Intuitionistic Q -Fuzzy Subalgebras of BCK/BCI-Algebras

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Abstract. The intuitionistic Q -fuzzification of the concept of subalgebras in BCK/BCI -algebra is considered, and some related properties are investigated.

Keywords: BCK/BCI-algebra, Q -fuzzy set, Intuitionistic Q -fuzzy set, intuitionistic Q -fuzzy subalgebra, preimage

Mathematics Subject Classification: 06F35, 03G25, 03E72

1. INTRODUCTION

The notion of BCK-algebras was proposed by Imai and Iséki in 1966. In the same year, Iséki [4] introduced the notion of a BCI-algebra which is a generalization of a BCK-algebra. After the introduction of the concept of fuzzy sets by Zadeh [6], several researches were conducted on the generalization of the notion of fuzzy sets. The idea of “intuitionistic fuzzy set” was first published by Atanassov [1, 2], as a generalization of the notion of fuzzy set. In this paper, using the Atanassov’s idea, we establish the intuitionistic Q -fuzzification of the concept of subalgebras in BCK/BCI-algebras, and investigate some of their properties.

2. PRELIMINARIES

In this section we include some elementary aspects that are necessary for this paper.

Recall that a *BCI-algebra* is an algebra $(X, *, 0)$ of type $(2, 0)$ satisfying the following axioms:

- (I) $((x * y) * (x * z)) * (z * y) = 0$,
- (II) $(x * (x * y)) * y = 0$,
- (III) $x * x = 0$, and
- (IV) $x * y = 0$ and $y * x = 0$ imply $x = y$

for every $x, y, z \in X$. A BCI-algebra X satisfying the condition

$$(V) \quad 0 * x = 0 \text{ for all } x \in X$$

is called a *BCK-algebra*. In any BCK/BCI-algebra X one can define a partial order " $\leq$ " by putting $x \leq y$ if and only if $x * y = 0$.

A BCK/BCI-algebra X has the following properties:

$$(2.1) \quad x * 0 = x,$$

$$(2.2) \quad (x * y) * z = (x * z) * y,$$

$$(2.3) \quad x \leq y \text{ implies that } x * z \leq y * z \text{ and } z * y \leq z * x,$$

$$(2.4) \quad (x * z) * (y * z) \leq x * y$$

for all $x, y, z \in X$.

A non-empty subset S of a BCK/BCI-algebra X is called a *subalgebra* of X if $x * y \in S$ whenever $x, y \in S$. A mapping $f : X \rightarrow Y$ of BCK/BCI-algebras is called a *homomorphism* if $f(x * y) = f(x) * f(y)$ for all $x, y \in X$. Let X

be a BCK/BCI-algebra. A fuzzy set f in X , i.e., a mapping $f : X \rightarrow [0, 1]$, is called a *fuzzy subalgebra* of X if $f(x * y) \geq f(x) \wedge f(y)$ for all $x, y \in X$. Note that if f is a fuzzy subalgebra of a BCK/BCI-algebra X , then $f(0) \geq f(x)$ for all $x \in X$.

3. INTUITIONISTIC Q -FUZZY SUBALGEBRAS

In what follows, let Q and X denote a set and a BCK/BCI-algebra, respectively, unless otherwise specified. A mapping $H : X \times Q \rightarrow [0, 1]$ is called a *Q -fuzzy set* in X .

A Q -fuzzy set $H : X \times Q \rightarrow [0, 1]$ is called a *fuzzy subalgebra* of X over Q (briefly, *Q -fuzzy subalgebra of X*) if $H(x * y, q) \geq H(x, q) \wedge H(y, q)$ for all $x, y \in X$ and $q \in Q$.

Definition 3.1. let Q and X denote a set and a BCK/BCI-algebra, respectively. An *intuitionistic Q -fuzzy set* (IQFS for short) A is an object having the form

$$A = \{(x, \mu_A(x, q), \gamma_A(x, q)) : x \in X, q \in Q\}$$

where the functions $\mu_A : X \times Q \rightarrow [0, 1]$ and $\gamma_A : X \times Q \rightarrow [0, 1]$ denote the degree of membership (namely $\mu_A(x, q)$) and the degree of nonmembership (namely $\gamma_A(x, q)$) of each element $(x, q) \in X \times Q$ to the set A , respectively, and $0 \leq \mu_A(x, q) + \gamma_A(x, q) \leq 1$ for all $x \in X$ and $q \in Q$.

For the sake of simplicity, we shall use the symbol $A = (\mu_A, \gamma_A)$ for the IQFS $A = \{(x, \mu_A(x, q), \gamma_A(x, q)) : x \in X, q \in Q\}$.

Definition 3.2. An IQFS $A = (\mu_A, \gamma_A)$ in X is called an *intuitionistic Q -fuzzy subalgebra* of X if

(IQF1) $\mu_A(x * y, q) \geq \mu_A(x, q) \wedge \mu_A(y, q)$ and $\gamma_A(x * y, q) \leq \gamma_A(x, q) \vee \gamma_A(y, q)$ for all $x, y \in X$ and $q \in Q$.

Example 3.3. Let $X = \{0, a, b, c\}$ be a BCK-algebra with the following Cayley table:

$*$	0	a	b	c
0	0	0	0	0
a	a	0	0	a
b	b	a	0	b
c	c	c	c	0

Define an $IQFS = (\mu_A, \gamma_A)$ in X as follows: for every $q \in Q$,

$$\mu_A(0, q) = \mu_A(b, q) = 0.6, \mu_A(a, q) = \mu_A(c, q) = 0.2$$

and

$$\gamma_A(0, q) = \gamma_A(b, q) = 0.3, \gamma_A(a, q) = \gamma_A(c, q) = 0.7$$

It is easy to verify that $A = (\mu_A, \gamma_A)$ is an intuitionistic Q -fuzzy subalgebra of X .

Example 3.4. Consider a BCI-algebra $X = \{0, x\}$ with Cayley table as follows (Iséki [2]):

$*$	0	x
0	0	x
x	x	0

Let $Q = \{1, 2\}$ and let $A = (\mu_A, \gamma_A)$ be an intuitionistic Q -fuzzy set in X defined by

$$\mu_A(0, 1) = \mu_A(0, 2) = 1, \mu_A(x, 1) = 0.8, \mu_A(x, 2) = 0.5$$

and

$$\gamma_A(0, 1) = \gamma_A(0, 2) = 0, \gamma_A(x, 1) = 0.1, \gamma_A(x, 2) = 0.2.$$

It is easy to verify that $A = (\mu_A, \gamma_A)$ is an intuitionistic Q -fuzzy subalgebra of X .

Proposition 3.5. If $A = (\mu_A, \gamma_A)$ in X is an intuitionistic fuzzy Q -subalgebra of X , then $\mu_A(0, q) \geq \mu_A(x, q)$ and $\gamma_A(0, q) \leq \gamma_A(x, q)$ for all $x \in X$ and $q \in Q$.

Proof. Let $x \in X$ and $q \in Q$. Then $\mu_A(0, q) = \mu_A(x * x, q) \geq \mu_A(x, q) \wedge \mu_A(x, q) = \mu_A(x, q)$ and $\gamma_A(0, q) = \gamma_A(x * x, q) \leq \gamma_A(x, q) \vee \gamma_A(x, q) = \gamma_A(x, q)$. $\square$

Proposition 3.6. Let $A = (\mu_A, \gamma_A)$ be an intuitionistic Q -fuzzy subalgebra of X . Define an intuitionistic Q -fuzzy set $B = (\mu_B, \gamma_B)$ in X by

$$\mu_B(x, q) = \frac{\mu_A(x, q)}{\mu_A(0, q)}, \gamma_B(x, q) = \frac{\gamma_A(x, q)}{\gamma_A(0, q)}$$

for all $x \in X$ and $q \in Q$. Then $B = (\mu_B, \gamma_B)$ is an intuitionistic Q -fuzzy subalgebra of X .

Proof. Let $x, y \in X$ and $q \in Q$. Then

$$\begin{aligned}\mu_B(x * y, q) &= \frac{\mu_A(x * y, q)}{\mu_A(0, q)} \geq \frac{1}{\mu_A(0, q)} \{\mu_A(x, q) \wedge \mu_A(y, q)\} \\ &= \left\{ \frac{\mu_A(x, q)}{\mu_A(0, q)} \wedge \frac{\mu_A(y, q)}{\mu_A(0, q)} \right\} = \mu_B(x, q) \wedge \mu_B(y, q)\end{aligned}$$

and

$$\begin{aligned}\gamma_B(x * y, q) &= \frac{\gamma_A(x * y, q)}{\gamma_A(0, q)} \leq \frac{1}{\gamma_A(0, q)} \{\gamma_A(x, q) \vee \gamma_A(y, q)\} \\ &= \left\{ \frac{\gamma_A(x, q)}{\gamma_A(0, q)} \vee \frac{\gamma_A(y, q)}{\gamma_A(0, q)} \right\} = \gamma_B(x, q) \vee \gamma_B(y, q).\end{aligned}$$

Hence $B = (\mu_B, \gamma_B)$ is an intuitionistic Q -fuzzy subalgebra of X . $\square$

Let $A = (\mu_A, \gamma_A)$ be an IQFS in a set X and let $\alpha, \beta \in [0, 1]$ be such that $\alpha + \beta \leq 1$. Then we define the set

$$X_A^{(\alpha, \beta)} = \{x \in X \mid \mu_A(x, q) \geq \alpha, \gamma_A(x, q) \leq \beta, q \in Q\}.$$

Theorem 3.7. *Let $A = (\mu_A, \gamma_A)$ be an intuitionistic Q -fuzzy subalgebra of X . Then $X_A^{(\alpha, \beta)}$ is a subalgebra of X for every $(\alpha, \beta) \in \text{Im}(\mu_A) \times \text{Im}(\gamma_A)$ with $\alpha + \beta \leq 1$.*

Proof. Let $x, y \in X_A^{(\alpha, \beta)}$ and $q \in Q$. Then $\mu_A(x, q) \geq \alpha, \gamma_A(x, q) \leq \beta, \mu_A(y, q) \geq \alpha, \gamma_A(y, q) \leq \beta$ which imply that

$$\mu_A(xy, q) \geq \mu_A(x, q) \wedge \mu_A(y, q) \geq \alpha$$

$$\gamma_A(xy, q) \leq \gamma_A(x, q) \vee \gamma_A(y, q) \leq \beta.$$

Thus $xy \in X_A^{(\alpha, \beta)}$. Therefore $X_A^{(\alpha, \beta)}$ is a subalgebra of X . $\square$

For the convenience of notation, we denote

$$(\dots((x * y_1) * y_2) * \dots) * y_n = x * \prod_{i=1}^n y_i, \text{ and}$$

$$y_n * (y_{n-1} * (\dots * (y_1 * x) \dots)) = x * \prod_{i=1}^n y_i.$$

Proposition 3.8. *Let $A = (\mu_A, \gamma_A)$ be an intuitionistic Q -fuzzy subalgebra of a BCK-algebra X and let $x_1, x_2, \dots, x_n$ be arbitrary elements of X . If there exists $k \in \{1, 2, \dots, n\}$ such that $x_k = x_1$, then $\mu_A(x_1 * \prod_{i=2}^n x_i, q) \geq \mu_A(x, q)$*

and $\gamma_A(x_1 * \prod_{i=2}^n x_i, q) \leq \gamma_A(x, q)$ for all $x \in X$ and $q \in Q$.

Proof. Let k be a fixed number in $\{1, 2, \dots, n\}$ such that $x_k = x_1$. Using (III), (V) and (2.2), one can deduce

$$\mu_A(x_1 * \prod_{i=2}^n x_i, q) = \mu_A(0, q) \text{ and } \gamma_A(x_1 * \prod_{i=2}^n x_i, q) = \gamma_A(0, q).$$

It follows from Proposition 3.5 that

$$\mu_A(x_1 * \prod_{i=2}^n x_i, q) \geq \mu_A(x, q) \text{ and } \gamma_A(x_1 * \prod_{i=2}^n x_i, q) \leq \gamma_A(x, q)$$

for all $x \in X$ and $q \in Q$. $\square$

Proposition 3.9. *Let $A = (\mu_A, \gamma_A)$ be an intuitionistic Q-fuzzy subalgebra of a BCK-algebra X . Then*

(i) $\mu_A(x * \prod_{i=1}^{2k-1} x, q) \geq \mu_A(x, q)$ and $\gamma_A(x * \prod_{i=1}^{2k-1} x, q) \leq \gamma_A(x, q)$ for all $x \in X, q \in Q$ and for $k = 1, 2, \dots$.

(ii) $\mu_A(x * \prod_{i=1}^{2k} x, q) = \mu_A(x, q)$ and $\gamma_A(x * \prod_{i=1}^{2k} x, q) = \gamma_A(x, q)$ for all $x \in X, q \in Q$ and for $k = 0, 1, 2, \dots$.

(iii) $\mu_A(x * \prod_{i=1}^{n-1} x, q) \geq \mu_A(x, q)$ and $\gamma_A(x * \prod_{i=1}^{n-1} x, q) \leq \gamma_A(x, q)$ for all $x \in X, q \in Q$ and for $n = 1, 2, \dots$.

Proof. It follows immediately from (III), (2.1) and Proposition 3.5. $\square$

Theorem 3.10. *If $\{A_i : i \in \Lambda\}$ is an arbitrary family of intuitionistic Q-fuzzy subalgebras of X , then $\cap A_i$ is an intuitionistic Q-fuzzy subalgebra of X where $\cap A_i = \{((x, \wedge \mu_{A_i}(x, q), \vee \gamma_{A_i}(x, q)) : x \in X, q \in Q)\}$.*

Proof. Let $x, y \in X, q \in Q$. Then

$$\wedge \mu_{A_i}(x * y, q) \geq \wedge (\mu_{A_i}(x, q) \wedge \mu_{A_i}(y, q)) = (\wedge \mu_{A_i}(x, q)) \wedge (\wedge \mu_{A_i}(y, q))$$

and

$$\vee \gamma_{A_i}(x * y, q) \leq \vee (\gamma_{A_i}(x, q) \vee \gamma_{A_i}(y, q)) = (\vee \gamma_{A_i}(x, q)) \vee (\vee \gamma_{A_i}(y, q)).$$

Hence $\cap A_i = (\wedge \mu_{A_i}, \vee \gamma_{A_i})$ is an intuitionistic Q-fuzzy subalgebra of X . $\square$

Theorem 3.11. *If an IQFS $A = (\mu_A, \gamma_A)$ in X is an intuitionistic Q-fuzzy subalgebra of X , then so is $\square A$, where $\square A = \{((x, q), \mu_A(x, q), 1 - \mu_A(x, q)) : x \in X, q \in Q\}$.*

Proof. It is sufficient to show that $\overline{\mu_A}$ satisfies the second condition in (IQF1). Let $x, y \in X$ and $q \in Q$. Then

$$\begin{aligned} \overline{\mu_A}(x * y, q) &= 1 - \mu_A(x * y, q) \leq 1 - (\mu_A(x, q) \wedge \mu_A(y, q)) \\ &= (1 - \mu_A(x, q)) \vee (1 - \mu_A(y, q)) \\ &= \overline{\mu_A}(x, q) \vee \overline{\mu_A}(y, q). \end{aligned}$$

Hence $\square A$ is an intuitionistic Q-fuzzy subalgebra of X . $\square$

Theorem 3.12. *If an IQFS $A = (\mu_A, \gamma_A)$ in X is an intuitionistic Q-fuzzy subalgebra of X , then the sets*

$$X_\mu := \{x \in X : \mu_A(x, q) = \mu_A(0, q)\}$$

and

$$X_\gamma := \{x \in X : \gamma_A(x, q) = \gamma_A(0, q)\}$$

are subalgebras of X for all $q \in Q$.

Proof. Let $x, y \in X_\mu$ and $q \in Q$. Then $\mu_A(x, q) = \mu_A(0, q) = \mu_A(y, q)$, and so

$$\mu_A(x * y, q) \geq \mu_A(x, q) \wedge \mu_A(y, q) = \mu_A(0, q).$$

By using Proposition 3.5, we know that $\mu_A(x * y, q) = \mu_A(0, q)$ or equivalently $x * y \in X_\mu$. Now let $x, y \in X_\gamma$. Then

$$\gamma_A(x * y, q) \leq \gamma_A(x, q) \vee \gamma_A(y, q) = \gamma_A(0, q),$$

and by applying Proposition 3.5, we conclude that $\gamma_A(x * y, q) = \gamma_A(0, q)$ and hence $x * y \in X_\gamma$. $\square$

Definition 3.13. Let $A = (\mu_A, \gamma_A)$ be an IQFS in X and let $\alpha \in [0, 1]$. Then the set $\mu_{A, \alpha}^\geq := \{x \in X : \mu_A(x, q) \geq \alpha, q \in Q\}$ is called a μ -level α -cut and $\gamma_{A, \alpha}^\leq := \{x \in X : \gamma_A(x, q) \leq \alpha, q \in Q\}$ is called a γ -level α -cut of A .

Theorem 3.14. If an IQFS $A = (\mu_A, \gamma_A)$ in X is an intuitionistic Q -fuzzy subalgebra of X , then the μ -level α -cut and γ -level α -cut of A are subalgebras of X for every $\alpha \in [0, 1]$ such that $\alpha \in \text{Im}(\mu_A) \cap \text{Im}(\gamma_A)$, which are called a μ -level subalgebra and a γ -level subalgebra respectively.

Proof. Let $x, y \in \mu_{A, \alpha}^\geq$ and $q \in Q$. Then $\mu_A(x, q) \geq \alpha$ and $\mu_A(y, q) \geq \alpha$. It follows that $\mu_A(x * y, q) \geq \mu_A(x, q) \wedge \mu_A(y, q) \geq \alpha$ so that $x * y \in \mu_{A, \alpha}^\geq$. Hence $\mu_{A, \alpha}^\geq$ is a subalgebra of X . Now let $x, y \in \gamma_{A, \alpha}^\leq$. Then $\gamma_A(x * y, q) \leq \gamma_A(x, q) \vee \gamma_A(y, q) \leq \alpha$ and so $x * y \in \gamma_{A, \alpha}^\leq$. Therefore $\gamma_{A, \alpha}^\leq$ is a subalgebra of X . $\square$

Theorem 3.15. Let $A = (\mu_A, \gamma_A)$ be an IQFS in X such that the sets $\mu_{A, \alpha}^\geq$ and $\gamma_{A, \alpha}^\leq$ are subalgebras of X . Then $A = (\mu_A, \gamma_A)$ is an intuitionistic Q -fuzzy subalgebra of X .

Proof. We need to show that $A = (\mu_A, \gamma_A)$ satisfies the condition (IQF1). If the first condition of (IQF1) is not true, then there exist $x_0, y_0 \in X$ such that $\mu_A(x_0 * y_0, q) < \mu_A(x_0, q) \wedge \mu_A(y_0, q)$ for all $q \in Q$. Let

$$\alpha_0 := \frac{1}{2} (\mu_A(x_0 * y_0, q) + (\mu_A(x_0, q) \wedge \mu_A(y_0, q))).$$

Then $\mu_A(x_0 * y_0, q) < \alpha_0 < \mu_A(x_0, q) \wedge \mu_A(y_0, q)$, and so $x_0 * y_0 \notin \mu_{A, \alpha_0}^\geq$, but $x_0, y_0 \in \mu_{A, \alpha_0}^\geq$. This leads to a contradiction. Now assume that the second condition of (IQF1) does not hold. Then $\gamma_A(x_0 * y_0, q) > \gamma_A(x_0, q) \vee \gamma_A(y_0, q)$ for some $x_0, y_0 \in X$ and $q \in Q$. Taking

$$\beta_0 := \frac{1}{2} (\gamma_A(x_0 * y_0, q) + (\gamma_A(x_0, q) \vee \gamma_A(y_0, q))),$$

then $\gamma_A(x_0 * y_0, q) > \beta_0 > \gamma_A(x_0, q) \vee \gamma_A(y_0, q)$. It follows that $x_0, y_0 \in \gamma_{A, \beta_0}^\leq$ and $x_0 * y_0 \notin \gamma_{A, \beta_0}^\leq$. This leads to a contradiction. This completes the proof. $\square$

Theorem 3.16. *Any subalgebra of X can be realized as both a μ -level subalgebra and a γ -level subalgebra of some intuitionistic Q -fuzzy subalgebra of X .*

Proof. Let S be a subalgebra of X and let μ_A and γ_A be Q -fuzzy sets in X defined by

$$\mu_A(x, q) := \begin{cases} \alpha, & \text{if } x \in S, \\ 0, & \text{otherwise,} \end{cases} \quad \text{and } \gamma_A(x, q) := \begin{cases} \beta, & \text{if } x \in S, \\ 1, & \text{otherwise,} \end{cases}$$

for all $x \in X$ and $q \in Q$ where α and β are fixed numbers in $(0, 1)$ such that $\alpha + \beta < 1$. Let $x, y \in X$ for all $q \in Q$. If $x, y \in S$, then $x * y \in S$. Hence $\mu_A(x * y, q) = \mu_A(x, q) \wedge \mu_A(y, q)$ and $\gamma_A(x * y, q) = \gamma_A(x, q) \vee \gamma_A(y, q)$. If at least one of x and y does not belong to S , then at least one of $\mu_A(x, q)$ and $\mu_A(y, q)$ is equal to 0, and at least one of $\gamma_A(x, q)$ and $\gamma_A(y, q)$ is equal to 1. It follows that $\mu_A(x * y, q) \geq 0 = \mu_A(x, q) \wedge \mu_A(y, q)$ and $\gamma_A(x * y, q) \leq 1 = \gamma_A(x, q) \vee \gamma_A(y, q)$. Hence $A = (\mu_A, \gamma_A)$ is an intuitionistic Q -fuzzy subalgebra of X . Obviously, $\mu_{A, \alpha}^{\geq} = S = \gamma_{A, \beta}^{\leq}$. This completes the proof. $\square$

Definition 3.17. Let f be a map from a set X to a set Y . If $A = (\mu_A, \gamma_A)$ and $B = (\mu_B, \gamma_B)$ are IQFSs in X and Y respectively, then the *preimage* of B under f , denoted by $f^{-1}(B)$, is an IQFS in X defined by

$$f^{-1}(B) = (f^{-1}(\mu_B), f^{-1}(\gamma_B)),$$

and the *image* of A under f , denoted by $f(A)$, is an IQFS of Y defined by

$$f(A) = (f_{\sup}(\mu_A), f_{\inf}(\gamma_A)),$$

where

$$f_{\sup}(\mu_A)(y, q) = \begin{cases} \sup_{x \in f^{-1}(y)} \mu_A(x, q), & \text{if } f^{-1}(y, q) \neq \emptyset, \\ 0, & \text{otherwise,} \end{cases}$$

and

$$f_{\inf}(\gamma_A)(y, q) = \begin{cases} \inf_{x \in f^{-1}(y)} \gamma_A(x, q), & \text{if } f^{-1}(y, q) \neq \emptyset, \\ 1, & \text{otherwise,} \end{cases}$$

for each $y \in Y$ and $q \in Q$.

Theorem 3.18. *Let $f : X \rightarrow Y$ be a homomorphism of BCK/BCI-algebras. If $B = (\mu_B, \gamma_B)$ is an intuitionistic Q -fuzzy subalgebra of Y , then the preimage $f^{-1}(B) = (f^{-1}(\mu_B), f^{-1}(\gamma_B))$ of B under f is an intuitionistic Q -fuzzy subalgebra of X .*

Proof. Assume that $B = (\mu_B, \gamma_B)$ is an intuitionistic Q -fuzzy subalgebra of Y and let $x_1, x_2 \in X$ and $q \in Q$. Then

$$\begin{aligned} f^{-1}(\mu_B)(x_1 * x_2, q) &= \mu_B(f(x_1 * x_2), q) = \mu_B(f(x_1) * f(x_2), q) \\ &\geq \mu_B(f(x_1), q) \wedge \mu_B(f(x_2), q) \\ &= f^{-1}(\mu_B)(x_1, q) \wedge f^{-1}(\mu_B)(x_2, q) \end{aligned}$$

and

$$\begin{aligned} f^{-1}(\gamma_B)(x_1 * x_2, q) &= \gamma_B(f(x_1 * x_2), q) = \gamma_B(f(x_1) * f(x_2), q) \\ &\leq \gamma_B(f(x_1), q) \vee \gamma_B(f(x_2), q) \\ &= f^{-1}(\gamma_B)(x_1, q) \vee f^{-1}(\gamma_B)(x_2, q). \end{aligned}$$

Therefore $f^{-1}(B) = (f^{-1}(\mu_B), f^{-1}(\gamma_B))$ is an intuitionistic Q -fuzzy subalgebra of X . $\square$

Theorem 3.19. *Let $f : X \rightarrow Y$ be a homomorphism from a BCK/BCI-algebra X onto a BCK/BCI-algebra Y . If $A = (\mu_A, \gamma_A)$ is an intuitionistic Q -fuzzy subalgebra of X , then the image $f(A) = (f_{\sup}(\mu_A), f_{\inf}(\gamma_A))$ of A under f is an intuitionistic Q -fuzzy subalgebra of Y .*

Proof. Let $A = (\mu_A, \gamma_A)$ be an intuitionistic Q -fuzzy subalgebra of X and let $y_1, y_2 \in Y$ and $q \in Q$. Noticing that

$$\begin{aligned} &\{(x_1 * x_2, q) \mid (x_1, q) \in f^{-1}(y_1, q) \text{ and } (x_2, q) \in f^{-1}(y_2, q)\} \\ &\subseteq \{(x, q) \in X \times Q \mid (x, q) \in f^{-1}(y_1 * y_2, q)\}, \end{aligned}$$

we have

$$\begin{aligned} &f_{\sup}(\mu_A)(y_1 * y_2, q) \\ &= \sup\{\mu_A(x, q) \mid (x, q) \in f^{-1}(y_1 * y_2, q)\} \\ &\geq \sup\{\mu_A(x_1 * x_2, q) \mid (x_1, q) \in f^{-1}(y_1, q) \text{ and } (x_2, q) \in f^{-1}(y_2, q)\} \\ &\geq \sup\{\mu_A(x_1, q) \wedge \mu_A(x_2, q) \mid (x_1, q) \in f^{-1}(y_1, q) \text{ and } (x_2, q) \in f^{-1}(y_2, q)\} \\ &= \sup\{\mu_A(x_1, q) \mid (x_1, q) \in f^{-1}(y_1, q)\} \wedge \sup\{\mu_A(x_2, q) \mid (x_2, q) \in f^{-1}(y_2, q)\} \\ &= f_{\sup}(\mu_A)(y_1, q) \wedge f_{\sup}(\mu_A)(y_2, q) \end{aligned}$$

and

$$\begin{aligned} &f_{\inf}(\gamma_A)(y_1 * y_2, q) \\ &= \inf\{\gamma_A(x, q) \mid (x, q) \in f^{-1}(y_1 * y_2, q)\} \\ &\leq \inf\{\gamma_A(x_1 * x_2, q) \mid (x_1, q) \in f^{-1}(y_1, q) \text{ and } x_2 \in f^{-1}(y_2, q)\} \\ &\leq \inf\{\gamma_A(x_1, q) \vee \gamma_A(x_2, q) \mid (x_1, q) \in f^{-1}(y_1, q) \text{ and } (x_2, q) \in f^{-1}(y_2, q)\} \\ &= \inf\{\gamma_A(x_1, q) \mid (x_1, q) \in f^{-1}(y_1, q)\} \vee \inf\{\gamma_A(x_2, q) \mid (x_2, q) \in f^{-1}(y_2, q)\} \\ &= f_{\inf}(\gamma_A)(y_1, q) \vee f_{\inf}(\gamma_A)(y_2, q). \end{aligned}$$

Hence $f(A) = (f_{\sup}(\mu_A), f_{\inf}(\gamma_A))$ is an intuitionistic Q -fuzzy subalgebra of Y . $\square$

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