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Revenue Malmquist Productivity Index And Application In Bank Branch

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1. Introduction

Productivity growth is one of the major sources of economic development. In recent years the owners and analysis of productivity have had several scientific developments in firm and industry performances. These studies have been focused on data gathering of productivity and their experiences. This has been resulted better efficiency and providing useful information for owners and designers of public and private sectors.In the last, only efficiency change was used for progress and regress studies [1], but it has been shown that the technical change has effects in productivity, too. Hereof MPI was determined [2]. Fare et al. (1992, 1994a) developed MPI, which was suggested initially by Malmquist (1953) [3]. He assimilated Fare's views for efficiency measurements and Caves et al. (1983) recommendations for productivity evaluation; and defined MPI for each unit based on inputs disposal and outputs products [4, 5]. Hereafter many researches were completed for calculation of this index and several applications were procured [6, 7]. Cost Malmquist productivity index was reported, where instead technical efficiency, the cost efficiency for every unit was prevented. This index is useful when the costs and the demand of any unit's input and the value of the output are available [8]. In this paper, the

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Revenue Malmquist productivity index is used. This index could be calculated when the price of each outputs is available and progress and regress of output revenue is the basis. At first section, basic definitions of Data Envelopment Analysis will be illustrated [9, 10, 11], then in section three, the definitions of MPI, and in section four RM are presented. In the last section for commercials Bank is used and the obtained data are reported and discussed.

2. Data Envelopment Analysis

For each manager, information regarding unit's efficiency is one of the important factors to measure the productivity using efficiency of units, because productivity of each system is a function of efficiency and impression. Before eighteenth century several researches have been investigated to measure the efficiency in a system. Here a system means a set from which Decision Making Units (DMU) will be chosen and a DMU could be efficient, when that unit procures the best benefits from the existing facilities of unit.

The amount of obtained DMUs is hypothesized in a system as n and DMU_j, $j = 1, \ldots, n$ is the j^{th} Decision Making Unit. This DMU includes m inputs $X_j = (x_{ij}, \ldots, x_{mj})$ and s outputs $Y_j = (y_{ij}, \ldots, y_{sj})$. The inputs and outputs of each DMU are non-negative and at least one of the inputs and outputs are positive. Production possibility set T_c is procured from non-empty, possibility, constant return to scale, and convexity is:

$$T_c = \{(X, Y)/X \geq \sum_{j=1}^{n} \lambda_j X_j, Y \leq \sum_{j=1}^{n} \lambda_j Y_j, \lambda_j \geq 0, j = 1, \ldots, n\} \qquad (1)$$

This set is named as production possibility set of CCR model. Frontier of T_c that is a piecewise linear surface, is named efficiency frontier. Each DMU on this frontier is relative efficiency; and the others are inefficiency. If DMU_o dose not place on frontier (inefficiencies ones), could be transferred with difference methods to the frontier. One of these methods is CCR model, with the definition of the input oriented as below:

$$Min \quad \theta s.t. \quad \sum_{j=1}^{n} \lambda_j X_j \leq \theta X_o \quad \sum_{j=1}^{n} \lambda_j Y_j \geq Y_o \quad \lambda_j \geq 0, \quad j = 1, \ldots, n$$

The unknowns of above problem are θ, $\lambda_1, \ldots, \lambda_n$. With the definition: θ is the value of relative efficiency of DMU_o.

3. Malmquist Productivity Index

Farell (1957) determined a suitable method to evaluate experimental production function for several inputs and outputs with using linear programming technique and Data Envelopment Analyses (DEA). By applying DEA, the best efficiency frontier will be calculated with a set of DMUs and omitting of any priority for inputs and outputs. The DMUs of efficiency frontier are the units with the maximum output and/or the minimum input levels. Using the efficient units and efficiency frontier, is the analysis of other inefficiency units possible. Malmquist Productivity Index is defined with assimilation efficiency changes of each unit and technology changes. MPI can be calculated via several functions, such as distance function:

$$D(X_o, Y_o) = \inf\{\theta/(\theta X_o, Y_o) \in PPS\} \tag{2}$$

This equation shows in special conditions, only the efficiency frontier change at time $t+1$ related to t; that could not be a suitable criterion to calculate the technology change. If $D^k(X^k, Y^k) = 1$, then k^{th} unit is hypothesized as efficient. This distance function dose not defines the inefficiency values. Fare decomposed MPI into two components, using linear inefficiency of technology frontier. The efficiency frontier will be specified for each DMU with DEA. Production function is tant t and $t+1$. Calculation of the MPI requires four linear programming problems as below: $O \in Q = \{1, 2, \dots, n\}$

$$\begin{aligned} D_o^t(X_o^t, Y_o^t) = Min \quad & \theta \\ s.t. \quad & \sum_{j=1}^{n} \lambda_j x_{ij}^t \leq \theta x_{io}^t, \quad i = 1, \dots, m \\ & \sum_{j=1}^{n} \lambda_j y_{rj}^t \geq y_{ro}^t, \quad r = 1, \dots, s \\ & \lambda_j \geq 0, \quad j = 1, \dots, n \end{aligned}$$

x_{io}^t is the i^{th} input and y_{ro}^t is the r^{th} output of DMU_o at time t. The value of efficiency $(D_o'(X_o^t, Y_o^t) = \theta_o^*)$ shows that how much can be decrease inputs of DMU_o to production that output.

Instead t, CCR problem (4), is calculated at time $t+1$ and is equal $D^{t+1}(X_o^{t+1}, Y_o^{t+1})$ and is the technical efficiency for DMU_o at time $t+1$. The value of $D^t(X_o^{t+1}, Y_o^{t+1})$ for DMU_o, is the distance of DMU_o at $t+1$ with the frontier of time t, calculated by below problem:

$$\begin{aligned} D^t(X_o^{t+1}, Y_o^{t+1}) = Min \quad & \theta \\ s.t. \quad & \sum_{j=1}^{n} \lambda_j x_{ij}^t \leq \theta x_{io}^{t+1}, \quad i = 1, \dots, m \\ & \sum_{j=1}^{n} \lambda_j y_{rj}^t \geq y_{ro}^{t+1}, \quad r = 1, \dots, s \\ & \lambda_j \geq 0, \quad j = 1, \dots, n \end{aligned}$$

$$
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& \sum_{j=1}^{n} \lambda_j y_{rj}^{t+1} \geq y_{ro}^t, \quad r = 1, \ldots, s \\
& \lambda_j \geq 0, \quad j = 1, \ldots, n
\end{aligned}
$$

The same model $D^{t+1}(X_o^t, Y_o^t)$ is calculated.

Fare hypotheses $D_o^{t+1}(X_o^{t+1}, Y_o^{t+1})$, $D_o^t(X_o^t, Y_o^t)$ must be equal to 1 to be efficient. Therefore he defined relative efficiency change as:

$$TEC_o = \frac{D_o^{t+1}(X_o^{t+1}, Y_o^{t+1})}{D_o^t(X_o^t, Y_o^t)} \tag{3}$$

He described one geometric compotation to determine technology change between t and $t+1$:

$$FS_o = \left[\frac{D_o^t(X_o^{t+1}, Y_o^{t+1})}{D_o^{t+1}(X_o^{t+1} Y_o^{t+1})} . \frac{D_o^t(X_o^t, Y_o^t)}{D_o^{t+1}(X_o^t, Y_o^t)} \right]^{\frac{1}{2}} \tag{4}$$

MPI will be calculated from multiplication efficiency change and technology change for each input oriented DMU_o at time t and $t+1$:

$$M_o = \frac{D_o^{t+1}(X_o^{t+1}, Y_o^{t+1})}{D_o^t(X_o^t, Y_o^t)} \left[\frac{D_o^t(X_o^{t+1}, Y_o^{t+1})}{D_o^{t+1}(X_o^{t+1} Y_o^{t+1})} . \frac{D_o^t(X_o^t, Y_o^t)}{D_o^{t+1}(X_o^t, Y_o^t)} \right]^{\frac{1}{2}} \tag{5}$$

The simple form of relation (9) is:

$$M_o = \left[\frac{D_o^t(X_o^{t+1}, Y_o^{t+1})}{D_o^t(X_o^t Y_o^t)} . \frac{D_o^{t+1}(X_o^{t+1}, Y_o^{t+1})}{D_o^{t+1}(X_o^t, Y_o^t)} \right]^{\frac{1}{2}} \tag{6}$$

This value defines geometric convex compotation, because it specified the smallest decrease of efficiencies and any small change in each efficiency effects in MPI. Three conditions are available:

1. $M_o > 1$, Increase productivity and observe progress.
2. $M_o < 1$, Decrease productivity and observe regress.
3. $M_o = 1$, No change in productivity at time $t+1$ in comparison to t.

4. Revenue Malmquist Productivity Index

Pursuant to previous section, RM is an index for signification progress and regress each unit based on consideration benefit as product Y^t. This section discusses revenue frontier change and reverie efficiency using MPI, as described

4.1. **Assumed.** At time period t, product output $Y^t \in \mathbb{R}^{\sim}$ with dispose of each input unit $X^t \in \mathbb{R}^{\geqslant}$. The production technology at time t is defined as output offer set witch is

$$L^t(X^t) = \{Y^t | Y^t \text{ can product } X^t\} \tag{7}$$

$L^t(X^t)$ contains all output vectors, which can be produced from X^t. This set is non-empty, closed, convex, bounded, and satisfies strong disposability of inputs and outputs. Bound of the set is named as output isoquant, that is:

$$IsoqL^t(X^t) = \{Y^t; Y^t \in L^t(X^t), \lambda Y^t \notin L^t(X^t) \text{ for } \lambda > 1\} \tag{8}$$

This set shows a boundary (frontier) to the output offer set in the sense that any radial expansion of output vectors that lie on the frontier is not possible within Lt(Xt). The output distance function is defined as:

$$D_o^t(X^t, Y^t) = \sup\{\varphi | (\varphi Y^t) \in L^t(X^t), \varphi > o\} \tag{9}$$

The subscript o denotes output orientation. $D_o^t(X^t, Y^t)$ in (13) is the highest possible demand, which can be multiplied whit Y^t remains in $L^t(X^t)$. If $D_o^t(X^t, Y^t) > 1$, then $Y^t \in intL^t(X^t)$. If $D_o^t(X^t, Y^t) = 1$, then $Y^t \in IsoqL^t(X^t)$. $D_o^t(X^t, Y^t)$ is similar with the definition of technical efficiency in output oriented:

$$TE_o^t(X^t, Y^t) = Max\{\varphi | (\varphi Y^t) \in L^t(X^t), \varphi > o\} \tag{10}$$

When output price $W^t \in \mathbb{R}^{\sim}$, are available, the revenue function is defined:

$$R^t(X^t, W^t) = Max\{W^t Y^t | Y^t \in L^t(X^t), W^t > o\} \tag{11}$$

$R^t(X^t, W^t)$ is the maximum revenue of producing outputs Y^t. Frontier of this set is:

$$IsoqR^t(X^t, W^t) = \{Y^t | W^t Y^t = R^t(X^t, W^t)\} \tag{12}$$

This boundary contains the output vectors that can have the maximum revenue with their price W^t. Therefore technical efficiency and distance function have the same definition. Overall efficiency defines:

$$OE_o^t(X^t, Y^t, W^t) = \frac{W^t Y^t}{R^t(X^t, W^t)} \tag{13}$$

Because technical efficiency is less than overall efficiency (revenue) for each unit, then:

$$TE_o^t(X^t, Y^t) \leq OE_o^t(X^t, Y^t, W^t) \tag{14}$$

According to technical efficiency is the same as distance function:

$$D_o^t(X^t, Y^t) \leq \frac{W^t Y^t}{R^t(X^t, W^t)} \tag{15}$$

Allocative efficiency defines as follows:

$$AE_o^t(X^t, Y^t, W^t) = \frac{W^t Y^t}{D_o^t(X^t, Y^t) R^t(X^t, W^t)} \tag{16}$$

Malmquist productivity index [3] with distance function is defined:

$$OM^t = \left[\frac{D_o^t(X^t, Y^t)}{D_o^t(X^{t+1}, Y^{t+1})}\right] \tag{17}$$

$$OM^{t+1} = \left[\frac{D_o^{t+1}(X^t, Y^t)}{D_o^{t+1}(X^{t+1}, Y^{t+1})}\right] \tag{18}$$

OM^t compose DMUs at time t and $t+1$ to frontier t. OM^{t+1} compose DMUs at time t and $t+1$ to frontier $t+1$. Malmquist productivity index (OM) is a geometric component of (21) and (22):

$$OM = \left[\frac{D_o^t(X^t, Y^t)}{D_o^t(X^{t+1}, Y^{t+1})} . \frac{D_o^{t+1}(X^t, Y^t)}{D_o^{t+1}, Y_o^{t+1})}\right]^{\frac{1}{2}} \tag{19}$$

OM is Malmquist productivity index and has inverse relative with M_o definition pervious section. Three conditions are exited:

1. $OM > 1$, observe progress.
2. $OM < 1$, observe regress.
3. $OM = 1$, do not observe any change in productivity.

4.2. **Revenue Malmquist Productivity Index.** By using Allocative and technical efficiency, output's price productivity changes are determined. To take care of (21) to (23) Revenue Malmquist Productivity Index (RM) is calculated as:

$$RM^t = \left[\frac{W^tY^t/R^t(X^t, W^t)}{W^tY^{t+1}/R^t(X^{t+1}, W^t)}\right] \tag{20}$$

$$RM^{t+1} = \left[\frac{W^{t+1}Y^t/R^{t+1}(X^t, W^t)}{W^{t+1}Y^{t+1}/R^{t+1}(X^{t+1}, W^{t+1})}\right] \tag{21}$$

$$RM = \left[\frac{W^tY^t/R^t(X^t, W^t)}{W^tY^{t+1}/R^t(X^{t+1}, W^t)} . \frac{W^{t+1}Y^t/R^{t+1}(X^t, W^{t+1})}{W^{t+1}Y^{t+1}/R^{t+1}(X^{t+1}, W^{t+1})}\right] \tag{22}$$

And $W^tY^t = \sum_{n=1}^{N} w_n^t y_n^t$, n is the n^{th} output and $R^t(X^t, W^t)$ is the maximum reverie which is calculated in (15). OM index discusses outputs quantity and RM index discusses outputs reverie. $\frac{W^tY^t}{R^t(X^t,W^t)}$ is the reverie efficiency to product Y^t at time period t with output price W^t. This fraction compares reverie of output Y^t and the maximum product reverie and its value is not less than 1. Value 1 means this output has the maximum reverie and value greater than one means this output can be decreased. This fraction is exactly overall efficiency as defined in (17). Therefore with using overall efficiency and OM, RM can be provided. RM is the value that shows which output's part can increase arrive reverie frontier. (Using constant return to scale is not necessary, but it is only for clear and distinction bench mark of reverie frontier).

Similarity OM index, for RM can say:

1. $RM > 1$, observe progress and decrease productivity.

2. $RM < 1$, observe regress and increase productivity.
3. $RM = 1$, no change in productivity.

In next section RM decomposed to compare this index and its application.

4.3. **Decomposition of Revenue Malmquist Productivity Index.** RM index mentioned in (26) can be decomposed easily in to OM index mentioned in (23). Results of this composition are overall efficiency change (OEC) and revenue technical change (RTC). Each of these components can be self decomposed as follow:

4.3.1. *First stage of decomposition.* As said before, RM index can be decomposed into OEC and RTC:

$$RM = \frac{W^tY^t/R^t(X^t,W^t)}{W^{t+1}Y^{t+1}/R^{t+1}(X^{t+1},W^{t+1})} \left[\frac{W^{t+1}Y^{t+1}/R^{t+1}(X^{t+1},W^{t+1})}{W^tY^{t+1}/R^t(X^{t+1},W^t)} . \frac{W^{t+1}Y^t/R^{t+1}(X^t,W^{t+1})}{W^tY^t/R^t(X^t,W^t)}\right]^{\frac{1}{2}} \quad (23)$$

in (27) the numerator and denominator of the component outside the square brackets are the value of overall efficiency change at two time periods t and $t+1$, that is OEC; and it value indicates whether the production unit catches up the revenue boundary when going from period t to period $t+1$ or not. The component inside the square brackets indicates RTC, which is reverie frontier change. RTC compares product revenue for each output to the maximum product.

4.3.2. *Second stage of decomposition.* Here the first component of decomposition can be decomposed again:
The composition of OEC: It can be decomposed into technical efficiency change (TEC) and allocative efficiency change (AEC):

$$OEC = \frac{D_o^t(X^t,W^t)}{D_o^{t+1}(X^{t+1},W^{t+1})} \times \frac{W^tY^t/[R^t(X^t,W^t).D_o^t(X^t,W^t)]}{W^{t+1}Y^{t+1}/[R^{t+1}(X^{t+1},W^{t+1}).D_o^{t+1}(X^{t+1},W^{t+1})]} \quad (24)$$

The first component on the right side of (28) indicates technical change. The second component in (28) is allocative efficiency change.

The decomposition of RTC: this component can be decomposed as follow:

$$RTC = \left[\frac{D_o^{t+1}(X^{t+1},Y^{t+1})}{D_o^t(X^{t+1},Y^{t+1})} . \frac{D^{t+1}(X^t,Y^t)}{D^t(X^t,Y^t)}\right]^{\frac{1}{2}}$$

$$\left[\frac{W^{t+1}Y^{t+1}/[R^{t+1}(X^{t+1},W^{t+1})D_o^{t+1}(X^{t+1},Y^{t+1})]}{W^tY^{t+1}/[R^t(X^{t+1},W^t)D_o^t(X^{t+1},Y^{t+1})]} \frac{W^{t+1}Y^t/[R^{t+1}(X^t,W^{t+1})D_o^{t+1}(X^t,Y^t)]}{W^tY^t/[R^t(X^t,W^t)D_o^t(X^t,Y^t)]}\right]^{\frac{1}{2}}$$

The first fraction on the right side indicates measure of technical change, which is one of the OM,s component. The second part in (29) is ratio of output's price change to the maximum revenue change, which represents (RE). The decomposition of RM can be summarized as:

RM = overall efficiency change (OEC) × revenue technical change (RTC)
= technical efficiency change (TEC) × allocative efficiency change (AEC) (*)
× revenue effect (RE)
= OM × allocative efficiency change (AEC) × revenue effect (RE)

4.4. **Calculation of RM index and its components.** Output distance function and revenue distance function are used to calculate allocative and technical efficiency. These components' calculation is by non-parametric programming and DEA technical possible of (x_{oi}, y_{or}) is calculated by follow model: The unit (x'_{oi}, y'_{or}) indicates i^{th} input and r^{th} output of unit o^{th} at time period t. Product revenue at time t for r^{th} component is W^tY^t and is calculated by $W^tY^t = \sum_{r=1}^{s} w_{or}^t y_{or}^t$. Therefore the revenues $W^tY^{t+1}, W^{t+1}Y^t, W^{t+1}Y^{t+1}$ are calculated with $\sum_{r=1}^{s} w_{or}^t y_{or}^{t+1}$, $\sum_{r=1}^{s} w_{or}^{t+1} y_{or}^{t+1}$, $\sum_{r=1}^{s} w_{or}^{t+1} y_{or}^{t+1}$. The value of $R^t(X^t, W^t)$ is calculated by follow model:

$$\begin{aligned} R^t(X^t, W^t) = Max \quad & w_{or}^t y_r \\ s.t. \quad & \sum_{j=1}^{n} z_j y_{jr}^t \geq y_r, && r = 1, \ldots, s \\ & \sum_{j=1}^{n} z_j x_{ij}^t \leq x_{ik}^t, && i = 1, \ldots, m \\ & z_j \geq 0, && j = 1, \ldots, n \\ & y_r \geq 0, && r = 1, \ldots, s \end{aligned}$$

$$\begin{aligned} R^t(X^{t+1}, W^t) = Max \quad & w_{or}^t y_r \\ s.t. \quad & \sum_{j=1}^{n} z_j y_{jr}^t \geq y_r, && r = 1, \ldots, s \\ & \sum_{j=1}^{n} z_j x_{ij}^t \leq x_{ik}^{t+1}, && i = 1, \ldots, m \\ & z_j \geq 0, y_j \geq 0, && j = 1, \ldots, n \end{aligned}$$

changing t to $t+1$ and converse. The distance functions are calculated: The values of $R_o^{t+1}(X^{t+1}, Y^{t+1})$ and $R_o^{t+1}(X^t, Y^t)$ could be available with the same models as (30) and (31) by

$$\begin{aligned} D_o^t(X^t, W^t) = Max \quad & \theta \\ s.t. \quad & \sum_{j=1}^{n} z_j y_{jr}^t \geq y_{kr}^t, && r = 1, \ldots, s \\ & \sum_{j=1}^{n} z_j x_{ij}^t \leq \theta x_{ik}^t, && i = 1, \ldots, m \\ & z_j \geq 0, && j = 1, \ldots, n \end{aligned}$$

$$D_o^t(X^{t+1}, W^t) = Max \quad \theta$$
$$s.t. \quad \sum_{j=1}^{n} z_j y_{jr}^t \geq y_{kr}^{t+1}, \quad r = 1, \ldots, s$$
$$\sum_{j=1}^{n} z_j x_{ij}^t \leq \theta x_{ik}^{t+1}, \quad i = 1, \ldots, m$$
$$z_j \geq 0, \quad j = 1, \ldots, n$$

The values $D_o^{t+1}(X^{t+1}, Y^{t+1})$ and $D_o^{t+1}(X^t, Y^t)$ are calculated, too, with the same model. By using of the above models, the RM index is calculated. This is a criterion to appointment revenue progress and regress of a system. This index is important for owners and public designers, because it discusses the revenue of product.

5. Application of Revenue Malmquist Productivity Index

In this section, Cost Malmquist productivity and revenue Malmquist productivity indexes will be studied from application view and the results of this consideration will be reported in form of tables.

5.1. **Data.** By using GAMS programming for 36 branches of Iranian commercial bank branches will be calculate Cost and Revenue Malmquist productivity index. Each unit has 3 inputs and 5 outputs as follow:

Inputs	Outputs
1. Payable interest	1. Public deposits
2. Personnel	2. Non-Public deposits
3. Non performing loans	3. Loans granted
	4. Received interest
	5. Fee

Table 1. Inputs and Outputs Indexes

Input indexes:
1. Payable interested (I1): It means the advantage that the bank pay to each customer and it is some of payable interest of payable interest of branches.
2. Personnel (I2): It means the total of the personnel who are working in each branch.
3. Non performing loans (I3): It is an index that creates when the customers don not pay their loans; summarize the non performing loans is the branch's none performing loans.

Output indexes:
1. Public deposits (O1): It is the total of four main deposits.
2. Non-Public deposits (O2): It is outer deposits which not note in 1.
3. Loans granted facilities (O3): It is any loans in a branch. All of them are loans granted (facilities) in a branch.
4. Received interest (O4): It is the advantage of loans granted facilities that

customers pay.
5. Fee (O5): It received when the branch does services to customer.

5.2. **Results analysis.** Each table the first, calculated efficiency value at time period t; and indicates efficiency changes and technical changes; in the end, determined Malmquist productivity index.

If efficiency change and technical change are greater than 1, then instance indexes is greater than 1 and observe progress; if two of them are less than 1 instance index are less than 1 and observe regress; otherwise if deficit in one of change amends with another change instance unit is progress. In table 5 notice that unit 1 has efficiency change and technical change greater than 1, therefore $RM > 1$, and this unit has reverie progress. Unit 10 has efficiency change less than 1 and technical change amends deficit, and $RM > 1$, observe progress. necessary equal 1 for efficiency. Because of efficiency is calculated for each unit at a time to another time's frontier it can be greater than 1and it is not Finally, observe that whit using this index, progress and regress appointment will be carefully and applicatory.

	I1	I2	I3	O1	O2	O3	O4	O5
1	1475.26	36.54	42810	2578287	356990	1627013	1360.72	439.73
2	2019.26	174.81	38840	917241	32086	590739	3826.03	524.65
3	11234.43	481.76	265915	5194917	451694	4226854	65005.41	5231.33
4	9959.5	508.21	301871	4586566	983999	3668742	11100.86	1213.14
5	3463.2	349.41	83723	1966426	454031	2727931	35486.25	3149.79
6	9078.58	278.44	500984	3664338	261773	55921	156717.53	1848.33
7	9522.39	397	82883	5269974	312655	2735519	28452.94	2649.24
8	4077.01	479.98	496950	4319584	1308445	13324914	29406.31	5182.72
9	2104.19	262.13	88015	1024665	63040	909081	4434.86	967.78
10	782.31	151.19	26641	565345	13220	156617	769.94	221.83
11	9921.01	750.58	51310	3506837	142287	1089604	11119.47	1469.9
12	3715.97	544.43	56717	1373245	42309	815105	25161.37	595.39
13	58.04	30.81	2227	86446	17204	15454	549.63	41.8
14	1831.06	181.94	18793	525142	19520	241729	1678.56	261.32
15	982.85	192.96	20917	489549	22567	310081	2086.39	402.61
16	7016.04	697.99	106258	2692686	128459	1667839	10924.76	2064.82
17	565.15	135.95	26461	374585	13912	282274	3943.99	744.73
18	80.73	49.2	18978	231898	5214	264384	1272.41	222.48
19	4522.28	429.17	130562	1610813	115385	968329	9964.54	2517.26
20	700.75	108.45	22675	346601	35473	3592	983429.27	457.37
21	599.92	151.85	24970	464746	29676	355414	3202.48	1101.66
22	987.86	150.26	38363	434916	14105	385666	3711.41	418.93
23	1997.14	428.61	46734	1080937	27965	639256	6120.45	1162.14
24	1691.12	469.31	78889	1440176	104670	1037745	15024.42	2121.56
25	351.75	135.64	26076	366844	8483	253671	2831.38	557.09
26	843.14	62.66	16393	177381	5361	262289	4014.31	305.49
27	437.24	125.9	22622	383170	7881	311718	3644.57	375.74
28	525.38	115.6	10337	458151	13203	322065	3311.05	573.73
29	421.89	169.42	33478	391168	17299	241369	2274.11	1014.67
30	1149.08	251.94	36926	916901	35075	646924	5899.33	885.41
31	1271.39	307.1	40706	786031	21968	678959	6434.53	978.82
32	8009.31	1099.03	355928	2622893	123394	2211448	19493.28	3411.81
33	1008.11	176.05	17896	592656	20038	987688	6522.43	1227.74
34	1262.37	154.26	18094	425496	12783	217203	1605.9	624.52
35	1039.59	214.56	25915	576971	15551	565672	3586.5	483.98
36	5376.56	527.31	7942	1315223	48781	1071052	9185.96	1143.34

Table 2. Inputs and outputs value, at time t

	I1	I2	I3	O1	O2	O3	O4	O5
1	1475.26	36.95	22222	2290697	426262	1712520	100502.95	439.73
2	2019.26	179.14	57375	933001	46912	564804	78316.56	524.65
3	11234.43	485.78	270980	5242723	600403	4279961	352119.81	5231.33
4	9959.5	524.01	211676	4682582	757950	3836221	1040739.02	1213.14
5	3463.2	355.05	84153	1946292	553593	2604435	213686.99	3149.79
6	9078.58	281.27	523793	618610	38201	5731482	1308473.46	1848.33
7	9522.39	404.9	82692	5347169	375537	2822805	24663	52649.24
8	4077.01	477.86	513554	4231800	12088	13816384	546829.47	5182.72
9	2104.19	265.41	89268	1027275	86315	944054	113445.9	967.78
10	782.31	150.29	29061	566499	15814	154700	21466.37	221.83
11	9921.01	761.17	51479	3459316	169549	1114642	169056.57	1469.9
12	3715.97	553.84	56541	1374064	48452	827562	119583.77	595.39
13	58.04	29.46	2343	83197	17519	16209	2244.13	41.8
14	1831.06	182.38	18318	523093	29096	243607	26645.94	261.32
15	982.85	190.37	20366	504142	24973	321492	37246.46	402.61
16	7016.04	708.43	107364	2700652	180729	1683119	327533.83	2064.82
17	565.15	137.66	25985	361286	15042	287590	52707.5	744.73
18	80.73	49.72	21218	230705	7242	268058	39370.76	222.48
19	4522.28	435.86	119588	1635426	115347	1004498	162240.29	2517.26
20	700.75	103.16	22599	355151	18081	364314	59032.05	457.37
21	599.92	155.07	24910	462702	30393	375502	43574.78	1101.66
22	987.86	148.66	38601	409545	18405	399412	81633.18	418.93
23	1997.14	431.55	47919	1068043	33964	661847	83459.26	1162.14
24	1691.12	470.94	80381	14434	63	118040	1072619	135881.08
25	351.75	137.62	23377	367605	11525	258178	42040.62	557.09
26	843.14	66.45	17179	169005	8137	263487	52821.64	305.49
27	437.24	127.9	22535	356899	7270	324076	63086.67	375.74
28	525.38	115.74	10471	466911	17487	332502	68716.39	573.73
29	421.89	166.52	33161	390301	21278	252887	42365.46	1014.67
30	1149.08	261.58	37686	908641	47018	695308	113375.87	885.41
31	1271.39	307.16	40839	781265	363878	694834	147125.75	978.82
32	8009.31	1103.82	364590	2574296	143821	2278791	27217	9.81
33	1008.11	179.82	17260	577114	19286	1019763	161825.69	1227.74
34	1262.37	154.12	15872	428666	17241	222096	53780.6	624.52
35	1039.59	219.58	28591	559179	18171	570138	75184.17	483.98
36	5376.56	534.13	10516;	1314954	57840	1077250	157648.19	1143.34

Table 3. Inputs and outputs value, at time t

MALM	$D^t(t)$	$D^{t+1}(t)$	$D^t(t+1)$	$D^{t+1}(t+1)$	Effic. Prog.	Tech. Prog.	Malmquist Ind.
1	1	1.1498	2.9548	1	1	1.603	1.60304
2	1	0.3706	0.4304	0.45	0.45	1.6064	0.72282
3	1	1.0061	1.0132	1	1	1.0035	1.00352
4	0.5175	0.3147	1.5125	1	1.9323	1.577	3.04725
5	1	1.165	1.2159	1	1	1.0216	1.02163
6	1	1.0087	1.0354	1	1	1.0132	1.01318
7	1	0.8231	1.078	0.8481	0.8481	1.2427	1.05392
8	1	1.0141	1.3492	1	1	1.1535	1.15348
9	0.5442	0.4437	0.4916	0.5893	1.0829	1.0116	1.09543
10	0.528	0.4584	0.4588	0.4493	0.851	1.0846	0.92291
11	1	0.6051	0.872	0.611	0.611	1.5357	0.93838
12	1	0.73	0.4545	0.4657	0.4657	1.1562	0.53846
13	1	1.0519	1.1105	1	1	1.0275	1.02748
14	0.4699	0.3257	0.4432	0.3393	0.7221	1.3727	0.99118
15	0.6375	0.4496	0.4951	0.5497	0.8623	1.13	0.97444
16	0.603	0.3979	0.5499	0.6937	1.1504	1.0961	1.26102
17	1	0.8432	0.7935	1	1	0.97	0.97003
18	1	1.0634	3.1402	1	1	1.7184	1.71839
19	0.6362	0.6397	0.6365	0.6406	1.0071	0.994	1.00101
20	0.6735	0.5733	0.638	0.8421	1.2504	0.9434	1.17961
21	1	1.0103	1.0545	1	1	1.0216	1.02163
22	0.4461	0.3754	0.5724	0.6865	1.5387	0.9955	1.53178
23	0.6952	0.5376	0.5403	0.5932	0.8532	1.0854	0.92603
24	0.9648	0.8081	0.8251	0.8327	0.8631	1.0877	0.93879
25	0.9145	0.851	0.9324	0.9314	1.0185	1.0372	1.05632
26	0.5928	0.5802	0.6564	0.7574	1.2777	0.941	1.20225
27	0.7971	0.6678	0.9211	1	1.2546	1.0485	1.3154
28	1	0.9889	1.0686	1	1	1.0395	1.03954
29	1	0.9966	1.1707	1	1	1.0839	1.08387
30	0.7514	0.6782	0.7936	0.7178	0.9553	1.1067	1.05732
31	0.6727	0.6091	0.769	0.7789	1.1578	1.0443	1.20903
32	0.3717	0.3729	0.382	0.39	1.0494	0.988	1.03683
33	1	1.006	2.4363	1	1	1.5562	1.5562
34	0.5825	0.5639	0.8132	0.6089	1.0453	1.1745	1.22768
35	0.574	0.5139	0.8323	0.5145	0.8962	1.3443	1.2048
36	1	1.3056	1.8916	1	1	1.2037	1.20368

Table 4. Malmquist productivity

RM	$D^t(t)$	$D^{t+1}(t)$	$D^t(t+1)$	$D^{t+1}(t+1)$	Effic. Prog.	Tech. Prog.	Rev. Malmq. Ind.
1	1	0.9228	2.1001	1	1	1.5085	1.50853
2	0.4148	0.2464	0.3176	0.268	0.6462	1.4122	0.91255
3	0.5048	0.3635	0.4921	0.3631	0.7194	1.3718	0.98692
4	0.3336	0.2879	0.56	0.3797	1.138	1.3074	1.48787
5	0.9177	0.6702	0.8384	0.6816	0.7427	1.2978	0.96388
6	0.4439	0.398	0.4358	0.4049	0.9123	1.0955	0.99945
7	1	0.4984	1.0185	0.5398	0.5398	1.9456	1.05027
8	1	0.9511	1.0554	1	1	1.0534	1.05342
9	0.3528	0.3056	0.3991	0.3537	1.0028	1.1412	1.14439
10	0.332	0.2859	0.3482	0.3044	0.917	1.1523	1.05669
11	0.9151	0.4179	0.6888	0.4578	0.5003	1.815	0.90809
12	0.4628	0.2332	0.3152	0.2093	0.4521	1.7292	0.78183
13	1	0.9052	0.9695	0.8769	0.8769	1.1052	0.96912
14	0.41	0.1995	0.3612	0.2306	0.5624	1.7942	1.00909
15	0.4595	0.3052	0.4261	0.3346	0.7283	1.3846	1.00837
16	0.4708	0.2357	0.4715	0.2801	0.595	1.8337	1.09097
17	0.6498	0.5752	0.6756	0.61	0.9387	1.1186	1.05003
18	1	0.8746	1.1914	1	1	1.1671	1.16712
19	0.3393	0.2848	0.379	0.3101	0.9138	1.2067	1.10269
20	0.4792	0.4087	0.5081	0.4321	0.9016	1.1743	1.05874
21	0.8498	0.7311	0.891	0.7668	0.9023	1.1622	1.04862
22	0.3238	0.2762	0.3801	0.3242	1.001	1.1725	1.17364
23	0.4958	0.3523	0.4617	0.3703	0.7469	1.3247	0.98947
24	0.742	0.6397	0.7642	0.6587	0.8877	1.16	1.02981
25	0.7328	0.6366	0.8237	0.7161	0.9772	1.1507	1.12448
26	0.3682	0.2254	0.3618	0.2488	0.6757	1.5413	1.04143
27	0.5913	0.5101	0.6561	0.5662	0.9576	1.1589	1.10947
28	1	0.6436	0.9533	0.7383	0.7383	1.4165	1.04574
29	0.8528	0.7416	0.9275	0.8062	0.9454	1.1502	1.08735
30	0.5806	0.4968	0.6606	0.5632	0.97	1.1708	1.1357
31	0.5125	0.4383	0.6138	0.5217	1.018	1.173	1.19403
32	0.2637	0.2247	0.2806	0.2392	0.907	1.1734	1.0642
33	1	0.6437	1.2403	0.7546	0.7546	1.598	1.20583
34	0.4944	0.2605	0.624	0.3511	0.7101	1.8366	1.30424
35	0.4647	0.3516	0.4847	0.3863	0.8313	1.2876	1.07045
36	1	1.1482	1.0317	1	1	0.9479	0.94791

Table 5. Reverie Malmquist productivity index

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