Effect of the Rotation on Waves in a Cylindrical Borehole Filled with Micropolar Fluid

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Abstract

The present investigation has concerned with propagation of surface waves in rotating cylindrical borehole filled with a micropolar viscous fluid and hosted in an infinite micropolar elastic solid medium. The dispersion equation for the propagation of surface wave has been obtained. These waves are found to be dispersive. The effects of fluid viscosity, rotation, and micropolarity of the fluid on the dispersion curve have been noticed and depicted graphically. It is found that there is significant effect of fluid viscosity and rotation on the dispersion curve, while no appreciable effect is observed by the micropolarity of the viscous fluid for a particular model. The frequency equation has been derived and dispersion curves giving natural frequency as a function of frequency ratio and wave number have been plotted graphically for a particular model. The results indicate that the effect of the viscous fluid and the rotation on phase velocity is very pronounced. Comparisons have been made in the absence of rotation.

Keywords: Cylindrical bore; Micropolar; Porous; Stress; Phase velocity; Rotation; Micropolar fluid

1- INTRODUCTION

The problem of propagation of waves along a cylindrical hole embedded in an infinite elastic medium is of great importance due to its manifold applications. In practice, the
cylindrical hole may be realized by a bore hole or a mine gallery. Bore hole studies are of
great help in exploration seismology, e.g., in exploration of oils, gases and hydro-
carbons, etc. In the oil industry, a coustic borehole logging is commonly practiced.
Mahmoud et al. [1] discussed effect of the rotation on the radial vibrations in a non-
homogeneous orthotropic hollow cylinder. Abd-Alla and Mahmoud [3,7] solved
magneto-thermoelastic problem in rotating non-homogeneous orthotropic hollow
cylindrical under the hyperbolic heat conduction model, and investigated effect of the
rotation on propagation of thermoelastic waves in a non-homogeneous infinite cylinder of
isotropic material. Abd-Alla et al [2,4,5,6] studied wave propagation modeling in
cylindrical human long wet bones with cavity, propagation of S-wave in a non-
homogeneous anisotropic incompressible and initially stressed medium under influence
of gravity field, also, effect of the rotation on a non-homogeneous infinite cylinder of
orthotropic material. Effect of the non-homogeneity on wave propagation on orthotropic
elastic media, the wave propagation in cylindrical poroelastic dry bones, effect of the
rotation on wave motion through cylindrical bore in a micropolar porous cubic crystal, is
investigated by Mahmoud [8-10]. Stilke[11] obtained solutions for the propagation of
elastic waves at the surface of a tunnel-like hole with a circular border embedded in a
three dimensional uniform elastic medium and found that the phase and group velocities
depend on the ratio between the wavelength and the circumference of the cylindrical
hole. Since then several problems concerning the cylindrical bore have been attempted by
several authors. Some of them are by Kumar and Deswal [12]. Bhujanga Rao and Rama
Recently, Cheng and Blanch [17] reviewed the methods of simulating elastic wave
propagation in a borehole by considering two different approaches, a quasi-analytic
approach known as the discrete wave number summation method and a finite difference
method. Recently, many authors studied effect of rotation and the peristaltic motion of
micropolar fluid [18-25].

In this paper, we have investigated a problem of propagation of surface waves in a
cylindrical borehole situated in an infinite micropolar elastic solid and filled with a
micropolar viscous fluid. The cylindrical bore is assumed to be of infinite extent and the
frequency equation for the propagation of surface waves is derived and then solved
numerically for a particular model. The effect of borehole radius, micropolarity and
viscosity of the contained fluid is noticed on the dispersion curves. The present model
may be viewed in the situation arises in oil well exploration. The oil inside the oil well is
generally found in crude form containing several impurities and therefore it can be best
modeled with muddy like/dusty viscous fluid of micropolar nature. Thus the present
problem may be of great help to oil companies.

2- FIELD EQUATIONS AND RELATIONS

Following Eringen [1], [4] the equations of motion and constitutive relations for
micropolar viscous fluid and micropolar solid media, in the absence of body forces and
surface stresses are given as follows: For micropolar rotational fluid medium,
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\[
\begin{aligned}
(c_{1f}^2 + c_{3f}^2)\nabla \cdot \mathbf{u}^f - (c_{2f}^2 + c_{4f}^2)\nabla \times \nabla \times \mathbf{u}^f + c_{5f}^2 \nabla \times \Phi_f^f = \mathbf{u}^f + \Omega_f^f \times (\Omega_f^f \times \mathbf{u}^f) \\
(c_{1s}^2 + c_{3s}^2)\nabla \cdot \Phi_s^f - c_{2s}^2 \nabla \times \nabla \times \Phi_s^f + c_{5s}^2 (\nabla \times \mathbf{u}^s - 2\Phi_s^s) = \ddot{\Phi}_s^f,
\end{aligned}
\]  

(1) (2)

and for micropolar rotational solid medium,

\[
\begin{aligned}
(c_{1s}^2 + c_{3s}^2)\nabla \cdot \mathbf{u}^s - (c_{2s}^2 + c_{4s}^2)\nabla \times \nabla \times \mathbf{u}^s + c_{5s}^2 \nabla \times \Phi_s^s = \mathbf{u}^s + \Omega_s^s \times (\Omega_s^s \times \mathbf{u}^s) \\
(c_{1s}^2 + c_{3s}^2)\nabla \cdot \Phi_s^s - c_{2s}^2 \nabla \times \nabla \times \Phi_s^s + c_{5s}^2 (\nabla \times \mathbf{u}^s - 2\Phi_s^s) = \ddot{\Phi}_s^s,
\end{aligned}
\]  

(3) (4)

where

\[
\begin{aligned}
c_{1R}^2 &= (\lambda^R + 2\mu^R) / \rho^R, & c_{2R}^2 &= \mu^R / \rho^R, & c_{3R}^2 &= K^R / \rho^R, \\
c_{4R}^2 &= (\alpha^R + \beta^R) / \rho^R \mu^R, & c_{5R}^2 &= \gamma^R / \rho^R \mu^R, & c_{6R}^2 &= \omega^R / \mu^R,
\end{aligned}
\]

\(\rho^R\) is the density of the medium, \(j^R\) is the micro-inertia, \(\mathbf{u}^R\) is the displacement and \(\Phi^R\) is the microrotation vectors. Here, the quantity having superscript \(R\) corresponds to the micropolar fluid or solid medium as per the following definition

\[R = \begin{cases} R^f & \text{for micropolar viscous fluid medium,} \\ R^s & \text{for micropolar elastic solid medium.} \end{cases}\]

The quantities \(\lambda^f, \mu^f, K^f\), are the fluid viscosity coefficients and the quantities \(\alpha^f, \beta^f, \gamma^f\) are the fluid viscosity coefficients responsible for gyrational dissipation of the micropolar fluid, while the quantities \(\lambda^s, \mu^s\) are the Lame's constant and the quantities \(K^s, \alpha^s, \beta^s, \gamma^s\) are the micropolar elastic constants for the micropolar elastic solid medium. The over dot represents the temporal derivative of the respective quantities.

The constitutive relations are given by,

\[
\begin{aligned}
\tau_{kl}^f &= \lambda^f (\dot{u}_{r,r}^f \delta_{kl} + \mu^f (\dot{u}_{s,s}^f + \dot{u}_{s,k}^f)) + K^f (\dot{u}_{l,l}^f - \varepsilon_{ilp} \dot{\phi}_p^f), \\
\tau_{kl}^s &= \lambda^s (\dot{u}_{r,r}^s \delta_{kl} + \mu^s (\dot{u}_{s,s}^s + \dot{u}_{s,k}^s)) + K^s (\dot{u}_{l,l}^s - \varepsilon_{ilp} \dot{\phi}_p^s), \\
m_{kl}^f &= \alpha^f \dot{\phi}_{r,r}^f \delta_{kl} + \beta^f \dot{\phi}_{s,s}^f + \gamma^f \dot{\phi}_{s,k}^f, \\
m_{kl}^s &= \alpha^s \dot{\phi}_{r,r}^s \delta_{kl} + \beta^s \dot{\phi}_{s,s}^s + \gamma^s \dot{\phi}_{s,k}^s,
\end{aligned}
\]

(5) (6) (7) (8)
where \( \tau_{ri} \) is the force stress tensor, \( m_{ri} \) is the couple stress tensor, the "comma" in the subscript denotes the spatial derivative, \( \delta_{ri} \) and \( e_{rij} \) are Kronecker delta and the alternating tensors respectively. Other symbols have their usual meanings.

3- FORMULATION OF THE PROBLEM AND ITS SOLUTION

Consider a circular cylindrical bore of radius \( 'a' \) through a micropolar elastic medium of infinite extent. Taking the cylindrical polar coordinates \((r,0,z)\) such that the z-axis is pointing vertically upward along the axis of the cylinder. Our aim is to investigate the frequency equation relevant to the propagation of axial symmetric waves which are harmonic along the axial direction. To discuss the surface waves at micropolar fluid/micropolar solid interface, we consider the following forms of the displacement and microrotation as

\[
\begin{align*}
\Phi &= (0, 0, \Delta), \\
\Phi' &= (0, 0, \Delta), \\
\psi &= (0, 0, \frac{\partial u}{\partial r}), \\
\psi' &= (0, 0, \frac{\partial u'}{\partial r}).
\end{align*}
\]

Since we are considering axially symmetric waves, therefore, the quantities would remain independent of \( \theta \). With these considerations, the above equations (1) and (2) become

\[
\begin{align*}
(\mu' + K') (\nabla^2 - \frac{1}{r^2}) u' + (\lambda' + \mu') \frac{\partial^2 u'}{\partial r^2} - K' \frac{\partial \phi'}{\partial z} &= \rho' \frac{\partial^2 u'}{\partial t^2} - \rho' (\Omega')^2 u', \\
(\mu' + K') (\nabla^2 - \frac{1}{r^2}) u'' + (\lambda' + \mu') \frac{\partial^2 u''}{\partial r^2} + K' \frac{\partial (r \phi')}{\partial r} &= \rho' \frac{\partial^2 u''}{\partial t^2} - \rho' (\Omega')^2 u',
\end{align*}
\]

and equations (3) and (4) become

\[
\begin{align*}
(\mu'' + K'') (\nabla^2 - \frac{1}{r^2}) u'' + (\lambda'' + \mu'') \frac{\partial^2 u''}{\partial r^2} - K'' \frac{\partial \phi''}{\partial z} &= \rho'' \frac{\partial^2 u''}{\partial t^2} - \rho'' (\Omega'')^2 u'', \\
(\mu'' + K'') (\nabla^2 - \frac{1}{r^2}) u''' + (\lambda'' + \mu'') \frac{\partial^2 u'''}{\partial r^2} + K'' \frac{\partial (r \phi'')}{\partial r} &= \rho'' \frac{\partial^2 u'''}{\partial t^2} - \rho'' (\Omega'')^2 u.',
\end{align*}
\]
where

\[ e^R = \frac{1}{r} \left( \frac{\partial u^R_r}{\partial r} + \frac{\partial u^R_z}{\partial z} \right), \]

\[ \nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2}. \]

Introducing the potentials \( \phi^R, \psi^R, \Gamma^R \) as follows

\[ u^R_r = \frac{\partial \phi^R}{\partial r} + \frac{\partial^2 \psi^R}{\partial r \partial z}, \]

\[ u^R_z = \frac{\partial \phi^R}{\partial z} - (\nabla^2 - \frac{\partial^2}{\partial z^2}) \psi^R, \]

\[ \phi^R_2 = -\frac{\partial \Gamma^R}{\partial r}. \]

into equations (9)-(14), we obtain

\[ [(c_{1R}^2 + c_{3R}^2) \nabla^2 + (\Omega^R)^2 \left( \begin{array}{c} 1 \\ c^2 \end{array} \right) \phi^R = 0, \]

\[ [(c_{2R}^2 + c_{3R}^2) \nabla^2 + (\Omega^R)^2 \left( \begin{array}{c} 1 \\ c^2 \end{array} \right) \psi^R + c_{3R}^2 \Gamma^R = 0, \]

\[ [c_{5R}^2 \nabla^2 - 2c_{6R}^2 \left( \begin{array}{c} \partial^2 \\ \partial t \end{array} \right) \Gamma^R - c_{6R}^2 \nabla^2 \psi^R = 0, \]

where

\[ \left( \begin{array}{c} \partial^2 \\ \partial t \end{array} \right) \quad \text{for} \quad R = s \]

\[ \left( \begin{array}{c} \partial^2 \\ \partial t \end{array} \right) \quad \text{for} \quad R = f \]

Let the solutions of equations (16),(17) and (18) in the form

\[ \phi^R(r, z, t) = \phi^R(r, z) e^{iot}, \]

\[ \psi^R(r, z, t) = \psi^R(r, z) e^{iot}, \]

\[ \Gamma^R(r, z, t) = \Gamma^R(r, z) e^{iot}, \]

for harmonic vibration motion and introduce the inversion of the Hankel transform which is defined by:

\[ \phi^R(r, z, \omega) = \int_0^\infty \hat{\phi}^R(\eta, z, \omega) J_0(\eta r) \eta d\eta \]

\[ \psi^R(r, z, \omega) = \int_0^\infty \hat{\psi}^R(\eta, z, \omega) J_0(\eta r) \eta d\eta \]

\[ \Gamma^R(r, z, \omega) = \int_0^\infty \hat{\Gamma}^R(\eta, z, \omega) J_0(\eta r) \eta d\eta \]
The solutions of these equations corresponding to the waves propagation along z-direction are given by:

\[
\phi^f = \int_0^\infty A_1^f(\eta) e^{(\xi^f_1 z + i \omega t)} J_0(\eta \rho) \eta d\eta,
\]

\[
\phi^s = \int_0^\infty A_1^s(\eta) e^{(\xi^s_1 z + i \omega t)} J_0(\eta \rho) \eta d\eta,
\]

\[
\psi^f = \int_0^\infty [A_2^f(\eta) e^{(-\xi^f_1 z + i \omega t)} + A_2^s(\eta) e^{(-\xi^s_1 z + i \omega t)}] J_0(\eta \rho) \eta d\eta,
\]

\[
\psi^s = \int_0^\infty [A_2^f(\eta) e^{(-\xi^f_1 z + i \omega t)} + A_2^s(\eta) e^{(-\xi^s_1 z + i \omega t)}] J_0(\eta \rho) \eta d\eta,
\]

\[
\Gamma^f = -\int_0^\infty (\eta^2 - \xi^f_1 + b_2^f) A_2^f(\eta) e^{(-\xi^f_1 z + i \omega t)} J_0(\eta \rho) \eta d\eta
\]

\[
\Gamma^s = -\int_0^\infty (\eta^2 - \xi^s_1 + b_2^s) A_2^s(\eta) e^{(-\xi^s_1 z + i \omega t)} J_0(\eta \rho) \eta d\eta.
\]

where \(\omega\) is the angular frequency, \(\xi^j\) \((j=0,1,2,3,4)\) is the wave number, \(c\) is the phase velocity, \(J_0(\ )\) is the Bessel function of zero order and first kind, \(J_1(\ )\) is the Bessel function of first order and first kind and

\[
b_2^f = \frac{i \omega - \Omega_2^f}{c_{2f} + c_{3f}}, \quad b_2^s = \frac{\omega^2 - \Omega_2^s}{c_{2s} + c_{3s}},
\]

\[
(\xi_0^f)^2 = \eta^2 - \frac{\partial^2}{\partial z^2} \phi^f \quad \text{and} \quad (\xi_0^s)^2 = \frac{(\Omega^f)^2 - i \omega^2}{c_{1f}^2 - c_{3f}^2},
\]

\[
(\xi_{1f})^2 = \eta^2 - \frac{\partial^2}{\partial z^2} \phi^f \quad \text{and} \quad (\xi_{2f})^2 = \frac{(\Omega^f)^2 - \omega^2}{c_{1s}^2 - c_{3s}^2},
\]

\[
(\xi_{1f})^2 = \eta^2 - \frac{\partial^2}{\partial z^2} \phi^f \quad \text{and} \quad (\xi_{2f})^2 = \frac{(\Omega^f)^2 - \omega^2}{c_{1s}^2 - c_{3s}^2},
\]

\[
\xi_{2f}^2 = \eta^2 - f_{2f}^2, \quad \xi_{2s}^2 = \eta^2 - f_{2s}^2.
\]
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\[ f_{1R} = \sqrt{\frac{b_R - (b_R^2 - 4a_R c_R)}{2a_R}}, \]
\[ f_{2R} = \sqrt{\frac{b_R - (b_R^2 - 4a_R c_R)}{2a_R}}, \]
\[ a' = c_{2f}^2 (c_{2f}^2 + c_{3f}^2), \]
\[ b' = i\omega c_{5f}^2 - c_{5f}^2 \Omega_f^2 + 2c_{6f}^2 c_{2f}^2 + i\omega (c_{2f}^2 + c_{3f}^2) + c_{6f}^2 c_{5f}^2, \]
\[ c' = -2i\omega c_{6f}^2 + 2c_{6f}^2 \Omega_f^2 + \omega^2 + i\omega \Omega_f^2, \]
\[ a' = c_{2s}^2 (c_{2s}^2 + c_{3s}^2), \]
\[ b' = -c_{5s}^2 \omega^2 - c_{5s}^2 \Omega_s^2 + 2c_{6s}^2 c_{2s}^2 - \omega^2 (c_{2s}^2 + c_{3s}^2) + c_{6s}^2 c_{3s}^2, \]
\[ c' = 2c_{6s}^2 \omega^2 + 2c_{6s}^2 \Omega_s^2 + \omega^4 - \omega^2 \Omega_s^2, \]

4- FREQUENCY EQUATION

Now, we intend to apply the boundary conditions at the fluid-solid interface. For the type of waves considered in a fluid-filled cylindrical borehole, there are three boundary conditions at the surface of the cylindrical borehole: (i) the displacement must remain finite at the center of the borehole, (ii) there are no incoming waves from infinity and (iii) the continuity of displacement, micro-rotation and stresses at the fluid-solid interface. In our present problem, the fluid column inside the micropolar solid formation is micropolar as well as viscous in nature. Since the micropolar viscous fluid can support couple stresses and shear stress, therefore, both shear and couple stresses must be taken into account while formulating the boundary conditions at the surface of cylindrical borehole. The choice of solutions considered in equations (19) - (21) is so that they automatically satisfy the above boundary conditions (i) and (ii). This is because the Bessel functions, \( J_n(\cdot) \) represent outgoing and incoming waves in cylindrical coordinates, Boundary condition (iii) implies that the radial displacement and microrotation, radial force, shear force stress and couple stress across the fluid-solid interface are continuous. Mathematically, these boundary conditions can be expressed as: at \( r = a \)
Using the equations (5)-(8), (15) and (19) - (21) into the above boundary conditions given in (22), we obtain a system of six homogeneous equations in six unknowns namely

\[
\begin{align*}
\{[(\lambda^{s}+2\mu^{s}+K^{s})\eta^{2}+\lambda^{s}\xi_{0s}^{2}]}J_{0}(\eta\alpha)+(2\mu^{s}+K^{s})\frac{\eta}{a}J_{1}(\eta\alpha)\}A_{1}^{s} \\
-(2\mu^{s}+K^{s})\xi_{0s}^{s}[\eta^{2}J_{0}(\eta\alpha)+\frac{\eta}{a}J_{1}(\eta\alpha)]A_{3}^{s} \\
-(2\mu^{s}+K^{s})\xi_{0s}^{s}[\eta^{2}J_{0}(\eta\alpha)+\frac{\eta}{a}J_{1}(\eta\alpha)]A_{2}^{s} \\
i\omega\{[(\lambda^{s}+2\mu^{s}+K^{s})\eta^{2}+\lambda^{s}\xi_{0s}^{2}]}J_{0}(\eta\alpha)+(2\mu^{s}+K^{s})\frac{\eta}{a}J_{1}(\eta\alpha)\}A_{1}^{s} \\
i\omega(2\mu^{s}+K^{s})\xi_{0s}^{s}[\eta^{2}J_{0}(\eta\alpha)+\frac{\eta}{a}J_{1}(\eta\alpha)]A_{2}^{s} \\
i\omega(2\mu^{s}+K^{s})\xi_{0s}^{s}[\eta^{2}J_{0}(\eta\alpha)+\frac{\eta}{a}J_{1}(\eta\alpha)]A_{2}^{s}=0 \\
(2\mu^{s}+K^{s})\xi_{0s}^{s}\eta J_{1}(\eta\alpha)A_{1}^{s} \\
+\eta[(\mu^{s}+K^{s})\eta^{2}-\mu^{s}\xi^{2}_{3s}-K^{s}(\eta^{2}-\xi^{2}_{2s}b_{2s})]\}J_{1}(\eta\alpha)A_{2}^{s} \\
+\eta[(\mu^{s}+K^{s})\eta^{2}-\mu^{s}\xi^{2}_{3s}-K^{s}(\eta^{2}-\xi^{2}_{2s}b_{2s})]\}J_{1}(\eta\alpha)A_{2}^{s} \\
i\omega(2\mu^{s}+K^{s})\eta\xi_{0s}^{s}J_{1}(\eta\alpha)A_{1}^{s} \\
i\omega(2\mu^{s}+K^{s})\eta\xi_{0s}^{s}J_{1}(\eta\alpha)A_{1}^{s} \\
-(2\mu^{s}+K^{s})\xi_{0s}^{s}[\eta^{2}J_{0}(\eta\alpha)+\frac{\eta}{a}J_{1}(\eta\alpha)A_{2}^{s}=0 \\
(\eta^{2}-\xi^{2}_{2s}+b_{2s})\eta[\nu^{s}\eta J_{0}(\eta\alpha)+\frac{\nu^{s}}{a}J_{1}(\eta\alpha)+\frac{\beta^{s}}{a}J_{1}(\eta\alpha)]A_{2}^{s}
\end{align*}
\]
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\[ + \left( \eta^2 - \xi^2 \right) \eta \left[ \nu^J \eta J_0(\eta a) + \frac{\nu^J}{a} J_1(\eta a) + \frac{\beta^J}{a} J_1(\eta a) \right] A_2' \]

\[ - i \omega \left( \eta^2 - \xi^2 \right) \eta \left[ \nu^J \eta J_0(\eta a) + \frac{\nu^J}{a} J_1(\eta a) + \frac{\beta^J}{a} J_1(\eta a) \right] A_2' \]

\[ - i \omega \left( \eta^2 - \xi^2 \right) \eta \left[ \nu^J \eta J_0(\eta a) + \frac{\nu^J}{a} J_1(\eta a) + \frac{\beta^J}{a} J_1(\eta a) \right] A_2' = 0 \quad (25) \]

\[ - \eta J_1(\eta a) A_2' + \eta \xi J_1(\eta a) A_2' \]

\[ + \eta \xi^J_1 J_1(\eta a) A_2' - \eta J_1(\eta a) A_2' \]

\[ + \eta \xi^J_1 J_1(\eta a) A_2' + \eta \xi^J_1 J_1(\eta a) A_2' = 0 \quad (26) \]

\[ - \eta^2 J_0(\eta a) A_2' - \eta^2 J_0(\eta a) A_2' \]

\[ - \eta^2 J_0(\eta a) A_2' + \eta^2 J_0(\eta a) A_2' \]

\[ + \eta^2 J_0(\eta a) A_2' e^{(-\xi^J_1 z + i \omega t)} + \eta^2 J_0(\eta a) A_2' = 0 \quad (27) \]

\[- \eta \left( \eta^2 - \xi^2 \right) J_1(\eta a) A_2' \]

\[ - \eta \left( \eta^2 - \xi^2 \right) J_1(\eta a) A_2' \]

\[ + \eta \left( \eta^2 - \xi^2 \right) J_1(\eta a) A_2' \]

\[ + \eta \left( \eta^2 - \xi^2 \right) J_1(\eta a) A_2' = 0 \quad (28) \]

For a nontrivial solution of these equations, the determinant of the coefficient matrix must vanish. The zero determinant of the coefficients matrix will give the frequency equation for the surface waves at micropolar solid/micropolar fluid interface. Thus, elimination of these unknowns would give the following frequency equation as

\[ D \left( \xi, \omega, F \right) = 0 \quad (29) \]

where \( D \) is the determinant of the system matrix \( \left[a_{mn} \right]_{6 \times 6} \) of the homogeneous linear system of equations (23)-(28). The parameter \( F \) contains the geometrical and material constants. The entries of matrix \( \left[a_{mn} \right] \) are given by:
\[
\begin{align*}
    a_{11} &= \left\{ \left( \lambda^r + 2 \mu^r + K^r \right) \eta^2 + \lambda^r \xi^r \right\} J_0(\eta a) + \left( 2 \mu^r + K^r \right) \frac{\eta}{a} J_1(\eta a) \\
    a_{12} &= -\left( 2 \mu^r + K^r \right) \xi^r \left[ \eta^2 J_0(\eta a) + \frac{\eta}{a} J_1(\eta a) \right] \\
    a_{13} &= -\left( 2 \mu^r + K^r \right) \xi^r \left[ \eta^2 J_0(\eta a) + \frac{\eta}{a} J_1(\eta a) \right] \\
    a_{14} &= -i \omega \left\{ \left( \lambda^r + 2 \mu^r + K^r \right) \eta^2 + \lambda^r \xi^r \right\} J_0(\eta a) + \left( 2 \mu^r + K^r \right) \frac{\eta}{a} J_1(\eta a) \\
    a_{15} &= i \omega \left( 2 \mu^r + K^r \right) \xi^r \left[ \eta^2 J_0(\eta a) + \frac{\eta}{a} J_1(\eta a) \right] \\
    a_{16} &= i \omega \left( 2 \mu^r + K^r \right) \xi^r \left[ \eta^2 J_0(\eta a) + \frac{\eta}{a} J_1(\eta a) \right] \\
    a_{21} &= (2 \mu^r + K^r) \xi^r \eta J_1(\eta a) \\
    a_{22} &= \eta \left\{ (\mu^r + K^r) \eta^2 - \mu^r \xi^r_{3r} - K^r (\eta^2 - \xi^2_{3r} b^r_{2r}) \right\} J_1(\eta a) \\
    a_{23} &= \eta \left\{ (\mu^r + K^r) \eta^2 - \mu^r \xi^r_{3r} - K^r (\eta^2 - \xi^2_{3r} b^r_{2r}) \right\} J_1(\eta a) \\
    a_{24} &= -i \omega (2 \mu^r + K^r) \eta \xi^r \eta J_1(\eta a) \\
    a_{25} &= -i \omega \eta \left\{ (\mu^r + K^r) \eta^2 - \mu^r \xi^r_{3r} - K^r (\eta^2 - \xi^2_{3r} b^r_{2r}) \right\} J_1(\eta a) \\
    a_{26} &= -i \omega \eta \left\{ (\mu^r + K^r) \eta^2 - \mu^r \xi^r_{3r} - K^r (\eta^2 - \xi^2_{3r} b^r_{2r}) \right\} J_1(\eta a) \\
    a_{31} &= 0 \\
    a_{32} &= (\eta^2 - \xi^2_{3r} b^r_{2r}) \eta \left\{ \nu^r \eta J_0(\eta a) + \frac{\nu^r}{a} J_1(\eta a) + \frac{\beta^r}{a} J_1(\eta a) \right\} \\
    a_{33} &= (\eta^2 - \xi^2_{3r} b^r_{2r}) \eta \left\{ \nu^r \eta J_0(\eta a) + \frac{\nu^r}{a} J_1(\eta a) + \frac{\beta^r}{a} J_1(\eta a) \right\}
\end{align*}
\]
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\[ a_{34} = 0 \]
\[ a_{35} = -i \omega (\eta^2 - \xi_{1f}^2 + b_2^f) \eta [\nu J_0(\eta \alpha) + \frac{\nu^f}{\alpha} J_1(\eta \alpha) + \frac{\beta^f}{\alpha} J_1(\eta \alpha) ] \]
\[ a_{36} = -i \omega (\eta^2 - \xi_{2f}^2 + b_2^f) \eta [\nu J_0(\eta \alpha) + \frac{\nu^f}{\alpha} J_1(\eta \alpha) + \frac{\beta^f}{\alpha} J_1(\eta \alpha) ] \]
\[ a_{41} = -\eta J_1(\eta \alpha) \]
\[ a_{42} = \eta \xi_{3}^{f} J_1(\eta \alpha) \]
\[ a_{43} = \eta \xi_{4}^{f} J_1(\eta \alpha) \]
\[ a_{44} = -\eta J_1(\eta \alpha) \]
\[ a_{45} = \eta \xi_{1}^{f} J_1(\eta \alpha) \]
\[ a_{46} = \eta \xi_{2}^{f} J_1(\eta \alpha) \]
\[ a_{51} = -\xi_{0}^{f} J_0(\eta \alpha) \]
\[ a_{52} = -\eta^2 J_0(\eta \alpha) \]
\[ a_{53} = -\eta^2 J_0(\eta \alpha) \]
\[ a_{54} = \xi_{0}^{f} J_0(\eta \alpha) \]
\[ a_{55} = \eta^2 J_0(\eta \alpha) \]
\[ a_{56} = \eta^2 J_0(\eta \alpha) \]
\[ a_{61} = 0 \]
\[ a_{62} = -\eta (\eta^2 - \xi_{3}^{2} + b_3^f) J_1(\eta \alpha) \]
\[ a_{63} = -\eta (\eta^2 - \xi_{4}^{2} + b_3^f) J_1(\eta \alpha) \]
\[ a_{64} = 0 \]
\[ a_{65} = \eta (\eta^2 - \xi_{1f}^2 + b_2^f) J_1(\eta \alpha) \]
\[ a_{66} = \eta (\eta^2 - \xi_{2f}^2 + b_2^f) J_1(\eta \alpha) \].
We notice from the above equation (29) that for a fixed parameter $F$, it is an implicit functional relationship between the wave number and natural frequency. Moreover, some of the coefficients are involving complex quantities, therefore it is expected that the relevant surface waves are dispersive. For the waves of very short wavelengths, i.e., for large of $\xi_0$, the dispersion equation (29) will converge to the dispersion equation of Stoneley-type surface waves at micropolar solid/micropolar fluid interface.

5- NUMERICAL RESULTS AND DISCUSSIONS
Here, we shall investigate the dispersion relation given by (29) numerically for a particular model. Since this equation is an implicit functional relation of wave number and natural frequency, therefore one can proceed to find the variation of natural frequency with wave number. For numerical computations, we take the following values of the relevant parameters for micropolar solid (aluminum epoxy) and micropolar fluid as

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Value</th>
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<th>Value</th>
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</thead>
<tbody>
<tr>
<td>$\lambda^f$</td>
<td>$1.0 \times 10^{10}$ dyne sec/cm$^2$</td>
<td>$\lambda^s$</td>
<td>$7.59 \times 10^{10}$ dyne/cm$^2$</td>
</tr>
<tr>
<td>$\mu^f$</td>
<td>$0.5 \times 10^{10}$ dyne sec/cm$^2$</td>
<td>$\mu^s$</td>
<td>$1.89 \times 10^{10}$ dyne/cm$^2$</td>
</tr>
<tr>
<td>$K^f$</td>
<td>$0.0110 \times 10^{10}$ dyne sec/cm$^2$</td>
<td>$K^s$</td>
<td>$0.0149 \times 10^{10}$ dyne/cm$^2$</td>
</tr>
<tr>
<td>$\beta^f$</td>
<td>$0.0122 \times 10^{10}$ dyne sec</td>
<td>$\beta^s$</td>
<td>$0.0226 \times 10^{10}$ dyne</td>
</tr>
<tr>
<td>$\gamma^f$</td>
<td>$0.0126 \times 10^{10}$ dyne sec</td>
<td>$\gamma^s$</td>
<td>$0.0263 \times 10^{10}$ dyne</td>
</tr>
<tr>
<td>$j^f$</td>
<td>$0.00140$ cm$^2$</td>
<td>$j^s$</td>
<td>$0.00196$ cm$^2$</td>
</tr>
<tr>
<td>$\rho^f$</td>
<td>$1.0$ cm$^3$</td>
<td>$\rho^s$</td>
<td>$2.192$ gm/cm$^3$</td>
</tr>
</tbody>
</table>

The radius of the cylindrical borehole is taken $a=10$ cm whenever not mentioned. Since some of the entries of the determinantal equation (29) are complex, therefore it is not analytically possible to find the roots of this equation for a given wavenumber. For a given wavenumber, equation (29) is solved numerically by taking the above numerical values of the physical parameters. Suppose the roots of equation (29) lie along a smooth curve $C$ in the phase velocity-wavenumber domain, then for a particular value of wavenumber $\xi=\xi_0$.

In order to study numerically the effect of viscosity, rotation and micropolarity on dispersion curves, we have solved the frequency equations (29) for the natural frequency by taking different values of wave number in non-dimensional form. In fact, the
numerical results could have been more informative if the values of elastic constants appearing in the text for soil were known. To understand the problem in greater detail numerically. Figures (1,4) show the effect of rotation and micropolarity on dispersion curves for two modes considered. It can be seen that the effect of micropolarity and rotation is significant for all values of wave number. Figure 1: shows the variation of natural frequency with the wave number in the absence of rotation ($\Omega=0$). It's obvious that the natural frequency in the absence of rotation is increasing with increases of wave number. Figures (2-4): show the variation of the natural frequency in the present rotation ($\Omega=0.5, 1.0, 1.5$) and it is for all values of the wave number, the dispersion curves for natural frequency in the bore hole through a micropolar elastic medium show by the solid curve (first mode) more than the dispersion for the natural frequency in the bore hole through an elastic medium shown by the dashed curve (second mode). Figure (5): shows the effect of rotation on the natural frequency ($\Omega=0.5, 1, 1.5$), it is notice that, the natural frequency decrease with increasing the rotation. Figure(6):shows the effect of viscosity on the dispersion curves. It is obvious that the natural frequency for highly viscous fluid is greater than that for the low viscous fluid up to certain values of the non-dimensional wave number. In Figures (5, 6) the variation of the natural frequency with the wave number, increase with increase of fluid viscosity. The results indicate that effect of rotation, viscosity and micropolarity on the natural frequency are very pronounced.

6- CONCLUSION

A mathematical study has been presented here to determine the effect of rotation, micropolarity and viscosity on surface waves dispersion in bore holes. The problem has been reduced to that of wave propagation in cylindrical bore holes situated in elastic media and filled with viscous fluid. The frequency equation for this problem has been obtained from the frequency equation of the present problem. Numerical computation have been performed to solve the frequency equations and the following inferences are made.

(i) It is seen that natural frequency of wave propagation depends on wave number, showing that the frequency equation is dispersive curves.

(ii) The effect of micropolarity and the rotation is significant for all values of wave numbers.

(iii) Very little effect of viscosity is found on the dispersion curve for low viscosity fluids, while for highly viscous fluids a strong effect is observed. For highly, the effect of viscosity on dispersion curves is seen for all values of wave numbers.
Figure 1: Variation of the non dimensional frequency versus the wave number, ($\Omega = 0$)

Figure 2: Variation of the non dimensional frequency versus the wave number, ($\Omega = 0.5$)

Figure 3: Variation of the non dimensional frequency versus the wave number, ($\Omega = 1$)
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Figure 4: Variation of the non dimensional frequency versus the wave number, \( \Omega = 1.5 \)

Figure 5: Variation of the non dimensional frequency of first mode versus the wave number, \( \Omega = (0.5, 1.0, 1.5) \)

Figure 6: Comparison of the non-dimensional frequency at very low and very high fluid viscosity \( \mu' \) of the micropolar fluid.
REFERENCES


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