On common fixed point theorems in fuzzy metric spaces

Seong Hoon Cho

Department of Mathematics,
Hanseu University,
Seosan, Chungnam, 356-820, South Korea

Ji Hong Jung

Department of Mathematics,
Hanseu University,
Seosan, Chungnam, 356-820, South Korea

Abstract

In this paper we introduce the notion of ϵ -chanable fuzzy metric space. We give some conditions of which four self mappings of ϵ -chanable fuzzy metric space have a unique common fixed point. Also we characterize the conditions for two self mappings of ϵ -chanable fuzzy metric space have a unique common fixed point.

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1 Introduction

The concept of fuzzy sets was introduced initially by Zadeh[21]. Since then, it was developed extensively by many authors and used in various fields. Especially, [2, 6, 12, 14] introduced the concept of fuzzy metric spaces in different ways. In [6, 7], George and Veeramani modified the concept of fuzzy metric space which introduced by Kramosil and Michalek[14]. They, also, obtained the Hausdorff topology for this kind of fuzzy metric spaces and showed that every metric induces a fuzzy metric.

Sessa[17] introduced a generalization of commutativity, so called weak commutativity. Further Jungck[10] introduced more generalized commutativity,

which is called compatibility in metric space and proved common fixed point theorems. Grabiec[8] proved fuzzy Banach contraction theorem on fuzzy metric space in the sense of [14]. Bijendra Singh and M. S. Chauhan[18] introduced the concept of compatibility in fuzzy metric space and proved some common fixed point theorems in fuzzy metric spaces in the sense of George and Veeramani. Jungck and Rhoades[11] introduced the notion of coincidentally commuting(or weakly compatible) mappings and obtained fixed point theorems for set-valued mapping. Also, Dhage[3] introduced this concept and proved common fixed point theorems in D-metric space.

Recently, Bijendra Singh and Shishir Jain[19] introduced the concept of weak compatibility in Menger space and proved common fixed point theorem in Menger space.

In this paper, we introduce the concept of ϵ -chainable fuzzy metric space and prove common fixed point theorems for four weakly compatible mappings of ϵ -chainable fuzzy metric space. We, also, characterize the conditions for self mappings of ϵ -chainable fuzzy metric space have a unique common fixed point which is an extension of the result of [5] and we extend a result of [15] to ϵ -chainable fuzzy metric space.

2 Preliminaries

In this section, we give some definitions and lemmas.

Definition 2.1. [16] A binary operation $*$: $[0, 1] \times [0, 1] \rightarrow [0, 1]$ is called a continuous t -norm if $([0, 1], *)$ is an abelian topological monoid with 1 such that $a * b \leq c * d$, whenever $a \leq c, b \leq d$ for all $a, b, c, d \in [0, 1]$.

Examples of t -norm are $a * b = ab$ and $a * b = \min\{a, b\}$.

Definition 2.2. [6] The 3-tuple $(X, M, *)$ is called a fuzzy metric space if X is an arbitrary set, $*$ is a continuous t -norm and M is a fuzzy set on $X^2 \times (0, \infty)$ satisfying the following conditions:

- (1) $M(x, y, t) > 0$,
 - (2) $M(x, y, t) = 1$ if and only if $x = y$,
 - (3) $M(x, y, t) = M(y, x, t)$,
 - (4) $M(x, y, t) * M(y, z, s) \leq M(x, z, t + s)$,
 - (5) $M(x, y, \cdot) : (0, \infty) \rightarrow [0, 1]$ is continuous,
- for all $x, y, z \in X$ and $t, s > 0$.

Let (X, d) be a metric space, and let $a * b = ab$ or $a * b = \min\{a, b\}$. Let $M(x, y, t) = \frac{t}{t + d(x, y)}$ for all $x, y \in X$ and $t > 0$. Then $(X, M, *)$ is a fuzzy metric space, and this fuzzy metric M induced by d is called the *standard fuzzy metric*[6].

Definition 2.3. [8] A sequence $\{x_n\}$ in a fuzzy metric space $(X, M, *)$ is said to be convergent to a point $x \in X$ (denoted by $\lim_{n \rightarrow \infty} x_n = x$), if for each $\epsilon > 0$ and each $t > 0$, there exists $n_0 \in \mathbb{N}$ such that $M(x_n, x, t) > 1 - \epsilon$ for all $n \geq n_0$.

George and Veeramani[6] show that a sequence $\{x_n\}$ in a fuzzy metric space $(X, M, *)$ converges to a point $x \in X$ if and only if $\lim_{n \rightarrow \infty} M(x_n, x, t) = 1$ for all $t > 0$.

A sequence $\{x_n\}$ in a fuzzy metric space $(X, M, *)$ is called Cauchy sequence[8] if for each $\epsilon > 0$ and each $t > 0$, there exists $n_0 \in \mathbb{N}$ such that $M(x_n, x_{n+p}, t) > 1 - \epsilon$ for all $n \geq n_0$ and all $t > 0$.

George and Veeramani[6] gave an example that $(\mathbb{R}, M, *)$ is not complete in the sense of [8], where M is the standard fuzzy metric with $d(x, y) = |x - y|$, and so to make $\mathbb{R}$ complete fuzzy metric space George and Veeramani redefine Cauchy sequence as follows.

Definition 2.4. [6] A sequence $\{x_n\}$ in a fuzzy metric space $(X, M, *)$ is called Cauchy if for each $\epsilon > 0$ and $t > 0$, there exists $n_0 \in \mathbb{N}$ such that $M(x_n, x_m, t) > 1 - \epsilon$ for all $n, m \geq n_0$.

A fuzzy metric space in which every Cauchy sequence is convergent is said to be complete.

Guangxing Song[20] gave an example that the definition of Cauchy sequence in the sense of [8] is incorrect and modified the definition of Cauchy sequence in the sense of [8] as follows.

A sequence $\{x_n\}$ in a fuzzy metric space $(X, M, *)$ is Cauchy sequence if and only if $M(x_n, x_{n+p}, t) \rightarrow 1$ (for each $t > 0$) as $n \rightarrow \infty$ uniformly on $p \in \mathbb{N}$.

Guangxing Song[20] proved that the meanings of definitions of Cauchy sequence in the sense of [20] and [6] are same.

Definition 2.5. [18] Self mappings A and B of a fuzzy metric space $(X, M, *)$ is said to be compatible if $\lim_{n \rightarrow \infty} M(ABx_n, BAx_n, t) = 1$ for all $t > 0$, whenever $\{x_n\}$ is a sequence in X such that $\lim_{n \rightarrow \infty} Ax_n = \lim_{n \rightarrow \infty} Bx_n = z$ for some $z \in X$.

Definition 2.6. [1] Self mappings A and B of a fuzzy metric space $(X, M, *)$ is said to be weakly compatible if $ABx = BAx$ when $Ax = Bx$ for some $x \in X$.

It is easy to see that if self mappings A and B of a fuzzy metric space $(X, M, *)$ is compatible then they are weakly compatible.

The following example shows that the converse of above statement does not hold.

Example Let $X = [0, 2]$ and $a * b = \min\{a, b\}$. Let M be the standard fuzzy metric induced by d , where $d(x, y) = |x - y|$ for $x, y \in X$.

Define two self mappings A and B of fuzzy metric space $(X, M, *)$ as follows:

$$Ax = \begin{cases} 2 - x, & (0 \leq x \leq 1) \\ 2, & (1 < x \leq 2) \end{cases} \quad \text{and} \quad Bx = \begin{cases} x, & (0 \leq x \leq 1) \\ 2, & (1 < x \leq 2). \end{cases}$$

Let $x_n = 1 - \frac{1}{n}$. Then $[A, B]$ is not compatible but $[A, B]$ is weakly compatible (see [1]).

Let $(X, M, *)$ be a fuzzy metric space and $\epsilon > 0$. A finite sequence $x = x_0, x_1, \dots, x_n = y$ is called ϵ -chain from x to y if $M(x_i, x_{i-1}, t) > 1 - \epsilon$ for all $t > 0$ and $i = 1, 2, \dots, n$.

A fuzzy metric space $(X, M, *)$ is called ϵ -chainable if for any $x, y \in X$, there exists an ϵ -chain from x to y .

Lemma 2.7. [8] *Let $(X, M, *)$ be a fuzzy metric space. Then for all $x, y \in X$, $M(x, y, \cdot)$ is nondecreasing*

From now on, let $(X, M, *)$ be a fuzzy metric space such that $\lim_{t \rightarrow \infty} M(x, y, t) = 1$ for all $x, y \in X$ and $t * t \geq t$ for all $t \in [0, 1]$.

Let T and T_M be two t -norm defined by for all $t, a, b \in [0, 1]$, $T(t, t) = t * t \geq t$ and $T_M(a, b) = \min\{a, b\}$, respectively. Then we know that the minimum T_M is the strongest t -norm (see [13]).

Guangxing Song considered as $T \neq T_M$ in [20], but it is well known that [13] $T = T_M$, that is, the only t -norm satisfying $T(t, t) = t * t \geq t$ for all $t \in [0, 1]$ is the minimum T_M .

Lemma 2.8. *Let $(X, M, *)$ be a fuzzy metric space. If there exists $q \in (0, 1)$ such that $M(x, y, qt) \geq M(x, y, t)$ for all $x, y \in X$ and $t > 0$, then $x = y$.*

Proof. Suppose that there exists $q \in (0, 1)$ such that $M(x, y, qt) \geq M(x, y, t)$ for all $x, y \in X$ and $t > 0$. Then $M(x, y, t) \geq M(x, y, \frac{t}{q})$ and so $M(x, y, t) \geq M(x, y, \frac{t}{q^n})$ for positive integer n . Taking limit as $n \rightarrow \infty$, $M(x, y, t) \geq 1$ and hence $x = y$. $\square$

3 Main Results

Theorem 3.1. *Let $(X, M, *)$ be a complete ϵ -chainable fuzzy metric space and let A, B, S and T be self mappings of X satisfying the following conditions:*

- (i) $AX \subset TX$ and $BX \subset SX$,
- (ii) A and S are continuous,
- (iii) the pairs $[A, S]$ and $[B, T]$ are weakly compatible,

(iv) there exists $q \in (0, 1)$ such that

$$M(Ax, By, qt) \geq M(Sx, Ty, t) * M(Ax, Sx, t) * M(By, Ty, t) * M(Ax, Ty, t)$$

for every $x, y \in X$ and $t > 0$.

Then A, B, S and T have a unique common fixed point in X .

Proof. As in [17], we can find a Cauchy sequence $\{y_n\}$ in X such that

$$y_{2n-1} = Tx_{2n-1} = Ax_{2n-2} \text{ and } y_{2n} = Sx_{2n} = Bx_{2n-1} \text{ for } n = 1, 2, \dots$$

From completeness, $y_n \rightarrow z$ for some $z \in X$, and so $\{Ax_{2n-2}\}, \{Sx_{2n}\}, \{Bx_{2n-1}\}$ and $\{Tx_{2n-1}\}$ also converges to z .

Since X is ϵ -chainable, there exists ϵ -chain from x_n to x_{n+1} , that is, there exists a finite sequence $x_n = y_1, y_2, \dots, y_l = x_{n+1}$ such that $M(y_i, y_{i-1}, t) > 1 - \epsilon$ for all $t > 0$ and $i = 1, 2, \dots, l$. Thus we have $M(x_n, x_{n+1}, t) \geq M(y_1, y_2, t/l) * M(y_2, y_3, t/l) * \dots * M(y_{l-1}, y_l, t/l) > (1 - \epsilon) * (1 - \epsilon) * \dots * (1 - \epsilon) \geq 1 - \epsilon$.

For $m > n$, $M(x_n, x_m, t) \geq M(x_n, x_{n+1}, t/m-n) * M(x_{n+1}, x_{n+2}, t/m-n) * \dots * M(x_{m-1}, x_m, t/m-n) > (1 - \epsilon) * (1 - \epsilon) * \dots * (1 - \epsilon) > 1 - \epsilon$, and so $\{x_n\}$ is a Cauchy sequence in X and hence there exists $x \in X$ such that $x_n \rightarrow x$. From (ii), $Ax_{2n-2} \rightarrow Ax$ and $Sx_{2n} \rightarrow Sx$. Since X is Hausdorff, $Ax = z = Sx$. Because $[A, S]$ is weakly compatible, $ASx = SAx$ and so $Az = Sz$. From (ii), $ASx_{2n} \rightarrow ASx$ and hence $ASx_{2n} \rightarrow Sz$. Also, from continuity of S , $SSx_{2n} \rightarrow Sz$.

From (iv),

$$\begin{aligned} & M(ASx_{2n}, Bx_{2n-1}, qt) \\ & \geq M(SSx_{2n} * Tx_{2n-1}, t) * M(ASx_{2n}, SSx_{2n}, t) \\ & \quad * M(Bx_{2n-1}, Tx_{2n-1}, t) * M(ASx_{2n}, Tx_{2n-1}, t). \end{aligned}$$

Taking limit as $n \rightarrow \infty$,

$$\begin{aligned} & M(Sz, z, qt) \\ & \geq M(Sz, z, t) * M(Sz, Sz, t) * M(z, z, t) * M(Sz, z, t) \\ & \geq M(Sz, z, t). \end{aligned}$$

From lemma 2.8, we get $Sz = z$, and hence

$$Az = Sz = z. \quad (3.1.1)$$

Since $AX \subset TX$, there exists $v \in X$ such that $Tv = Az = z$.

From (iv),

$$\begin{aligned} & M(Ax_{2n}, Bv, qt) \\ & \geq M(Sx_{2n}, Tv, t) * M(Ax_{2n}, Sx_{2n}, t) \\ & \quad * M(Bv, Tv, t) * M(Ax_{2n}, Tv, t). \end{aligned}$$

Letting $n \rightarrow \infty$, we have

$$\begin{aligned}
 & M(z, Bv, qt) \\
 & \geq M(z, Tv, t) * M(z, z, t) \\
 & * M(Bv, Tv, t) * M(z, Tv, t) \\
 & = M(z, z, t) * M(z, z, t) \\
 & * M(Bv, z, t) * M(z, z, t) \\
 & \geq M(Bv, z, t)
 \end{aligned}$$

and so $Bv = z$ and hence $Tv = Bv = z$. Since $[B, T]$ is weakly compatible, $TBv = BTv$ and hence

$$Tz = Bz. \quad (3.1.2)$$

From (iv),

$$\begin{aligned}
 & M(Ax_{2n}, Bz, qt) \\
 & \geq M(Sx_{2n}, Tz, t) * M(Ax_{2n}, Sx_{2n}, t) \\
 & * M(Bz, Tz, t) * M(Ax_{2n}, Tz, t)
 \end{aligned}$$

which implies that taking limit as $n \rightarrow \infty$,

$$\begin{aligned}
 & M(z, Bz, qt) \\
 & \geq M(z, Tz, t) * M(z, z, t) \\
 & * M(Bz, Tz, t) * M(z, Tz, t) \\
 & = M(z, Bz, t) * M(z, z, t) \\
 & * M(Bz, Bz, t) * M(z, Bz, t) \\
 & \geq M(z, Bz, t)
 \end{aligned}$$

which implies that

$$Bz = z. \quad (3.1.3)$$

From (3.1.1), (3.1.2) and (3.1.3), A, B, S and T have common fixed point z in X .

For uniqueness, let w be another common fixed point of A, B, S and T . Then

$$\begin{aligned}
 & M(z, w, qt) = M(Az, Bw, qt) \\
 & \geq M(Sz, Tw, t) * M(Az, Sz, t) * M(Bw, Tw, t) * M(Az, Tw, t) \\
 & \geq M(z, w, t).
 \end{aligned}$$

From lemma 2.8, $z = w$. □

Corollary 3.2. *Let $(X, M, *)$ be a complete ϵ -chanable fuzzy metric space and let A, B, S and T be self mappings of X satisfying (i) – (iii) of theorem 3.1 and there exists $q \in (0, 1)$ such that $M(Ax, By, qt) \geq M(Sx, Ty, t) * M(Ax, Sx, t) * M(Sx, By, 2t) * M(By, Ty, t) * M(Ty, Ax, t)$, for every $x, y \in X$ and $t > 0$.*

Then A, B, S and T have a unique common fixed point in X .

Proof. From definition, we have that $M(Sx, Ty, t) * M(Ax, Sx, t) * M(By, Ty, t) * M(By, Sx, 2t) * M(Ax, Ty, t) \geq M(Sx, Ty, t) * M(Ax, Sx, t) * M(By, Ty, t) * M(Sx, Ty, t) * M(Ty, By, t) * M(Ax, Ty, t) \geq M(Sx, Ty, t) * M(Ax, Sx, t) * M(By, Ty, t) * M(Ax, Ty, t)$ and hence, from theorem 3.1, A, B, S and T have a unique fixed point in X . $\square$

Corollary 3.3. *Let $(X, M, *)$ be a complete ϵ -chanable fuzzy metric space and let A, B, S and T be self mappings of X satisfying (i) – (iii) of theorem 3.1 and there exists $q \in (0, 1)$ such that $M(Ax, By, qt) \geq M(Sx, Ty, t)$, for every $x, y \in X$ and $t > 0$.*

Then A, B, S and T have a unique common fixed point in X .

Proof. e have that $M(Sx, Ty, t) = M(Sx, Ty, t) * 1 = M(Sx, Ty, t) * M(Ax, Ax, 5t) \geq M(Sx, Ty, t) * M(Ax, Sx, t) * M(Sx, By, 2t) * M(By, Ty, t) * M(Ty, Ax, t)$ and hence, from corollary 3.2, A, B, S and T have a unique fixed point in X . $\square$

Let S and T be the identity mapping on X in corollary 3.3. Then we get the next result.

Corollary 3.4. *Let $(X, M, *)$ be a complete ϵ -chanable fuzzy metric space and let A and B be self mappings of X satisfying the following condition: there exists $q \in (0, 1)$ such that $M(Ax, By, qt) \geq M(x, y, t)$, for every $x, y \in X$ and $t > 0$.*

Then A and B have a unique common fixed point in X .

In corollary 3.4, if we take $A = B$, then this result becomes to fuzzy Banach contraction theorem.

Corollary 3.5. *Let $(X, M, *)$ be a complete ϵ -chanable fuzzy metric space and let A be self mapping of X satisfying the following condition:*

there exists $q \in (0, 1)$ such that $M(Ax, Ay, qt) \geq M(x, y, t)$ for every $x, y \in X$ and $t > 0$.

Then A has a unique fixed point in X .

Let $(X, M, *)$ be a complete ϵ -chanable fuzzy metric space. For any $x, y \in X$ and $t > 0$, let $M_\epsilon(x, y, t) = \sup_c \{M(x_0, x_1, t) * M(x_1, x_2, t) * \dots * M(x_{n-1}, x_n, t) \mid c : x = x_0, x_1, \dots, x_n = y \text{ is an } \epsilon\text{-chain from } x \text{ to } y\}$. Then it is easy to show that M_ϵ is fuzzy metric satisfying $M(x, y, t) \geq M_\epsilon(x, y, t)$ and $M(x, y, t) = M_\epsilon(x, y, t)$, whenever $M(x, y, t) > 1 - \epsilon$.

Lemma 3.6. *Let $(X, M, *)$ be a complete ϵ -chanable fuzzy metric space. Then $(X, M_\epsilon, *)$ is complete ϵ -chanable.*

Proof. Let $x, y \in X$. Since $(X, M, *)$ is ϵ -chanable, there exists a finite sequence $x = x_0, x_1, \dots, x_n = y$ such that $M(x_{i-1}, x_i, t) > 1 - \epsilon$, and hence $M_\epsilon(x_{i-1}, x_i, t) = M(x_{i-1}, x_i, t) > 1 - \epsilon$. Thus $(X, M_\epsilon, *)$ is ϵ -chanable.

Now, we show that $(X, M_\epsilon, *)$ is complete.

Let $\{x_n\}$ be a Cauchy sequence in $(X, M_\epsilon, *)$. Then $M_\epsilon(x_m, x_n, t) > 1 - \epsilon$ for $m > n$. Since $(X, M, *)$ is ϵ -chanable, there exists ϵ -chain $x_n = y_1, y_2, \dots, y_l = x_m$ such that $M(y_{i-1}, y_i, y) > 1 - \epsilon$ for $i = 1, 2, \dots, l$. Hence $M(x_n, x_m, t) \geq M(y_1, y_2, t/l) * \dots * M(y_{l-1}, y_l, t/l) > (1 - \epsilon) * \dots * (1 - \epsilon) \geq 1 - \epsilon$, and hence $M_\epsilon(x_n, x_m, t) = M(x_n, x_m, t)$. Since $(X, M, *)$ is complete, there exists $z \in X$ such that $M(x_n, z, t) > 1 - \epsilon$ and hence $M_\epsilon(x_n, z, t) = M(x_n, z, t)$ and $M_\epsilon(x_n, z, t) > 1 - \epsilon$. Hence $\{x_n\}$ converges to z and $(X, M_\epsilon, *)$ is complete. $\square$

Theorem 3.7. *Let $(X, M, *)$ be a complete ϵ -chanable fuzzy metric space and let T_1 and T_2 be self mappings of X satisfying the following condition:*

If $M(x, y, t) > 1 - \epsilon$, there exists $q \in (0, 1)$ such that for $i, j = 1, 2$, $M(T_i x, T_j y, qt) \geq M(x, y, t)$ for all $x, y \in X$ and $t > 0$. Then T_1 and T_2 have a unique common fixed point in X .

Proof. For any $x, y \in X$ and $t > 0$, let $M_\epsilon(x, y, t) = \sup_c \{M(x_0, x_1, t) * M(x_1, x_2, t) * \dots * M(x_{n-1}, x_n, t) | c : x = x_0, x_1, \dots, x_n = y \text{ is an } \epsilon\text{-chain from } x \text{ to } y\}$. Then we know that $M(x, y, t) = M_\epsilon(x, y, t)$, whenever $M(x, y, t) > 1 - \epsilon$.

Given $x, y \in X$ and ϵ -chain $c : x = x_0, x_1, \dots, x_n = y$ such that $M(x_i, x_{i-1}, t) > 1 - \epsilon$, we have that $M(T_1 x_0, T_1 x_1, qt) \geq M(x_0, x_1, t) > 1 - \epsilon$ and $M(T_2 x_2, T_1 x_1, qt) \geq M(x_2, x_1, t) > 1 - \epsilon$, and hence $T_1 x_0, T_1 x_1, T_2 x_2$ form an ϵ -chain from $T_1 x_0$ to $T_2 x_2$.

Similarly, if n is odd (say $n=3$), we can show that $T_1 x_0, T_2 x_1, T_1 x_2$ and $T_2 x_3$ form an ϵ -chain from $T_1 x_0$ to $T_2 x_3$. Thus we have $M_\epsilon(T_1 x_0, T_2 x_3, qt) \geq M(T_1 x_0, T_2 x_1, qt) * M(T_2 x_1, T_1 x_2, qt) * M(T_1 x_2, T_2 x_3, qt) \geq M(x_0, x_1, t) * M(x_1, x_2, t) * M(x_2, x_3, t)$. Hence it follows that $M_\epsilon(T_1 x, T_2 y, qt) \geq M(x_0, x_1, t) * M(x_1, x_2, t) * \dots * M(x_{n-1}, x_n, t)$, where $x = x_0, x_1, \dots, x_n = y$ is an arbitrary ϵ -chain from x to y . Thus we have $M_\epsilon(T_1 x, T_2 y, qt) \geq M_\epsilon(x, y, t)$. From lemma 3.6 and corollary 3.4, T_1 and T_2 have a unique common fixed point in X . $\square$

In the following, we extend the result of [5] in metric space to ϵ -chanable fuzzy metric space.

Theorem 3.8. *Let $(X, M, *)$ be a complete ϵ -chanable fuzzy metric space and let S be continuous self mapping of X and T be self mapping of X . Then S and T have a common fixed point in X if and only if there exists a continuous self mapping A of X such that the following conditions are satisfied:*

(i) $AX \subset TX \cap SX$,

- (ii) the pairs $[A, S]$ and $[A, T]$ are weakly compatible,
 (iii) there exists $q \in (0, 1)$ such that for every $x, y \in X$ and $t > 0$,

$$M(Ax, Ay, qt) \geq M(Sx, Ty, t) * M(Ax, Sx, t) * M(Ay, Ty, t) * M(Ax, Ty, t).$$

 In fact A, S and T have a unique common fixed point in X .

Proof. First, we show that the necessity of the conditions (i)-(iii). Suppose that S and T have a common fixed point in X , say z . Then $Sz = z = Tz$.

Let $Ax = z$ for all $x \in X$. Then $AX \subset TX \cap SX$. Since $ASz = SAz$ and $ATz = TAz$, $[A, S]$ and $[A, T]$ are weakly compatible and hence the conditions (i) and (ii) are satisfied.

For some $q \in (0, 1)$, we have that

$$M(Ax, Ay, qt) = 1 \geq M(Sx, Ty, t) * M(Ax, Sx, t) * M(Ay, Ty, t) * M(Ax, Ty, t)$$

 for every $x, y \in X$ and $t > 0$, and hence the condition (iii) is satisfied.

Now, for the sufficiency of the conditions, let $A = B$ in theorem 3.1. Then A, S and T have a unique common fixed point in X . $\square$

Corollary 3.9. *Let $(X, M, *)$ be a complete ϵ -chanable fuzzy metric space and let S be continuous self mapping of X and T be self mapping of X . Then S and T have a common fixed point in X if and only if there exists a continuous self mapping A of X satisfying (i)-(ii) of theorem 3.8 and there exists $q \in (0, 1)$ such that for every $x, y \in X$ and $t > 0$,*

$$M(Ax, Ay, qt) \geq M(Sx, Ty, t) * M(Ax, Sx, t) * M(Ay, Ty, t) * M(Ay, Sx, 2t) * M(Ax, Ty, t).$$

In fact A, S and T have a unique common fixed point in X .

Corollary 3.10. *Let $(X, M, *)$ be a complete ϵ -chanable fuzzy metric space and let S be continuous self mapping of X and T be self mapping of X . Then S and T have a common fixed point in X if and only if there exists a continuous self mapping A of X satisfying (i) – (ii) of theorem 3.8 and there exists $q \in (0, 1)$ such that for every $x, y \in X$ and $t > 0$, $M(Ax, Ay, qt) \geq M(Sx, Ty, t)$.*

In fact A, S and T have a unique common fixed point in X .

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