The d-OR Gate Problem in Dynamic Fault Trees and its Solution in Markov Analysis

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Abstract

This paper points out a “d-OR gate problem” inherent in dynamic fault trees (FTs) with priority AND (PAND) gates and restorable basic events with repeating occurrences and restorations. The problem is that owing to the existence of a d-OR gate, an OR gate located in the subtree with a PAND gate as its topmost element, it is impossible to properly understand the occurrences and restorations of the top event. It is clarified in this paper for the first time that unlike a dynamic FT with non-restorable basic events, the state of such a dynamic FT with restorable basic events cannot be described correctly by the occurring order of basic events. This paper also presents an extended transition rule used in Markov analysis to solve the d-OR gate problem. This rule succeeds in establishing the applicability of Markov analysis to such dynamic FTs.

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1 Introduction

Fault tree (FT) analysis [5] only with static gates, such as AND/OR gates, has been widely used as a tool for quantitative reliability or safety assessment. However, gates representing dynamics, such as transient recovery and sequence dependency, are necessary to describe complex behavior of systems. A priority AND (PAND) gate is the typical example of such dynamic gates that show the dependency of an event sequence [5]. Because its inputs must occur in a prescribed order, dynamic FTs including PAND gates are difficult to
analyze quantitatively. Recently, several analysis frameworks have been presented for dynamic FTs with non-restorable basic events, e.g., Markov analysis with modularization [2] and with binary decision diagrams [4], a Bayesian network approach [1], a minimal cut set or sequence approach [10, 11], and an algebraic approach [9]. However, hardly any research has been made on dynamic FTs with restorable basic events with repeating occurrences and restorations. Quantitative analysis of such FTs, which can describe the behavior of restorable systems, has never been examined.

This paper for the first time points out a “d-OR gate problem” inherent in dynamic FTs with PAND gates and restorable basic events. A d-OR gate is an OR gate located in the subtree with a PAND gate as its topmost element. A restoration of the d-OR gate can cause an apparent change in its position in an occurring order and preserve its occurrence state contrary to the actuality. This apparent change can influence the positions of the inputs of the PAND gate. Because the order dominates the occurrences and restorations of the PAND gate, they can contradict the actuality. In such a case, it is impossible to properly understand the occurrences and restorations of the top event of the overall FT, and the correct analysis result, such as the top-event probability, cannot be obtained. Thus, it is clear that unlike a dynamic FT with non-restorable basic events, the state of a dynamic FT with restorable basic events cannot be described correctly by the occurring order of basic events.

Markov analysis [6] has been widely considered as one of the practical analysis techniques for FTs [3]. However, the d-OR gate problem reveals that Markov analysis cannot be generally applied to dynamic FTs with PAND gates and restorable basic events unless the problem is solved. The problem also implies that the analysis based on the minimal cut sets or sequences cannot be extended and applied to such dynamic FTs. It is an indisputable fact that the d-OR gate problem has a significant effect on the study of such dynamic FTs.

This paper presents an extended transition rule used in Markov analysis to solve the d-OR gate problem. In particular, on a state transition diagram considered in Markov analysis, the post-transition state against a restoration of the input of a d-OR gate is determined as contradictory to the ordinary transition rule, so as to avoid an apparent change in the position of the d-OR gate in an occurring order. This extended transition rule is very useful for potential applications of Markov analysis to such dynamic FTs.

2 Dynamic FT and its Markov analysis

2.1 Dynamic FT

Assume the following (a)–(e) on the analyzed dynamic FT.
(a) There are \( m \) gates, \( G_1, G_2, \ldots, G_m \), where each gate is either an AND gate, an OR gate, or a PAND gate.

(b) The top event is the output of \( G_1 \).

(c) There are \( n \) basic events, \( B_1, B_2, \ldots, B_n \).

(d) Each basic event, the top event, and each gate assume two states, a non-occurrence state and an occurrence state. The following two events can arise in each of them.

- **Occurrence**: Each occurs and falls into its occurrence state from its non-occurrence state.
- **Restoration**: Each is restored from its occurrence state to its non-occurrence state.

(e) \( B_k \) occurs [is restored] probabilistically in accordance with the exponential distribution with the occurrence rate \( \lambda_k \) [the restoration rate \( \mu_k \)], \( k = 1, 2, \ldots, n \).

2.2 Distinction between d- and s- basic events/gates

All basic events and gates can be classified into the following two [8]:

- **d- basic events/gates**: They are located in the subtree with a PAND gate as its topmost element. Not only their occurrence states but also the occurring order influences on the top event. A d-basic event \( B_k \) and a d-gate \( G_j \) is written as \( d-B_k \) and \( d-G_j \), respectively, as necessary.

- **s- basic events/gates**: It includes all other basic events/gates that cannot be characterized as d-type. Only their states influence the top event. An s-basic event \( B_k \) and an s-gate \( G_j \) is written as \( s-B_k \) and \( s-G_j \), respectively, as necessary.

A repeated basic event located both in the subtree with a PAND gate as its topmost element and elsewhere is a d-basic event. Note that \( G_1 \) is an s-gate.

Without loss of generality, \( B_1, B_2, \ldots, B_{n_d} \) are assumed to be the d-basic events, and \( B_{n_d+1}, B_{n_d+2}, \ldots, B_n \) to be the s-basic events.

2.3 State description on Markov diagram

On the basis of the distinction between d- and s- basic events, the state on a state transition diagram is described by \( (i_1, i_2, \ldots, i_{n_d}; i_{n_d+1}, i_{n_d+2}, \ldots, i_n) \),
where

\[ i_k = \begin{cases} 
0, & \text{d-}B_k \text{ is in non-occurrence state} \\
 j (= 1, 2, \ldots, n_d), & \text{d-}B_k \text{ has been in occurrence state} \\
 & \text{from } j \text{-th earliest time out of} \\
 & \text{d-basic events in occurrence states} \\
& (k = 1, 2, \ldots, n_d) 
\end{cases} \quad (1) \]

and

\[ i_k = \begin{cases} 
0, & \text{s-}B_k \text{ is in non-occurrence state} \\
1, & \text{s-}B_k \text{ is in occurrence state} \\
(k = n_d + 1, n_d + 2, \ldots, n). & \quad (k = n_d + 1, n_d + 2, \ldots, n). \quad (2) 
\end{cases} \]

The value \( i_k \) \((k = 1, 2, \ldots, n)\) represents the state of \( B_k \), which is hereafter referred as the (basic event) value of \( B_k \).

Note that \( \max_{1 \leq k \leq n_d} i_k \) and \( \sum_{n_d+1 \leq k \leq n} i_k \) indicate the number of d- and s- basic events in their occurrence states, respectively.

### 2.4 Gate values

Suppose that the gate \( G_j \) has \( n_j \) inputs from the left on the FT, \( X_{j1}, X_{j2}, \ldots, X_{jn_j} \). The situations where each input \( X_{j\ell} \) \((\ell = 1, 2, \ldots, n_j)\) is connected directly to the output of a gate \( G_k \) \((k \neq 1, j)\) or to a basic event \( B_k \) are written as \( X_{j\ell} = G_k \) or \( X_{j\ell} = B_k \), respectively. The state of \( G_j \) is represented by the value

\[ g_j = f_{T_j}(x_{j1}, x_{j2}, \ldots, x_{jn_j}), \quad (3) \]

where \( T_j \in \{ \text{d-AND, d-OR, d-PAND, s-AND, s-OR, s-PAND} \} \) denotes the gate type of \( G_j \), and

\[ x_{j\ell} = \begin{cases} 
g_k, & X_{j\ell} = G_k \\
i_k, & X_{j\ell} = B_k 
\end{cases} \quad (\ell = 1, 2, \ldots, n_j). \quad (4) \]
The function $f_{T_j}$ is chosen from the following (5) – (10) by the gate type $T_j$.

\[
\begin{align*}
    f_{d-\text{AND}}(x_1, x_2, \ldots, x_\ell) &= \begin{cases} 
        \max_{1 \leq j \leq \ell} x_j, & \prod_{j=1}^{\ell} x_j > 0 \\
        0, & \text{otherwise}
    \end{cases} \\
    f_{d-\text{OR}}(x_1, x_2, \ldots, x_\ell) &= \begin{cases} 
        \min_{1 \leq j \leq \ell} \{x_j \mid x_j > 0\}, & \sum_{j=1}^{\ell} x_j > 0 \\
        0, & \text{otherwise}
    \end{cases} \\
    f_{d-\text{PAND}}(x_1, x_2, \ldots, x_\ell) &= \begin{cases} 
        x_\ell, & 0 < x_1 < x_2 < \cdots < x_\ell \\
        0, & \text{otherwise}
    \end{cases} \\
    f_{s-\text{AND}}(x_1, x_2, \ldots, x_\ell) &= \begin{cases} 
        1, & \prod_{j=1}^{\ell} x_j > 0 \\
        0, & \text{otherwise}
    \end{cases} \\
    f_{s-\text{OR}}(x_1, x_2, \ldots, x_\ell) &= \begin{cases} 
        1, & \sum_{j=1}^{\ell} x_j > 0 \\
        0, & \text{otherwise}
    \end{cases} \\
    f_{s-\text{PAND}}(x_1, x_2, \ldots, x_\ell) &= \begin{cases} 
        1, & 0 < x_1 < x_2 < \cdots < x_\ell \\
        0, & \text{otherwise}.
    \end{cases}
\end{align*}
\]

The value $g_j$ ($j = 1, 2, \ldots, m$) representing the state of $G_j$ is referred hereafter as the gate value of $G_j$. Note that $(g_1, g_2, \ldots, g_m)$ in the state $(i_1, i_2, \ldots, i_{n_d}; i_{n_d+1}, i_{n_d+2}, \ldots, i_n)$ is uniquely determined by

\[
\begin{align*}
    g_1 &= f_{T_1}(x_{11}, x_{12}, \ldots, x_{1n_1}) \\
    g_2 &= f_{T_2}(x_{21}, x_{22}, \ldots, x_{2n_2}) \\
    \vdots \\
    g_m &= f_{T_m}(x_{m1}, x_{m2}, \ldots, x_{mn_m}).
\end{align*}
\]

### 2.5 Set of top-event occurrence states

The state of the top event is equivalent to that of $G_1$. Thus, the necessary and sufficient condition for the top event to be in its occurrence state is $g_1 = 1$, where $g_1$ is obtained from (11). The set of all top-event occurrence states can be obtained by

\[
S = \{ (i_1, i_2, \ldots, i_{n_d}; i_{n_d+1}, i_{n_d+2}, \ldots, i_n) \mid g_1 = 1 \}.
\]

### 2.6 Ordinary transition rule

Using the values of the d-basic events, $(i_1, i_2, \ldots, i_{n_d})$, obtained from the state description, $(i_1, i_2, \ldots, i_{n_d}; i_{n_d+1}, i_{n_d+2}, \ldots, i_n)$, a transition caused by an oc-
occurrence or a restoration of a d-basic event is considered\(^1\). The values of the s-basic events, \((i_{n_d+1}, i_{n_d+2}, \ldots, i_n)\), are preserved against the transition.

The transition can be divided into two phases as follows:

\[
\begin{align*}
\text{Pre-transition state } & (i_1, i_2, \ldots, i_{n_d}) \\
\xrightarrow{\text{First-half phase}} & \text{Intermediate state } (\tilde{i}_1, \tilde{i}_2, \ldots, \tilde{i}_{n_d}) \\
\xrightarrow{\text{Second-half phase}} & \text{Post-transition state } (i'_1, i'_2, \ldots, i'_{n_d}), \quad (13)
\end{align*}
\]

where \(\tilde{i}_k \geq 0 (k = 1, 2, \ldots, n_d)\) and \(\tilde{i}_{k_1} \neq \tilde{i}_{k_2} (\tilde{i}_{k_1}, \tilde{i}_{k_2} > 0, k_1 \neq k_2)\) in the intermediate state.

### 2.6.1 First-half phase

In the ordinary transition rule, the intermediate state \((\tilde{i}_1, \tilde{i}_2, \ldots, \tilde{i}_{n_d})\) is obtained from the pre-transition state \((i_1, i_2, \ldots, i_{n_d})\) and the information on the event as follows.

- **Occurrence of d-B\(_k\)** (assume \(i_k = 0\)):
  \[
  \tilde{i}_\ell = \begin{cases} 
  n_d, & \ell = k \\
  i_\ell, & \ell \neq k.
  \end{cases} \quad (14)
  \]

- **Restoration of d-B\(_k\)** (assume \(i_k > 0\)):
  \[
  \tilde{i}_\ell = \begin{cases} 
  0, & \ell = k \\
  i_\ell, & \ell \neq k.
  \end{cases} \quad (15)
  \]

### 2.6.2 Second-half phase

For \(z_j \geq 0 (j = 1, 2, \ldots, \ell)\) and \(p = 1, 2, \ldots, \ell\), define the following function:

\[
\begin{align*}
\quad h((z_1, z_2, \ldots, z_\ell), p) = \\
\quad \begin{cases} 
  q, & z_p > 0 \text{ and } z_p \text{ is } q-\text{th smallest} \\
  \quad \text{value in } \{z_j | z_j > 0\} \\
  0, & z_p = 0.
  \end{cases} \quad (16)
\end{align*}
\]

In the ordinary transition rule, the post-transition state \((i'_1, i'_2, \ldots, i'_{n_d})\) is obtained from the intermediate state \((\tilde{i}_1, \tilde{i}_2, \ldots, \tilde{i}_{n_d})\) by renumbering as follows:

\[
i_k' = h((\tilde{i}_1, \tilde{i}_2, \ldots, \tilde{i}_{n_d}), k), \quad k = 1, 2, \ldots, n_d. \quad (17)
\]

\(^1\)The d-OR gate problem pointed out in this paper is inherent in the dynamics of PAND gates. Thus, a transition caused by an occurrence or a restoration of an s-basic event has no connection with the subject of this paper.
3 The d-OR gate problem

The state description similar to that in Section 2.3, which is based on the occurring order of basic events in their occurrence states, has been widely used in the analysis of dynamic FTs. For example, a sequence of basic events in their occurrence states, itself, such as “B₃B₁B₄,” is used in [11]. However, if there is a d-OR gate, an OR gate located in the subtree with a PAND gate as its topmost element, the set of all top-event occurrence states \( S \) in (12) has less significance. In particular,

- **Issue 1:** There is the possibility that the top event does not occur even against a transition from a state outside \( S \) to a state belonging to \( S \), and
- **Issue 2:** There is the possibility that the top event is not restored even against a transition from a state belonging to \( S \) to a state outside \( S \).

Thus, it is impossible to properly understand the occurrences and restorations of the top event by \( S \), and the correct analysis result, such as the top-event probability, cannot be obtained. This is the “d-OR gate problem.” The problem implies that unlike a dynamic FT with non-restorable basic events in [11], the state of a dynamic FT with restorable basic events cannot be described correctly by the occurring order of basic events.

**Remark 3.1** As pointed out in [7], there is the possibility that a PAND gate does not actually occur even if its occurrence condition on the description is satisfied before an OR gate located in the subtree with the PAND gate as its topmost element is restored. The basis of this problem is equivalent to Issue 1, however it indicates one aspect of the d-OR gate problem.

The following simple example will suffice to illustrate the d-OR gate problem.

### 3.1 Example

Consider a dynamic FT shown in Figure 1, where the shaded gate and the shadowed basic events denote a d-gate and d-basic events, respectively. The state on a state transition diagram in Section 2.3 is described by \((i₁, i₂, i₃, i₄)\), where \(i_k\) \((k = 1, 2, 3, 4)\) represents the state of \(B_k\) as (1). The gate values are as follows:

\[
g₁ = f_{s-PAND}(i₁, g₂, i₂) = \begin{cases} 
1, & 0 < i₁ < g₂ < i₂ \\
0, & \text{otherwise}
\end{cases} \quad (18)
\]

\[
g₂ = f_{d-OR}(i₃, i₄) = \begin{cases} 
\min_{k=3,4}\{i_k \mid i_k > 0\}, & i₃ + i₄ > 0 \\
0, & \text{otherwise}
\end{cases} \quad (19)
\]
(see Section 2.4). The set of all top-event occurrence states is obtained by
\[
S = \{(i_1, i_2, i_3, i_4) \mid g_1 = 1\}
= \{(1, 3, 2, 0), (1, 3, 0, 2), (1, 3, 2, 4), (1, 4, 2, 3), (1, 3, 4, 2), (1, 4, 3, 2)\}.
\]

(20)

\[\text{Figure 1: Example of dynamic FT for d-OR gate problem.}\]

3.1.1 Issue 1

Consider the following transition:
\[
(2, 4, 3, 1) \not\in S \xrightarrow{\text{Restoration of } B_4} (1, 3, 2, 0) \in S.
\]

(21)

In the pre-transition state \((2, 4, 3, 1)\), \(G_2\) (by \(B_4\)), \(B_1\), and \(B_2\) have occurred in that order. Thus, the true occurrence condition of the PAND gate \(G_1\) on the FT that \(B_1\), \(G_2\), and \(B_2\) have occurred in this order is not satisfied, and the top event is not in its occurrence state. Even if \(B_4\) is restored, the d-OR gate \(G_2\) preserves its occurrence state by \(B_3\). Because there is no change in that occurring order, the true occurrence condition is not yet satisfied and the top event does not occur. This is the actuality. However, there is a contradiction with the appearance that the post-transition state \((1, 3, 2, 0)\) belongs to \(S\).

The transition (21) can be seen from a viewpoint of the ordinary transition rule in Section 2.6 as follows:

\[
\begin{align*}
\text{Pre-transition state } (2, 4, 3, 1) & \xrightarrow{\text{First-half phase}} \text{Intermediate state } (2, 4, 3, 0) \\
& \xrightarrow{\text{Second-half phase}} \text{Post-transition state } (1, 3, 2, 0).
\end{align*}
\]

(22)
In the pre-transition state \((2, 4, 3, 1)\), the value of the d-OR gate \(G_2\) is \(g_2 = 1\), thus the occurrence condition of the PAND gate \(G_1\) on the description, \(0 < i_1 < g_2 < i_2\) in (18), is not satisfied. However, in the intermediate state \((2, 4, 3, 0)\), \(\tilde{g}_2 = f_{d-OR}(\tilde{i}_3, \tilde{i}_4) = 3\), and therefore, the occurrence condition of \(G_1\) on the description, \(0 < \tilde{i}_1 < \tilde{g}_2 < \tilde{i}_2\), is satisfied. The magnitude relation between the values of \(G_2\) and \(B_1\) present on its left is apparently changed contrary to the actuality in the first-half phase of the ordinary transition rule. Note that in the second-half phase this apparent magnitude relation is preserved because only renumbering identical to that in (17) is performed.

### 3.1.2 Issue 2

Consider the following transition:

\[
(1, 3, 2, 4) \in S \xrightarrow{\text{Restoration of } B_3} (1, 2, 0, 3) \notin S. \tag{23}
\]

In the pre-transition state \((1, 3, 2, 4)\), \(B_1, G_2\) (by \(B_3\)), and \(B_2\) have occurred in that order. Thus, the true occurrence condition of the PAND gate \(G_1\) on the FT that \(B_1, G_2,\) and \(B_2\) have occurred in this order is satisfied, and the top event is in its occurrence state. Even if \(B_3\) is restored, the d-OR gate \(G_2\) preserves its occurrence state by \(B_4\). Because there is no change in that order, the top event is still in its occurrence state. This is the actuality. However, there is a contradiction with the appearance that the post-transition state \((1, 2, 0, 3)\) does not belong to \(S\).

The transition (23) can be seen from a viewpoint of the ordinary transition rule in Section 2.6 as follows:

\[
\begin{array}{c}
\text{Pre-transition state } (1, 3, 2, 4) \\
\xrightarrow{\text{First-half phase}} \text{Intermediate state } (1, 3, 0, 4) \\
\xrightarrow{\text{Second-half phase}} \text{Post-transition state } (1, 2, 0, 3).
\end{array} \tag{24}
\]

In the pre-transition state \((1, 3, 2, 4)\), the value of \(G_2\) is \(g_2 = 2\), and thus the occurrence condition of the PAND gate \(G_1\) on the description, \(0 < i_1 < g_2 < i_2\) in (18), is satisfied. However, in the intermediate state \((1, 3, 0, 4)\), \(\tilde{g}_2 = f_{d-OR}(\tilde{i}_3, \tilde{i}_4) = 4\), and therefore, the occurrence condition of \(G_1\) on the description, \(0 < \tilde{i}_1 < \tilde{g}_2 < \tilde{i}_2\), is not satisfied. The magnitude relation between the values of \(G_2\) and \(B_2\) present on its right is apparently changed contrary to the actuality in the first-half phase of the ordinary transition rule.
3.2 Essence of d-OR gate problem

3.2.1 Other possibilities of occurrence and restoration of PAND gate

A PAND gate on a dynamic FT is in its occurrence state if and only if all its inputs have occurred in the order from left to right. Owing to this true occurrence condition of a PAND gate on the FT, its actual occurrence and restoration are restricted to the following possibilities.

(a) Actual occurrence: A PAND gate ordinarily occurs when the extreme right input occurs at the last under the situation where the rest have occurred in the order from left to right.

(b) Actual restoration: A PAND gate is ordinarily restored when its input is restored.

On the other hand, there are other possibilities of an occurrence and a restoration of a PAND gate according to the occurrence condition on the description. Suppose that the PAND gate $G_j$ has $n_j$ inputs connected directly to basic events or (the outputs of) gates, $X_{j1}, X_{j2}, \ldots, X_{jn_j}$, from the left on the FT and let $x_{j\ell}$ and $x'_{j\ell}$ ($\ell = 1, 2, \ldots, n_j$) denote the value of the basic event/gate $X_{j\ell}$ in the pre- and post-transition states, respectively, against an occurrence or a restoration of a basic event located in the subtree with $G_j$ as its topmost element. The occurrence condition of $G_j$ on the description in the pre-transition state is

$$0 < x_{j1} < x_{j2} < \cdots < x_{jn_j}. \quad (25)$$

The occurrence condition of $G_j$ on the description in the post-transition state, $0 < x'_{j1} < x'_{j2} < \cdots < x'_{jn_j}$, is equivalent to that in the intermediate state,

$$0 < \tilde{x}_{j1} < \tilde{x}_{j2} < \cdots < \tilde{x}_{jn_j}, \quad (26)$$

because renumbering by (17) in the second-half phase of the ordinary transition rule does not change the magnitude relation among $\tilde{x}_{j1}, \tilde{x}_{j2}, \ldots, \tilde{x}_{jn_j}$. As a result, there are the following possibilities of an occurrence and a restoration of $G_j$ other than (a) and (b).

(a)’ Another occurrence: In the pre-transition state where the condition (25) is not satisfied, one input $X_{jk}$ changes its value from $x_{jk}(> 0)$ to $\tilde{x}_{jk}(> 0)$ so that the condition (26) is satisfied.

(b)’ Another restoration: In the pre-transition state where the condition (25) is satisfied, one input $X_{jk}$ changes its value from $x_{jk}(> 0)$ to $\tilde{x}_{jk}(> 0)$ so that the condition (26) is not satisfied.
Note that multiple inputs cannot change their values simultaneously against an occurrence or a restoration of a non-repeated basic event located in the subtree with $G_j$ as its topmost element. The Issue 1 is the epitome of $(a)'$. In the example in Section 3.1.1, the value of the second input of the PAND gate $G_1$ from the left becomes $\tilde{g}_2 = 3$ from $g_2 = 1$ so that the occurrence condition in the intermediate state, $0 < \tilde{i}_1 < \tilde{g}_2 < \tilde{i}_2$, is satisfied. Also, Issue 2 is the epitome of $(b)'$. In the example in Section 3.1.2, the value of the second input becomes $\tilde{g}_2 = 4$ from $g_2 = 2$ so that the occurrence condition is not satisfied.

### 3.2.2 Apparent change in position of d-OR gate in occurring order

In the other possibilities of an occurrence and a restoration of a PAND gate, $(a)'$ and $(b)'$ in Section 3.2.1, the key is that the input of a PAND gate changes its value between different natural numbers in the first-half phase to derive another occurring order of the inputs. The important points are as follows:

(i) Such a change between different natural numbers can be first to arise only in the value of a d-OR gate.

(ii) Another position of the d-OR gate in an occurring order derived from such a change in its value contradicts the actuality.

Thus, all these possibilities are apparent only on the description, however contradict the actuality. Hereafter we will explain (i) and (ii).

(i) In the first-half phase, a d-basic event changes its value between zero and a natural number, as shown in (14) and (15). From (5) and (7), it is understood that a d-AND gate and a d-PAND gate cannot change their values between different natural numbers against a change in their inputs between zero and a natural number. On the other hand, a d-OR gate can change its value between different natural numbers even against a change in its input between zero and a natural number, as seen in the aforementioned above. Hereafter, let “$j$-th-ranked input” denote the input of a d-OR gate, which has been in its occurrence state from the $j$-th earliest time. A d-OR gate can increase its value in the first-half phase if its first-ranked input is restored in the pre-transition state where its multiple inputs are in their occurrence states. That is, the value changes from the (smallest) natural number of its first-ranked input in the pre-transition state to the (second smallest) natural number of its second-ranked input (see (6)). Such a change between different natural numbers arising first in a d-OR gate can cause a similar change in the input of an s-PAND gate, where the d-OR gate is located in the subtree with the s-PAND gate as its topmost element, through similar changes in d-AND and/or d-PAND gates located between the s-PAND gate and the d-OR gate, if they exist.
(ii) A d-OR gate has been in its occurrence state since its first-ranked input occurred. Even if its first-ranked input is restored and its value changes to the (second smallest) value of its second-ranked input, its occurrence state has been preserved and the time when it occurred has not changed until all its inputs are restored. That is, its position in the occurring order against other basic events/gates does not actually change, while such an increase in the value of the d-OR gate implies that its position apparently changes as if it has the second smallest value. In conclusion, (a)' and (b)' in Section 3.2.1 are only apparent possibilities on the description, however they contradict the actuality because they can be caused only by a change in the value of a d-OR gate between different natural numbers. Such a change causes an apparent change in the position of the d-OR gate in the occurring order with preserving its occurrence state, which contradicts the actuality.

4 Extended transition rule

The discussion in Section 3.2 implies that in order to solve the d-OR gate problem, it suffices to avoid an apparent change in the position of a d-OR gate in an occurring order with preserving its occurrence state in the first-half phase of the ordinary transition rule. In particular, when the first-ranked input of a d-OR gate is restored and there exists another second-ranked input, the second-ranked input takes over the position of the d-OR gate in the occurring order corresponding to the value of the first-ranked input, which is lost in the first-half phase of the ordinary transition rule.

Note that an extended transition rule is considered, as mentioned below, for the transitions caused by restorations of d-basic events. The ordinary transition rule can be applied to the transitions caused by occurrences of d-basic events, because the d-OR gate problem does not arise in their cases.

For simplicity, assume the following.

Assumption 4.1 There is only one d-OR gate $G_\ell$ in the analyzed dynamic FT.

Suppose that the d-OR gate $G_\ell$ is located in the subtree with an s-PAND gate, $G_p$, as its topmost element, where $1 \leq p < \ell$.

Assumption 4.2 No repeated basic events exist in the subtree with $G_p$ as its topmost element.

Remark 4.3 In the case where multiple d-OR gates exist in the analyzed dynamic FT, it suffices to apply the extended transition rule, explained below, to each d-OR gate under the condition similar to Assumption 4.2 on repeated basic events.
Suppose that the d-OR gate $G_\ell$ has $n_\ell$ inputs from the left on the FT, $X_{\ell 1}, X_{\ell 2}, \ldots, X_{\ell n_\ell}$. Under Assumptions 4.1 and 4.2, the post-transition state $(i'_1, i'_2, \ldots, i'_{n_d})$ by a restoration of d-B$_k$ ($k = 1, \ldots, n_d$) is obtained from the pre-transition state $(i_1, i_2, \ldots, i_{n_d})$ by the following extended transition rule, which applies an overriding transition rule to solve the d-OR gate problem, if necessary.

**Algorithm 4.4 (Extended transition rule)**

**Step 1:** Check the necessity of overriding transition rule.

**Step 1-1:** Substitute

$$x_{\ell j} = \begin{cases} 
i_q, & X_{\ell j} = B_q \\ g_q, & X_{\ell j} = G_q \ (q \neq 1, \ell), \end{cases}$$

where $g_q$ is the value of $G_q$ in the pre-transition state

$$g_q = f_{T_q}(x_{q 1}, x_{q 2}, \ldots, x_{qn_q}).$$

Then obtain the value of $G_\ell$ in the pre-transition state $(i_1, i_2, \ldots, i_{n_d})$ by

$$g_\ell = f_{d-OR}(x_{\ell 1}, x_{\ell 2}, \ldots, x_{\ell n_\ell})$$

$$\begin{align*}
= & \begin{cases} 
\min_{1 \leq j \leq n_\ell} \{ x_{\ell j} \mid x_{\ell j} > 0 \}, & \sum_{j=1}^{n_\ell} x_{\ell j} > 0 \\
0, & \text{otherwise.}
\end{cases}
\end{align*}$$

**Step 1-2:** Obtain the intermediate state $(\tilde{i}_1, \tilde{i}_2, \ldots, \tilde{i}_{n_d})$ in the first-half phase of the ordinary transition rule (15).

**Step 1-3:** Obtain the value $\tilde{g}_\ell$ of $G_\ell$ in the intermediate state $(\tilde{i}_1, \tilde{i}_2, \ldots, \tilde{i}_{n_d})$ by a manner completely analogous to Step 1-1.

**Step 1-4:** If $g_\ell$ and $\tilde{g}_\ell$ satisfy

$$0 < g_\ell < \tilde{g}_\ell,$$

then proceed to Step 2. Otherwise, proceed to Step 3.

**Step 2:** Overriding transition rule.

**Step 2-1:** Obtain $j_1$ and $j_2$ such that

$$h((x_{\ell 1}, x_{\ell 2}, \ldots, x_{\ell n_\ell}), j_1) = 1, \quad h((x_{\ell 1}, x_{\ell 2}, \ldots, x_{\ell n_\ell}), j_2) = 2.$$
In the pre-transition state \((i_1, i_2, \ldots, i_{nd})\), \(X_{\ell j_1}\) is the first-ranked input, and thus the value of \(G_\ell\) is given by

\[
g_\ell = x_{\ell j_1}. \tag{32}
\]

Also, \(X_{\ell j_2}\) is the second-ranked input.

**Step 2-2:** Follow either (i) or (ii) to partially change the intermediate state \((\tilde{i}_1, \tilde{i}_2, \ldots, \tilde{i}_{nd})\) obtained in Step 1-2.

(i) If \(X_{\ell j_2} = B_q\), assign the following value to \(B_q\):

\[
\tilde{i}_q = g_\ell + \frac{1}{2}. \tag{33}
\]

(ii) If \(X_{\ell j_2} = G_q\), assign values in \([g_\ell, g_\ell + \frac{1}{2}]\) to all the inputs of \(G_q\) by Algorithm 4.5 (below), i.e., all the basic events/gates connected directly to the inputs.

**Step 2-3:** Proceed to Step 3.

**Step 3:** Renumbering. Set the post-transition state \((i'_1, i'_2, \ldots, i'_{nd})\) from the intermediate state \((\tilde{i}_1, \tilde{i}_2, \ldots, \tilde{i}_{nd})\) by (17).

Algorithm 4.5 used in Step 2-2 assigns values in \([z_1, z_2)\) \((0 \leq z_1 < z_2 < \infty)\) to all the inputs of the d-gate \(X_{jk}\) \((k = 1, 2, \ldots, n_j)\), i.e., all the basic events/gates connected directly to the inputs of \(G_j\), as follows.

**Algorithm 4.5** (Value assignment to the inputs of a gate)

Follow either (i) or (ii) for \(X_{jk}\) \((k = 1, 2, \ldots, n_j)\).

(i) If \(X_{jk} = B_q\), assign the following value to \(B_q\):

\[
\tilde{i}_q = z_1 + \frac{z_2 - z_1}{n_j} k. \tag{34}
\]

(ii) If \(X_{jk} = G_q\), assign values in \(\left[z_1 + \frac{z_2 - z_1}{n_j} (k - 1), z_1 + \frac{z_2 - z_1}{n_j} k\right]\) to all the inputs of \(G_q\) by the recursive use of Algorithm 4.5.

*Brief proof of the validity of Algorithm 4.4.* Under Assumption 4.2, the following (a) and (b) can be supposed without loss of generality.

(a) There are the d-basic events, \(B_{\ell'}, B_{\ell'+1}, \ldots, B_{nd}\), and the d-gates, \(G_{\ell+1}, G_{\ell+2}, \ldots, G_r\), in the subtree with the d-OR gate \(G_\ell\) as its topmost element, where \(\ell \leq r \leq m\).
(b) There are the d-basic events, \( B_{p'}, B_{p'+1}, \ldots, B_{e-1} \), and d-gates, \( G_{p+1}, G_{p+2}, \ldots, G_{\ell-1} \), in the subtree with the s-PAND gate \( G_p \) as its topmost element except the subtree considered in (a).

Thus, by defining an appropriate functions \( F_p \), we can write

\[
g_p = F_p(i_{p'}, i_{p'+1}, \ldots, i_{e-1}, g_e).
\]  

If the condition (30) is satisfied, then the multiple inputs of the d-OR gate \( G_\ell \) are in their occurrence states in the pre-transition state. Thus, there exist \( j_1 \) and \( j_2 \) satisfying the equation (31). By a restoration of \( B_k \), the first-ranked input of \( G_\ell \) corresponding to \( j_1 \) is restored, and the value of the second-ranked input corresponding to \( j_2 \) will be the value of \( G_\ell \), i.e., an apparent change in the position of the d-OR gate in an occurring order is caused, by the ordinary transition rule.

In such a case, Algorithm 4.4 changes the basic event values \( \tilde{i}_{p'}, \tilde{i}_{p'+1}, \ldots, \tilde{i}_{n_d} \) so that \( \tilde{g}_\ell = g_\ell + \frac{1}{2} \). Then,

\[
\tilde{g}_p = F_p(\tilde{i}_{p'}, \tilde{i}_{p'+1}, \ldots, \tilde{i}_{e-1}) = F_p(i_{p'}, i_{p'+1}, \ldots, i_{e-1}, g_\ell + \frac{1}{2})
\]  

(36)

because \( i_{k'} = \tilde{i}_{k'} \) \( (k' = p', p'+1, \ldots, \ell - 1) \). The values \( i_{p'}, i_{p'+1}, \ldots, i_{e-1} \) are non-negative integers, and thus the magnitude relation between \( g_\ell + \frac{1}{2} \) and them is equivalent to that between \( g_\ell \) and them. It implies that \( \tilde{g}_p \) is determined, i.e., whether the s-PAND gate \( G_p \) is in its occurrence or non-occurrence state is judged, from the occurring order in the pre-transition state, where the d-OR gate \( G_\ell \) has its correct position (without a possible apparent change by the ordinary transition rule). Thus,

\[
\tilde{g}_p = F_p(i_{p'}, i_{p'+1}, \ldots, i_{e-1}, g_\ell).
\]  

(37)

As a result,

\[
\tilde{g}_p = g_p.
\]  

(38)

In such a case where there is the possibility that the d-OR gate problem is caused by the ordinary transition rule, the s-PAND gate does not occur or is not restored by the extended transition rule, which is in perfect agreement with the actuality.

Note that the overriding transition rule assigns new values to the basic events located in the subtree with the d-OR gate \( G_\ell \) as its topmost element. Then, the values of such basic events do not represent their positions in an occurring order, as shown in (1). However, the overriding transition rule preserves the distinction between their occurrence or non-occurrence states. Thus, in Assumption 4.2, the basic events located in the subtree with the s-PAND gate \( G_p \) as its topmost element can also be located elsewhere except in the subtrees with other PAND gates as their topmost elements.
Remark 4.6 In [7], an overriding transition rule was presented to solve Issue 1 in the simplest case, where a d-OR gate has only two inputs connected directly to basic events. In order to solve the d-OR gate problem, this paper generalizes it to the extended transition rule, Algorithm 4.4, as follows.

- The new overriding transition rule, as introduced above, is based on the gate values, and can be applied to a more general and complicated structure of the subtree with a d-OR gate as its topmost element, where more than two basic events and AND/PAND gates exist.

- The new overriding transition rule and the ordinary one are unified, where the necessity of using the new overriding transition rule is checked in the extended transition rule.

5 Example

Consider a dynamic FT shown in Figure 2.

![Figure 2: Example of dynamic FT.](image)

The true occurrence condition of the s-PAND gate $G_1$ on the FT is that $B_1$, $G_2$, $B_2$ have occurred in this order.
There are nine d-basic events, B₁, B₂, ..., B₉, in the dynamic FT, and thus the state on a state transition diagram in Section 2.3 is described by (i₁, i₂, ..., i₉). Then, there are 986410 states on the Markov diagram [7].

The gate values are as follows:

\[
g₁ = f_{s-PAND}(i₁, g₂, i₂) \quad (39)\\
g₂ = f_{d-OR}(i₃, g₃, g₅) \quad (40)\\
g₃ = f_{d-AND}(i₄, g₄, i₅) \quad (41)\\
g₄ = f_{d-PAND}(i₆, i₇) \quad (42)\\
g₅ = f_{d-PAND}(i₈, i₉). \quad (43)
\]

The set \( S = \{(i₁, i₂, ..., i₉) \mid g₁ = 1\} \) contains 130844 top-event occurrence states. Note that the occurrence condition of G₁ on the description is \( 0 < i₁ < g₂ < i₂ \) in (39).

5.1 Solution to d-OR gate problem

It will be shown in Sections 5.1.1 and 5.1.2 that the d-OR gate problem can be solved by applying the extended transition rule in Section 4 to the d-OR gate G₂.

5.1.1 Issue 1

Consider a transition from a state \((5, 7, 4, 3, 6, 1, 7, 0, 2) \notin S\) against a restoration of B₃.

In the pre-transition state, G₂ (by B₃), B₁, and B₂ have occurred in that order. Thus, the true occurrence condition of the PAND gate G₁ on the FT is not satisfied, and the top event is not in its occurrence state. Even if B₄ is restored, the d-OR gate G₂ preserves its occurrence state by G₃. Because there is no change in that order, the true occurrence condition is not yet satisfied and the top event does not occur.

Hereafter, we will obtain the post-transition state by the extended transition rule, Algorithm 4.4.

**Step 1:** Check the necessity of overriding transition rule.

**Step 1-1:** We have \( g₄ = 7, g₅ = 0, g₃ = 7, \) and \( g₂ = 4 \) in the pre-transition state \((i₁, i₂, ..., i₉) = (5, 7, 4, 3, 6, 1, 7, 0, 2)\).

**Step 1-2:** Because \( \tilde{i₃} = 0 \) in the first-half phase of the ordinary transition rule (15), the intermediate state is \((\tilde{i₁}, \tilde{i₂}, ..., \tilde{i₉}) = (5, 8, 0, 3, 6, 1, 7, 0, 2)\).

**Step 1-3:** We have \( \tilde{g₄} = 7, \tilde{g₅} = 0, \tilde{g₃} = 7, \) and \( \tilde{g₂} = 7 \) in the intermediate state obtained in Step 1-2.
Step 1-4: Because $0 < g_2 < \bar{g}_2$, we proceed to Step 2 for the overriding transition rule\(^2\).

Step 2: Overriding transition rule.

Step 2-1: We can obtain $j_1 = 1$ such that $h((i_3, g_3, g_5), j_1) = 1$, i.e., the first input of $G_2$ from the left, $B_3$, is the first-ranked input in the pre-transition state, then $g_2 = i_3 = 4$. We can also obtain $j_2 = 2$ such that $h((i_3, g_3, g_5), j_2) = 2$, i.e., the second input of $G_2$ from the left, $G_3$, is the second-ranked input.

Step 2-2: In order to make the second-ranked input $G_3$ take over the position of the d-OR gate in the occurring order corresponding to the value $g_2 = 4$, we assign values in $(4, 4 + \frac{1}{2}]$ to all the inputs of $G_3$ by Algorithm 4.5 as follows.

1. We assign the value $\widetilde{i}_4 = 4\frac{1}{6}$ to the first input of $G_3$ from the left, $B_4$.
2. We consider the second input of $G_3$ from the left, $G_4$, and assign values in $(4\frac{1}{6}, 4\frac{1}{3}]$ to all the inputs of $G_4$ by Algorithm 4.5 as follows.
   1. We assign the value $\widetilde{i}_6 = 4\frac{1}{4}$ to the first input of $G_4$ from the left, $B_6$.
   2. We assign the value $\widetilde{i}_7 = 4\frac{1}{3}$ to the second input of $G_4$ from the left, $B_7$.
3. We assign the value $\widetilde{i}_5 = 4\frac{1}{2}$ to the third input of $G_3$ from the left, $B_5$.

As a result, we can obtain the intermediate state $(\widetilde{i}_1, \widetilde{i}_2, \ldots, \widetilde{i}_9) = (5, 8, 0, 4\frac{1}{6}, 4\frac{1}{2}, 4\frac{1}{4}, 4\frac{1}{3}, 0, 2)$.

Step 3: Renumbering. Using (17), we can obtain the post-transition state $(i'_1, i'_2, \ldots, i'_9) = (6, 7, 0, 2, 5, 3, 4, 0, 1)$.

In the post-transition state obtained above, $g'_2 = 5$, and thus the occurrence condition of $G_1$ on the description, $0 < i'_1 < g'_2 < i'_2$, is not satisfied, i.e., this state does not belong to $S$. It implies that the top event does not occur, which is in perfect agreement with the actuality.

This example indicates that Issue 1 is solved.

\(^2\)If the overriding transition rule is not applied, i.e., by the ordinary transition rule, from renumbering of the intermediate state $(i_1, i_2, \ldots, i_9) = (5, 8, 0, 3, 6, 1, 7, 0, 2)$, the post-transition state is obtained by $(i'_1, i'_2, \ldots, i'_9) = (4, 7, 0, 3, 5, 1, 6, 0, 2)$. In this state, $g'_2 = 6$, and thus the occurrence condition of $G_1$ on the description, $0 < i'_1 < g'_2 < i'_2$, is satisfied, i.e., this state belongs to $S$. It implies that the top event occurs contrary to the actuality (see Section 3.1.1).
5.1.2 Issue 2

Consider a transition from a state \((3, 6, 5, 7, 2, 1, 0, 4, 8) \in S\) against a restoration of \(B_3\).

In the pre-transition state, \(B_1, G_2\) (by \(B_3\)), and \(B_2\) have occurred in that order. Thus, the true occurrence condition of the PAND gate \(G_1\) on the FT is satisfied, and the top event is in its occurrence state. Even if \(B_3\) is restored, the d-OR gate \(G_2\) preserves its occurrence state by \(G_5\). Because there is no change in that order, the true occurrence condition is still satisfied and the top event is not restored.

Hereafter, we will obtain the post-transition state by the extended transition rule, Algorithm 4.4.

Step 1: Check the necessity of overriding transition rule.

Step 1-1: We have \(g_4 = 0, g_5 = 8, g_3 = 0,\) and \(g_2 = 5\) in the pre-transition state \((i_1, i_2, \ldots, i_9) = (3, 6, 5, 7, 2, 1, 0, 4, 8)\).

Step 1-2: Because \(\tilde{i}_3 = 0\) in the first-half phase of the ordinary transition rule (15), the intermediate state is \((\tilde{i}_1, \tilde{i}_2, \ldots, \tilde{i}_9) = (3, 6, 0, 7, 2, 1, 0, 4, 8)\).

Step 1-3: We have \(\tilde{g}_4 = 0, \tilde{g}_5 = 8, \tilde{g}_3 = 0,\) and \(\tilde{g}_2 = 8\) in the intermediate state obtained in Step 1-2.

Step 1-4: Because \(0 < g_2 < \tilde{g}_2\), we proceed to Step 2 for the overriding transition rule\(^3\).

Step 2: Overriding transition rule.

Step 2-1: We can obtain \(j_1 = 1\) such that \(h((i_3, g_3, g_5), j_1) = 1\), i.e., the first input of \(G_2\) from the left, \(B_3\), is the first-ranked input in the pre-transition state, then \(g_2 = i_3 = 5\). We can also obtain \(j_2 = 3\) such that \(h((i_3, g_3, g_5), j_2) = 2\), i.e., the third input of \(G_2\) from the left, \(G_5\), is the second-ranked input.

Step 2-2: In order to make the second-ranked input \(G_5\) take over the position of the d-OR gate in the occurring order corresponding to the value \(g_2 = 5\), we assign values in \((5, 5 + \frac{1}{2})\) to all the inputs of \(G_5\) by Algorithm 4.5 as follows.

---

\(^3\)If the overriding transition rule is not applied, i.e., by the ordinary transition rule, from renumbering of the intermediate state \((\tilde{i}_1, \tilde{i}_2, \ldots, \tilde{i}_9) = (3, 6, 0, 7, 2, 1, 0, 4, 8)\), the post-transition state is obtained by \((i'_1, i'_2, \ldots, i'_9) = (3, 5, 0, 6, 2, 1, 0, 4, 7)\). In this state, \(g'_2 = 7\), then the occurrence condition of \(G_1\) on the description, \(0 < i'_1 < g'_2 < i'_2\), is not satisfied, i.e., this state does not belong to \(S\). It implies that the top event is restored contrary to the actuality (see Section 3.1.2).
(1) We assign the value \( \tilde{i}_8 = 5 \frac{1}{4} \) to the first input of \( G_5 \) from the left, \( B_8 \).

(2) We assign the value \( \tilde{i}_9 = 5 \frac{1}{2} \) to the second input of \( G_5 \) from the left, \( B_9 \).

As a result, we can obtain the intermediate state \((\tilde{i}_1, \tilde{i}_2, \ldots, \tilde{i}_9) = (3, 6, 0, 7, 2, 1, 0, 5 \frac{1}{4}, 5 \frac{1}{2})\).

**Step 3:** Renumbering. Using (17), we can obtain the post-transition state \((i'_1, i'_2, \ldots, i'_9) = (3, 6, 0, 7, 2, 1, 0, 4, 5)\).

In the post-transition state obtained above, \( g'_2 = 4 \), and thus the occurrence condition of \( G_1 \) on the description, \( 0 < i'_1 < g'_2 < i'_2 \), is still satisfied, i.e., this state belongs to \( S \). It implies that the top event is not restored, which is in perfect agreement with the actuality.

This example indicates that Issue 2 is solved.

### 5.2 Markov analysis and its verification by Monte Carlo simulation

Consider a state transition diagram based on the extended transition rule applied to the d-OR gate \( G_2 \). There are 986410 states, \( S_1 (= (0, 0, \ldots, 0)) \), \( S_2 \), \ldots, \( S_{986410} \), on the diagram. Let \( p_i(t) \) denote the probability that the system described by the dynamic FT is in \( S_i \) on the diagram at the time \( t \). Define the probabilities vector \( p(t) = [p_1(t) \ p_2(t) \ \cdots \ p_{986410}(t)] \). Then, we can derive the differential equation \( \frac{dp(t)}{dt} = p(t)Q \) from the diagram, where \( Q \) is the transition rates matrix. The initial condition is \( p(0) = [1 \ 0 \ \cdots \ 0] \). The Markov analysis based on the differential equation provides the following result [8].

**(a)** The mean time to first occurrence (MTTFO [6]) of the top event is obtained by the mean time from the start to a first transition into one of the states belonging to the set \( S \).

**(b1)** The steady probability that the top event is in its occurrence state is obtained by \( \sum_{S_i \in S} p_i(\infty) \).

**(b2)** The steady frequency of occurrences of the top event is obtained by the total sum of the steady frequencies of transitions into \( S_i \in S \) from \( S_j \not\in S \).
The occurrence and restoration rates of the basic events are as follows:

\[
\begin{align*}
\lambda_1 &= 0.2 \text{ [1/hr]} , \quad \mu_1 = 0.05 \text{ [1/hr]} , \\
\lambda_2 &= 0.1 \text{ [1/hr]} , \quad \mu_2 = 0.07 \text{ [1/hr]} , \\
\lambda_3 &= 0.3 \text{ [1/hr]} , \quad \mu_3 = 0.08 \text{ [1/hr]} , \\
\lambda_4 &= 0.4 \text{ [1/hr]} , \quad \mu_4 = 0.09 \text{ [1/hr]} , \\
\lambda_5 &= 0.5 \text{ [1/hr]} , \quad \mu_5 = 0.10 \text{ [1/hr]} , \\
\lambda_6 &= 0.1 \text{ [1/hr]} , \quad \mu_6 = 0.06 \text{ [1/hr]} , \\
\lambda_7 &= 0.2 \text{ [1/hr]} , \quad \mu_7 = 0.05 \text{ [1/hr]} , \\
\lambda_8 &= 0.3 \text{ [1/hr]} , \quad \mu_8 = 0.09 \text{ [1/hr]} , \\
\lambda_9 &= 0.4 \text{ [1/hr]} , \quad \mu_9 = 0.10 \text{ [1/hr]} .
\end{align*}
\]

On the other hand, to verify the result obtained by the Markov analysis with

<table>
<thead>
<tr>
<th>Table 1: Analysis result for dynamic FT shown in Figure 2.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Mean time to first top-event occurrence</strong></td>
</tr>
<tr>
<td>---------------------------------------------</td>
</tr>
<tr>
<td>Top-event probability</td>
</tr>
<tr>
<td>Top-event frequency</td>
</tr>
</tbody>
</table>

the extended transition rule, we performed Monte Carlo simulation based on the ordinary transition rule with the following constraint conditions.

- If once $G_2$ occurs when $B_1$ is not in its occurrence state, $G_1$ can occur only after a restoration of $G_2$. The top event does not occur, even against a transition from a state outside $S$ to a state belonging to $S$, until $G_2$ is restored.

- If once the top event occurs, it can be restored only by a restoration of $B_1$, $B_2$, or $G_2$. Its occurrence state is preserved, even against a transition from a state belonging to $S$ to a state outside $S$, until one of $B_1$, $B_2$, and $G_2$ is restored.

We performed the simulation until the top event occurred one million times both for (a) and for (b1), (b2).

Table 1 shows the results obtained by the Markov analysis and the Monte Carlo simulation. There is good agreement between these results. This implies that the extended transition rule solves the d-OR gate problem and provides the correct analysis result.

### 6 Conclusions

The d-OR gate problem, as pointed out in this paper for the first time, is inherent in dynamic FTs with PAND gates and restorable basic events.
is no other way to obtain the correct analysis result for dynamic FTs that includes d-OR gates. Although more efficient solutions can be further studied, it has been shown that the extended transition rule succeeds in establishing the applicability of Markov analysis to such dynamic FTs.

References


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