1}^n \lambda_j^* \leq 1$, it follows that there exist at least one t such that $\lambda_t < 0$, this contradicts $\lambda \geq 0$. So $\lambda_j \leq 1$ for every j , especially for $j = p$. Therefore $(1 - \lambda_p^*) \geq 0$.

Theorem 3.3 In models (3) and (7) and (8) we always have: $e_p^l \leq e_p \leq e_p^u$.

Proof: Suppose that (θ, λ) is the optimal solution to model (9), with respect to

$y_{rj}^u \geq y_{rj} \geq y_{rj}^l$ for every r and $x_j^l \leq x_j \leq x_j^u$ and by theorem (4) and (5) we have:

$$\sum_{j=1, j \neq p}^n \lambda_j \left(\frac{y_{rj}^u}{x_j^l} \right) + \theta \left(\frac{y_{rI}^u}{x_I^l} \right) \geq \sum_{j=1, j \neq p}^n \lambda_j \left(\frac{y_{rj}}{x_j} \right) + \theta \left(\frac{y_{rI}}{x_I} \right) \geq (1 - \lambda_p) \frac{y_{rp}}{x_p} \geq (1 - \lambda_p) \frac{y_{rp}^l}{x_p^u}$$

Which means (θ, λ) is also a feasible solution to model (10), so we get $e_p^l \leq e_p$.

Now, if (θ'', λ'') is the optimal solution to model (11) similarly, is proved that $\theta'' \geq 0$ and $(1 - \lambda_p'') \geq 0$. Then the following inequalities hold:

$$\sum_{j=1, j \neq p}^n \lambda_j'' \left(\frac{y_{rj}}{x_j} \right) + \theta'' \left(\frac{y_{rI}}{x_I} \right) \geq \sum_{j=1, j \neq p}^n \lambda_j'' \left(\frac{y_{rj}^l}{x_j^u} \right) + \theta'' \left(\frac{y_{rI}^l}{x_I^u} \right) \geq (1 - \lambda_p'') \frac{y_{rp}^u}{x_p^l} \geq (1 - \lambda_p'') \frac{y_{rp}}{x_p}$$

It is obvious that (θ'', λ'') is a feasible solution of model (9). Thus, we have the inequality relation: $e_p \leq e_p^u$. Therefore $e_p^l \leq e_p \leq e_p^u$ and complete the proof.

3.2 Case (5) with interval data

The upper and lower bounds of the relative efficiency of DMU_p in (5) are obtained by solving the following problems:

$$\begin{aligned}
 e_p^l = \max \quad & \frac{\sum_{r=1}^s a_r y_{rp}^l}{\sum_{i=1}^m b_i x_{ip}^l} & e_p^u = \max \quad & \frac{\sum_{r=1}^s a_r y_{rp}^u}{\sum_{i=1}^m b_i x_{ip}^u} \\
 \text{s.t.} \quad & \frac{\sum_{r=1}^s a_r y_{rI}^u}{\sum_{i=1}^m b_i x_{iI}^l} = 1, & \text{s.t.} \quad & \frac{\sum_{r=1}^s a_r y_{rI}^l}{\sum_{i=1}^m b_i x_{iI}^u} = 1, \\
 & \frac{\sum_{r=1}^s a_r y_{rj}^u}{\sum_{i=1}^m b_i x_{ij}^l} \leq 1, \quad j \neq p, & & \frac{\sum_{r=1}^s a_r y_{rj}^l}{\sum_{i=1}^m b_i x_{ij}^u} \leq 1, \quad j \neq p, \\
 & \frac{\sum_{r=1}^s a_r y_{rp}^l}{\sum_{i=1}^m b_i x_{ip}^u} \leq 1, & & \frac{\sum_{r=1}^s a_r y_{rp}^u}{\sum_{i=1}^m b_i x_{ip}^l} \leq 1, \\
 & a_r \geq 0 \quad r = 1, \dots, s, & & a_r \geq 0 \quad r = 1, \dots, s, \\
 & b_i \geq 0 \quad i = 1, \dots, m. & & b_i \geq 0 \quad i = 1, \dots, m.
 \end{aligned} \tag{12} \tag{13}$$

Consider dual of models:

$$\begin{aligned}
 e_p = \min \quad & \theta \\
 \text{s.t.} \quad & \mu y_{rI} + \sum_{j=1, j \neq p}^n \lambda_j y_{rj} + \lambda_p y_{rp} \geq y_{rp}, \quad r = 1, \dots, s, \\
 & \theta x_{ip} - \mu x_{iI} - \sum_{j=1, j \neq p}^n \lambda_j x_{ij} - \lambda_p x_{ip} \geq 0, \quad i = 1, \dots, m, \\
 & \lambda_j \geq 0 \quad j = 1, \dots, n, \\
 & \theta, \mu \quad \text{free.}
 \end{aligned} \tag{14}$$

$$\begin{aligned}
 e_p^l = \min \quad & \theta' \\
 \text{s.t.} \quad & \mu' y_{rI}^u + \sum_{j=1, j \neq p}^n \lambda'_j y_{rj}^u + \lambda'_p y_{rp}^l \geq y_{rp}^l, \quad r = 1, \dots, s, \\
 & \theta' x_{ip}^u - \mu' x_{iI}^l - \sum_{j=1, j \neq p}^n \lambda'_j x_{ij}^l - \lambda'_p x_{ip}^u \geq 0, \quad i = 1, \dots, m, \\
 & \lambda'_j \geq 0 \quad j = 1, \dots, n, \\
 & \theta', \mu' \quad \text{free.}
 \end{aligned} \tag{15}$$

$$\begin{aligned}
 e_p^u = \min \quad & \theta'' \\
 \text{s.t.} \quad & \mu'' y_{rI}^l + \sum_{j=1, j \neq p}^n \lambda''_j y_{rj}^l + \lambda''_p y_{rp}^u \geq y_{rp}^u, \quad r = 1, \dots, s, \\
 & \theta'' x_{ip}^l - \mu'' x_{iI}^u - \sum_{j=1, j \neq p}^n \lambda''_j x_{ij}^u - \lambda''_p x_{ip}^l \geq 0, \quad i = 1, \dots, m, \\
 & \lambda''_j \geq 0 \quad j = 1, \dots, n, \\
 & \theta'', \mu'' \quad \text{free.}
 \end{aligned} \tag{16}$$

Theorem 3.4 If (θ, λ, μ) be an optimal solution of model (14), then $\mu \geq 0$.

Proof: Consider linear form of (5), since in optimal solution we have:
 $\sum_{r=1}^s a_r^* y_{rI} - \sum_{i=1}^m b_i^* x_{iI} = 0$, the linear form can be written as follows:

$$\begin{aligned} e_p &= \max && \sum_{r=1}^s a_r y_{rp} \\ \text{s.t.} &&& \sum_{r=1}^s a_r y_{rI} - \sum_{i=1}^m b_i x_{iI} \leq 0, \\ &&& \sum_{r=1}^s a_r y_{rj} - \sum_{i=1}^m b_i x_{ij} \leq 0, && j = 1, \dots, n, \\ &&& \sum_{i=1}^m b_i x_{ip} = 1, \\ &&& a_r, b_i \geq 0 && r = 1, \dots, s, i = 1, \dots, m. \end{aligned} \quad (17)$$

Suppose to the contrary that (a^*, b^*) be an optimal solution such that satisfies $a^* y_I - b^* x_I < 0$, but we know that there exist t such that $a^* y_t - b^* x_t = 0$, if $t = I$ the proof is completed. Other wise since $y_I \geq y_t$ and $x_I \leq x_t$ then $b^* x_I \leq b^* x_t$ and $a^* y_I \geq a^* y_t$. It follows that $0 > a^* y_I - b^* x_I \geq a^* y_t - b^* x_t$ where this is a contradiction. Hence ideal constraint's form in optimal solution is equality, then $\mu \geq 0$.

Theorem 3.5 *If (θ, λ, μ) be an optimal solution of (14), then $(\theta - \lambda_p) \geq 0$.*

Proof: Since $X_p \neq 0$ there exist t such that $x_{tp} > 0$. Now consider

$$\mu x_{tI} + \sum_{j=1, j \neq p}^n \lambda_j x_{tj} \leq (\theta - \lambda_p) x_{tp}$$

Suppose to the contrary that $(\theta - \lambda_p) < 0$, then $\mu x_{tI} + \sum_{j=1, j \neq p}^n \lambda_j x_{tj} < 0$. This means that we must have $\mu x_{tI} < 0$ or there exist l such that $\lambda_l x_{tl} < 0$, that with respect to $\mu \geq 0$ and $X \geq 0$ and $\lambda \geq 0$ this is contradiction. Therefore $(\theta - \lambda_p) \geq 0$.

Theorem 3.6 *In models (5) and (12) and (13) we always have: $e_p^l \leq e_p \leq e_p^u$.*

Proof: Assume that (θ, λ, μ) be an optimal solution of model (14), it is proved as same as the theorem (5) that $0 \leq \lambda_p \leq 1$, by theorem (7) and (8) we have:

$$\begin{aligned} \mu y_{rI}^u + \sum_{j=1, j \neq p}^n \lambda_j y_{rj}^u &\geq \mu y_{rI} + \sum_{j=1, j \neq p}^n \lambda_j y_{rj} \geq (1 - \lambda_p) y_{rp} \geq (1 - \lambda_p) y_{rp}^l \\ \mu x_{iI}^l + \sum_{j=1, j \neq p}^n \lambda_j x_{ij}^l &\leq \mu x_{iI} + \sum_{j=1, j \neq p}^n \lambda_j x_{ij} \leq (\theta - \lambda_p) x_{ip} \leq (\theta - \lambda_p) x_{ip}^u \end{aligned}$$

Then (θ, λ, μ) is a feasible solution to model (15) and since e_p^l is optimal solution we have $e_p^l \leq e_p$.

Now let $(\theta'', \lambda'', \mu'')$ be an optimal solution of model (16), similarly is proved that $\mu'' \geq 0$ and $(1 - \lambda_p'') \geq 0$ and $(\theta'' - \lambda_p'') \geq 0$. Then:

$$\begin{aligned} \mu'' y_{rI} + \sum_{j=1, j \neq p}^n \lambda_j'' y_{rj} &\geq \mu'' y_{rI}^l + \sum_{j=1, j \neq p}^n \lambda_j'' y_{rj}^l \geq (1 - \lambda_p'') y_{rp}^u \geq (1 - \lambda_p'') y_{rp} \\ \mu'' x_{iI} + \sum_{j=1, j \neq p}^n \lambda_j'' x_{ij} &\leq \mu'' x_{iI}^u + \sum_{j=1, j \neq p}^n \lambda_j'' x_{ij}^u \leq (\theta'' - \lambda_p'') x_{ip}^l \leq (\theta'' - \lambda_p'') x_{ip} \end{aligned}$$

Thus $(\theta'', \lambda'', \mu'')$ is a feasible solution of model (14), hence $e_p \leq e_p^u$. Therefore

we have $e_p^l \leq e_p \leq e_p^u$, and the proof is completed.

3.3 Integer programming model for bid selection with interval data

The following pair of IP models have been constructed to select an optimal set of bids for models (7) and (8) or (12) and (13).

$$\begin{array}{ll}
 \min & z_0 = \sum_{j=1}^n z_j \\
 \text{s.t.} & \sum_{j=1}^n e_j^l z_j - e_{pavg}^l \sum_{j=1}^n z_j \geq 0, \\
 & \sum_{j=1}^n q_j = D, \\
 & q_j - c_{jmax} z_j \leq 0, \quad j = 1, \dots, n, \\
 & q_j - o_{jmin} z_j \geq 0, \quad j = 1, \dots, n, \\
 & q_j \geq 0 \quad j = 1, \dots, n, \\
 & z_j \in \{0, 1\} \quad j = 1, \dots, n.
 \end{array} \quad (19)$$

$$\begin{array}{ll}
 \min & z_0 = \sum_{j=1}^n z_j \\
 \text{s.t.} & \sum_{j=1}^n e_j^u z_j - e_{pavg}^u \sum_{j=1}^n z_j \geq 0, \\
 & \sum_{j=1}^n q_j = D, \\
 & q_j - c_{jmax} z_j \leq 0, \quad j = 1, \dots, n, \\
 & q_j - o_{jmin} z_j \geq 0, \quad j = 1, \dots, n, \\
 & q_j \geq 0 \quad j = 1, \dots, n, \\
 & z_j \in \{0, 1\} \quad j = 1, \dots, n.
 \end{array} \quad (20)$$

With comparison of the solutions, if z_j^* in this models have a constant value, in final optimal solution z_j^* 's value is constant. But if for some vendors z_j^* 's value is variable, in final solution we defined z_j^* as $z_j^* \in \{0, 1\}$, where in this case buyer should select the vendor, itself and model isn't definitely determined.

4 Application example

We now apply this approach to a byer of food's products (for one goods) in Iran. There are 4 vendors in this district. Product price, quality and service time are considered to be the three most important factors in evaluating vendors. Price is utilized as the input, and quality and service time are considered as outputs.

In table 1 the interval input and interval outputs for these DMU_s are given.

Table 1: The set of interval input and outputs

DMU_j	x_j^l	x_j^u	y_{1j}^l	y_{1j}^u	y_{2j}^l	y_{2j}^u	MOR	C
1	19250	19250	1	1	0.7	1	500	4000
2	18000	18000	2	2	0.2	0.8	500	4000
3	22000	22000	1	1	0.7	1	500	4000
4	14000	14450	7	7	0	0.3	500	4000

MOR: minimum order requirement, C: capacity repressed.

The result of case (3) analysis are shown in table 2.

Table 2: Vendors efficiencies

DMU_j	e_p^l	e_p^u
1	0.51	1.00
2	0.22	0.92
3	0.45	0.94
4	0.97	1.00

The corresponding input and output weights are shown in table 3.

Table 3: Output weights in lower and upper bound of efficiencies

DMU_j	a_1^l	a_1^u	a_2^l	a_2^u
1	0.000000	154.761905	1.40000E+4	1.90952E+4
2	2.00000E+3	0.000000	0.000000	2.06429E+4
3	0.000000	0.000000	1.40000E+4	2.06429E+4
4	2.00000E+3	1.95179E+3	0.000000	1.12500E+3

It is evident from these result that vendor 4 is the best performer.

The demand for the buyer is 1500 kg. We utilize the efficiency scores obtained in a 0 - 1 integer programming models shown in expressions (19) and (20). The results are shown in Table 4 and 5.

Table 4: Vendor selection and order quantities for model 7

vendor	Selection	Order quantity
1	0	0
2	0	0
3	0	0
4	1	1500

Table 5: Vendor selection and order quantities for model 8

vendor	Selection	Order quantity
1	1	1500
2	0	0
3	0	0
4	0	0

Then $Z = (0, 0, 0, 1)$ or $Z = (1, 0, 0, 0)$ and $Q = (0, 0, 0, 1500)$ or $Q = (1500, 0, 0, 0)$. In the present case scenario, one vendor needs to be selected because of the demand requirements.

5 Conclusions

In this paper, we have reviewed models of Talluri (2002) and Zhu (2004). Some theorems about this models are proved. We have developed a new pair of models for dealing with interval data in every case. By solving these models an interval will be defined for the efficiency score. The model evaluations are integrated into a new pair of 0 - 1 integer programming formulation in determining the optimal set of vendors to be selected in meeting the demand requirements of the buyer without violating the minimum order necessities of the vendors. If case (2) and case (4) have interval input and output, for finding each vendor's interval efficiency can solve one problem that optimizes the sum of all of the efficiencies in their lower and upper bounds. One can prevent a big difference between the weights by applying weight control on models made in this paper.

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