Common Fixed Point Theorem for Two, Three and Four Maps in Fuzzy Metric Spaces

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Abstract

In this paper we have tried to prove the existence of common fixed point for a pair of self mapping under strict contractive and occasionally weakly compatibility condition under the concept of fuzzy metric space. Also we have proved, common fixed point theorem without completeness and E-A property for a pair, triplet and quadruplet of self mappings in fuzzy metric space. The results improve Vijayraju and Sajath [17].

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1. Introduction

In 1982, Sessa [15] introduced the concept of weakly commuting mappings which extends the notion of commuting mappings.

After 4 years, Jungck [6] defined compatible mappings as an extension of weakly commuting mappings.

Later on, the same author with Murthy and Cho [7] gave another extension of weakly commuting mappings under the name of compatible mappings of type (A).

Again, Pathak and Khan [13] extended compatible of type (A) mappings to compatible mappings of type (B).

On this direction, Pathak et al., [14] introduced the new concept as compatible type of (C) as an another extension of compatible type of (A) and proved a common fixed point theorem in a Banach space.
In their paper [8], Jungck and Rhoades defined the notion of weakly compatible mappings as an extension of all the results of above all them.

Again recently, Al-Thagafi and Shahzad [2] generalized the results of above authors by excluding the concept weakly compatible mappings to occasionally weakly compatible mappings.

In this paper we have extended the results of Vijayraju P and Sajath Z M.I. [17] in more general space fuzzy metric space and establish the existence of common fixed point for a pair, triplet and quadruplet of self mapping.

2. Preliminaries

Definition 2.1 Self mappings $f$ and $g$ of a metric space $(X, d)$ are said to be weakly commuting pair if,
$$d(fgx, gfx) \leq d(fx, gx).$$
for all $x \in X$

Definition 2.2 Self mappings $f$ and $g$ of a metric space $(X, d)$ are said to be compatible if,
$$\lim_{n \to \infty} d(fgx_n, gfx_n) = 0.$$ 
whenever $\{x_n\}$ is a sequence in $X$ such that
$$\lim_{n \to \infty} fx_n = \lim_{n \to \infty} gx_n = t$$
for some $t \in X$.

Definition 2.3 Self mappings $f$ and $g$ of a metric space $(X, d)$ are said to be compatible of type (A) if,
$$\lim_{n \to \infty} d(fgx_n, g^2x_n) = 0 \quad \text{and} \quad \lim_{n \to \infty} d(gfx_n, f^2x_n) = 0,$$
whenever $\{x_n\}$ is a sequence in $X$ such that
$$\lim_{n \to \infty} fx_n = \lim_{n \to \infty} gx_n = t$$
for some $t \in X$.

Definition 2.4 Self mappings $f$ and $g$ of a metric space $(X, d)$ are said to be compatible of type (B) if,
$$\lim_{n \to \infty} d(fgx_n, g^2x_n) \leq \frac{1}{2} \left[ \lim_{n \to \infty} d(fgx_n, ft) + \lim_{n \to \infty} d(ft, f^2x_n) \right]$$
and
\[ \lim_{n \to \infty} d(fgx_n, f^2x_n) \leq \frac{1}{2} \left[ \lim_{n \to \infty} d(gfx_n, gt) + \lim_{n \to \infty} d(gt, g^2x_n) \right] \]

whenever \( \{x_n\} \) is a sequence in \( X \) such that
\[ \lim_{n \to \infty} f_{x_n} = \lim_{n \to \infty} g_{x_n} = t \]
for some \( t \in X \).

**Definition 2.5** Self mappings \( f \) and \( g \) of a metric space \( (X, d) \) are said to be compatible of type (C) if,
\[ \lim_{n \to \infty} d(fgx_n, g^2x_n) \leq \frac{1}{3} \left[ \lim_{n \to \infty} d(fgx_n, ft) + \lim_{n \to \infty} d(ft, f^2x_n) + \lim_{n \to \infty} d(ft, g^2x_n) \right] \]
and
\[ \lim_{n \to \infty} d(gfx_n, f^2x_n) \leq \frac{1}{3} \left[ \lim_{n \to \infty} d(gfx_n, gt) + \lim_{n \to \infty} d(gt, g^2x_n) + \lim_{n \to \infty} d(gt, f^2x_n) \right] \]
whenever \( \{x_n\} \) is a sequence in \( X \) such that
\[ \lim_{n \to \infty} f_{x_n} = \lim_{n \to \infty} g_{x_n} = t \]
for some \( t \in X \).

**Definition 2.6** Self mappings \( f \) and \( g \) of a metric space \( (X, d) \) are said to be weakly compatible if they commute at their coincidence points.

**Definition 2.7** Two self mappings \( f \) and \( g \) of a set \( X \) are occasionally weakly compatible (shortly (owc)) iff, there is a point \( t \) in \( X \) which is a coincidence point of \( f \) and \( g \) at which \( f \) and \( g \) commute.

**Definition 2.8** A binary operation \( * : [0, 1] \times [0, 1] \to [0, 1] \) is called a continuous t norm if \( ( [0,1] , *) = 0 \) is an abelian topological monoid with unit 1 such that \( a * b \leq c * d \) whenever \( a \leq c \) and \( b \leq d \) for all \( a, b, c, d \in [0,1] \).
Examples : t-norms are \( a * b = ab \) and \( a * b = \min\{a,b\} \)

**Definition 2.9[4]** The three tuple \( (X, M, *) \) is called a fuzzy metric space, if \( X \) is an arbitrary set, \( * \) is a continuous t-norm and \( M \) is a fuzzy set in \( X^2 \times [0, \infty] \) satisfying the following condition : for all \( x, y, z \) in \( X \) and \( s, t > 0 \).
Common fixed point theorem

(i) $M(x, y, 0) = 0$.
(ii) $M(x, y, t) = 1$, for all $t > 0$, if and only if $x = y$.
(iii) $M(x, y, t) = M(y, x, t)$
(iv) $M(x, y, t) * M(y, z, s) \leq M(x, z, t + s)$
(v) $M(x, y, .) : [0, \infty) \rightarrow [0, 1]$ is left continuous.
(vi) $\lim_{t \to \infty} M(x, y, t) = 1$.

Lemma 2.10 [5] $M(x, y, .)$ is non-decreasing for all $x, y$ in $X$.

Lemma 2.11 [12] Let $\{x_n\}$ be a sequence in fuzzy metric space $(X, M, *)$ with $t * t \geq t$ for all $t \in [0,1]$ and condition (vi). If there exists a number $q \in (0,1)$ such that

$M(x_{n+2}, x_{n+1}, qt) \geq M(x_{n+1}, x_n, t)$

for all $t > 0$ and $n = 1, 2, \ldots$ then $\{x_n\}$ is a Cauchy sequence in $X$.

Lemma 2.12 [12] If for all $x, y \in X$, $t > 0$ with positive number $q \in (0,1)$ and $M(x, y, qt) \geq M(x, y, t)$ then $x = y$.

Definition 2.13 Two self mappings $S$ and $T$ of a fuzzy metric space are said to be commuting if $M(STx, TSx, t) = 1$ for all $t > 0$ and for all $x \in X$.

Definition 2.14 Two self mappings $S$ and $T$ of a fuzzy metric space are said to be commuting if $M(STx, TSx, t) \geq M(Sx, Tx, t)$ for all $t > 0$ and for all $x \in X$.

Clearly two commuting mappings are weakly commuting.

Definition 2.15 Let $T$ and $S$ be two self mappings of a fuzzy metric space $(X, M, *)$. Then $S$ and $T$ are said to be compatible if $\lim_{n \to \infty} M(STx_n, TSx_n, t) = 1$ whenever $\{x_n\}$ is a sequence in $X$ such that $\lim_{n \to \infty} Sx_n = \lim_{n \to \infty} Tx_n = x_0$.

Obviously two weakly commuting mappings are compatible.

Definition 2.16 Let Two self mappings $T$ and $S$ of a fuzzy metric space $(X, M, *)$. Then $S$ and $T$ are said to be weakly compatible if they commute at their
coincidence points; i.e.; if $M(Tu, Su, t) = 1$ implise $M(TSu, STu, t) = 1$ for $t > 0$.

It is easy to see that two compatible maps are weakly compatible.

**Definition 2.17** Let Two self mappings $T$ and $S$ of a fuzzy metric space $(X, M, \ast)$. Then $S$ and $T$ are said to be occasionally weakly compatible if there is a point $u$ for which $M(Tu, Su, t) = 1$ implise $M(TSu, STu, t) = 1$ for $t > 0$.

It is easy to see that two weakly compatible maps are occasionally weakly compatible.

**Definition 2.18** Let $X$ be a set. A symmetric on $X$ is a mapping $d : X \times X \rightarrow [0, \infty)$ such that $d(x, y) = 0$. iff $x = y$ and $d(x, y) = d(y, x)$ for all $x, y$ in $X$.

**Definition 2.19** The 3-tuple $(X, M, \ast)$ is called fuzzy symmetric space if $X$ is an arbitrary Set, $\ast$ is a continuous t-norm and $M$ is a fuzzy set in $X^2 \times [0, \infty]$ satisfying the following condition : for all $x, y, z$ in $X$ and $s, t > 0$.

- (i) $M(x, y, 0) = 0$.
- (ii) $M(x, y, t) = 1$, for all $t > 0$, if and only if $x = y$.
- (iii) $M(x, y, t) = M(y, x, t)$
- (iv) $M(x, y, .) : [0, \infty) \rightarrow [0, 1]$ is left continuous.
- (v) $\lim_{t \rightarrow \infty} M(x, y, t) = 1$.

3 Main Result

Now, we give our main result.

**Common fixed point for two mappings:**

**Theorem 3.1** Let $A$ and $S$ be two self owc mappings of a fuzzy metric space $(X, M, \ast)$ with $t \ast t \geq t$ such that for each $x \neq y$ in $X$, $t > 0$ and for $0 < q < 1$.

\[
M(Ax, Ay, qt) \geq \min \{M(Sx, Sy, t), M(Sx, Ay, t), M(Sy, Ay, t), M(Ax, Sx, t), M(Ax, Sy, t)\}
\]

Then $A$ and $S$ have a unique common fixed point.
Common fixed point theorem

Proof: Since A and S are owc so there exit a \( \in X \) such that \( Aa = Sa \) implies \( ASa = SAa \).

That is there exist a \( \in X \) such that \( M(Aa, Sa, t) = 1 \) implies \( M(ASa, SAa, t) = 1 \) for \( t > 0 \).

and since \( Sa = Aa \Rightarrow SSa = SAa \) and \( ASa = AAa \)

Thus \( SSa = SAa = ASa = AAa \).

Now we show that \( AAa = Sa \) is common fixed point of A and S. Suppose that \( AAa \neq Sa \). Then by inequality (3.1.1)

\[
M(Aa, AAa, qt) \geq \min \{ M(Sa, AAa, t), M(Sa, AAa, t), M(AAa, AAa, t), M(Aa, Sa, t), M(Aa, AAa, t) \}
\]

\[
= \min \{ M(Aa, AAa, t), M(Aa, AAa, t), M(AAa, AAa, t), M(Aa, Aa, t), M(Aa, AAa, t) \}
\]

\[
= \min \{ M(Aa, AAa, t), M(Aa, AAa, t), 1, 1, M(Aa, AAa, t) \}
\]

\[
M(Aa, AAa, qt) \geq M(Aa, AAa, t)
\]

Then by lemma 2.12

\( Aa = AAa \)

Thus \( AAa = SAa = Aa \).

Hence \( AAa = Sa \) is common fixed point of A and S.

Finally we show that the fixed point is unique.

Let \( x_0 \) and \( y_0 \) be two common fixed points of A and S.

Then \( Ax_0 = Sx_0 = x_0 \) and \( Ay_0 = Sy_0 = y_0 \) and by (3.1.1)

\[
M(Ax_0, Ay_0, qt) \geq \min \{ M(Sx_0, Ay_0, t), M(Sx_0, Ay_0, t), M(Sy_0, Ay_0, t), M(Sy_0, Ay_0, t) \}
\]

\[
= \min \{ M(Ax_0, Ay_0, t), M(Ax_0, Ay_0, t), M(Ay_0, Ay_0, t), M(Ax_0, Ay_0, t) \}
\]

\[
= \min \{ M(Ax_0, Ay_0, t), M(Ax_0, Ay_0, t), 1, 1, M(Ax_0, Ay_0, t) \}
\]

\[
M(Ax_0, Ay_0, qt) \geq M(Ax_0, Ay_0, t)
\]

Then by lemma (2.12) \( Ax_0 = Ay_0 \) \( i.e. \ x_0 = y_0 \) Proved.

Example: Let \( X = [1, \infty] \) define A, S : \( X \rightarrow X \) by \( Ax = x^2 \) and \( Sx = 3x - 2 \) for all \( x \in X \).

Let the fuzzy metric \( M(x, y, t) = \frac{1}{t + |x - y|} \) Then for all \( x \neq y \)

A and S satisfy the condition (3.1.1) and \( Ax = Sx \) iff \( x = 1 \) or \( x = 2 \).

\( AS(1) = SA(1) = 1 \) but \( AS(2) \neq SA(2) \) \( i.e. \ 1 \) is common fixed point A and S.
Common fixed point for three mappings:

**Theorem 3.2** Let A, B and S be three self-mappings of a fuzzy metric $(X, M, *)$ with $t * t \geq t$ such that for each $x \neq y$ in $X$, $t > 0$ and for $0 < q < 1$.

\[
M(Ax, By, qt) \geq \min \{M(Sx, Sy, t), M(Sx, By, t), M(Sy, By, t),
M(Ax, Sx, t), M(Sy, Ax, t)\} \quad \text{...(3.2.1)}
\]

And pair $(A, S)$ or $(B, S)$ is owc pair. \quad \text{...(3.2.2)}

Then A, B and S have a unique common fixed point.

**Proof:** Since $(A, S)$ is owc pair [from 3.2.2]

Then there is an element $u \in X$ such that $Au = Su$ and $ASu = SAu$.

First, we prove that $Au = Bu = Su$.

Indeed, by inequality (3.2.1) we get

\[
M(Au, Bu, qt) \geq \min \{M(Su, Su, t), M(Su, Bu, t), M(Su, Bu, t),
M(Au, Su, t), M(Su, Au, t)\}
= \min \{1, M(Au, Bu, t), M(Au, Bu, t), M(Au, Au, t), M(Au, Au, t)\}
= \min \{1, M(Au, Bu, t), M(Au, Bu, t), 1, 1\}
\]

\[
M(Au, Bu, qt) \geq M(Au, Bu, t)
\]

Then by lemma 2.12 $Au = Bu$ i.e. $Au = Bu = Su$.

Thus $ASu = SAu = ABu = SBu = AAu$

Now suppose that $BAu \neq AAu$. Then from (3.2.1) we get

\[
M(AAu, BAu, qt) \geq \min \{M(SAu, SAu, t), M(SAu, BAu, t), M(SAu, BAu, t),
M(AAu, SAu, t), M(SAu, AAu, t)\}
= \min \{M(AAu, AAu, t), M(AAu, BAu, t), M(AAu, BAu, t),
M(AAu, AAu, t), M(AAu, AAu, t)\}
= \min \{1, M(AAu, AAu, t), M(AAu, AAu, t), 1, 1\}
\]

\[
M(AAu, BAu, qt) \geq M(AAu, BAu, t)
\]

Hence by lemma 2.12 $AAu = BAu$ Thus $AAu = BAu = SAu$

If $AAu \neq Bu$ Again we have from (3.2.1)

\[
M(AAu, Bu, qt) \geq \min \{M(SAu, Su, t), M(SAu, Bu, t), M(SAu, Bu, t),
M(AAu, SAu, t), M(Su, AAu, t)\}
= \min \{M(AAu, Bu, t), M(AAu, Bu, t), M(AAu, Bu, t),
M(AAu, AAu, t), M(Bu, AAu, t)\}
= \min \{M(AAu, Bu, t), M(AAu, Bu, t), M(AAu, Bu, t), 1,
M(AAu, Bu, t)\}
\]

\[
M(AAu, Bu, qt) \geq M(AAu, Bu, t)
\]
Then by lemma 2.12
AAu = Bu.

i.e. AAu = Au = Bu = Su.
or AAu = BAu = SAu = Au = a (Let)
So, a = Au is common fixed point of mappings A, B and S.

**Uniqueness:**

Now Let $x_0, y_0$ be two distinct common fixed points of mappings A, B and S.

i.e. $Ax_0 = Bx_0 = Sx_0 = x_0$
and $Ay_0 = By_0 = Sy_0 = y_0$

So by condition (3.2.1)

$$M(Ax_0, By_0, qt) \geq \min \{M(Sx_0, Sy_0, t), M(Sx_0, By_0, t), M(Sy_0, By_0, t),$$
$$M(Ax_0, Sx_0, t), M(Sy_0, Ax_0, t)\}$$
$$\quad = \min \{M(Ax_0, By_0, t), M(Ax_0, By_0, t), M(By_0, By_0, t),$$
$$M(Ax_0, Ax_0, t), M(By_0, Ax_0, t)\}$$
$$\quad = \min \{M(Ax_0, By_0, t), M(Ax_0, By_0, t), 1, 1, M(Ax_0, By_0, t)\}$$

Then by lemma 2.12 $x_0 = y_0$.

**Common fixed point for four mappings:**

**Theorem 3.3** Let $A, B, S$ and $T$ be four self mappings of a fuzzy metric space $(X, M, *)$ with $t * t \geq t$ such that for each $x \neq y$ in $X$, $t > 0$ and for $0 < q < 1$.

$$M(Ax, By, qt) \geq \min \{M(Sx, Ty, t), M(Sx, By, t), M(Ty, By, t),$$
$$M(Sx, Ax, t), M(Ty, Ax, t)\}$$

...(3.3.1)

And pairs $(A, S)$ and $(B, T)$ are owc. ...(33.2)

Then $A, B, S$ and $T$ have a unique common fixed point.

Proof: Since $(A, S), (B, T)$ is owc pair [from 3.3.2]

Then there is an element $u, v \in X$ such that $Au = Su$ and $ASu = SAu,$ $Bv = Tv$ and $BTv = TBv.$

First, we prove that $Au = Bv$

Indeed, by inequality (3.3.1) we get

$$M(Au, Bv, qt) \geq \min \{M(Su, Tv, t), M(Su, Bv, t), M(Tv, Bv, t),$$
$$M(Su, Au, t), M(Tv, Au, t)\}$$
\[= \min \{M(Au, Bv, t), M(Au, Bv, t), M(Bv, Bv, t),
M(Au, Au, t), M(Bv, Au, t)\}\]
\[= \min \{M(Au, Bv, t), M(Au, Bv, t), 1, 1, M(Au, Bv, t)\}\]
\[M(Au, Bv, qt) \geq M(Au, Bv, t).\]

Then from lemma 2.12

Hence \(Au = Su = Bv = Tv.\)

New suppose that \(AAu \neq Au.\) By using inequality (2.3.1) we obtain

\[Au = Bv\]

\[M(AAu, Bv, qt) \geq \min \{M(SAu, Tv, t), M(SAu, Bv, t), M(Tv, Bv, t),
M(SAu, AAu, t), M(Tv, AAu, t)\}\]
\[= \min \{M(ASu, Bv, t), M(ASu, Bv, t), M(Bv, Bv, t),
M(ASu, AAu, t), M(Bv, AAu, t)\}\]
\[= \min \{M(AAu, Bv, t), M(AAu, Bv, t), M(Bv, Bv, t),
M(AAu, AAu, t), M(Bv, AAu, t)\}\]
\[= \min \{M(AAu, Bv, t), M(AAu, Bv, t), 1, 1, M(AAu, Bv, t)\}\]
\[M(AAu, Bv, qt) \geq M(AAu, Bv, t).\]

So by lemma (2.12)

\(AAu = Bv\)

Since \(Au = Bv\) So, \(AAu = Au = ASu = SAu\)

Similarly \(BAu = TAu = Au\)

Therefore \(Au = Su = Bv = Tv\) is a common fixed point of mapping \(A, B, S\) and \(T.\)

Put \(Au = Su = Bv = Tv = x,\) then \(Ax = Sx = Bx = Tx = x.\)

Uniqueness:

Let \(x_0\) and \(y_0\) are two common fixed points of \(A, B, S\) and \(T\) such that \(x_0 \neq y_0\) then

\(x_0 = Ax_0 = Sx_0 = Bx_0 = Tx_0\) and \(y_0 = Ay_0 = Sy_0 = By_0 = Ty_0\)

From condition (3.3.1) we have

\[M(Ax_0, By_0, qt) \geq \min \{M(Sx_0, Ty_0, t), M(Sx_0, By_0, t), M(Ty_0, By_0, t),
M(Sx_0, Ax_0, t), M(Ty_0, Ax_0, t)\}\]
\[= \min \{M(Ax_0, By_0, t), M(Ax_0, By_0, t), M(By_0, By_0, t),
M(Ax_0, Ax_0, t), M(By_0, Ax_0, t)\}\]
\[= \min \{M(Ax_0, By_0, t), M(Ax_0, By_0, t), 1, M(Ax_0, By_0, t)\}\]

\[M(Ax_0, By_0, qt) \geq M(Ax_0, By_0, t)\]

Then by lemma 2.12. \(Ax_0 = By_0\)

Thus \(A, S, B, T\) have unique common fixed point.
Remark 3.4:
If we put $S = T$ in the statement of theorem of 3.3 then we can get statement of theorem 3.2.

Remark 3.5:
If we put $A = B$ and $S = T$ in the statement of theorem of 3.4 then we can get statement of theorem 3.1.

Remark 3.6:
If we put $A = B$ in the statement of theorem of 3.2 then we can get statement of theorem 3.1.

Thus theorem 3.2 and theorem 3.3 are generalizations of theorem 3.1.

The next theorem involves a function $F : [0,1] \rightarrow [0,1]$ satisfying the following conditions:

(i) $F$ is increasing on $[0,1]$

(ii) $F(t) > t$, $\forall \ t \in (0,1]$ and $F(1) = 1$.

Theorem 3.7:
Let $A$ and $S$ be two self owc mappings of a fuzzy symmetric space $(X, M, *)$ with $t * t \geq t$ such that for each $x \neq y$ in $X$, $t > 0$.

(i) a function $F : [0,1] \rightarrow [0,1]$ satisfying the following conditions:

(a) $F$ is increasing on $[0,1]$

(b) $F(t) > t$, $\forall \ t \in (0,1]$ and $F(1) = 1$.

and $M(Ax, Ay, t) > F[min \{M(Sx, Sy, t), M(Sx, Ay, t), M(Sy, Ay, t), M(Ax, Sx, t), M(Ax, Sy, t)\}]$ ...(3.7.1)

Then $A$ and $S$ have a unique common fixed point.

Theorem 3.8:
Let $A$, $B$ and $S$ be three self mappings of a fuzzy symmetric space $(X, M, *)$ with $t * t \geq t$ such that for each $x \neq y$ in $X$, $t > 0$ and function $F : [0,1] \rightarrow [0,1]$ satisfying the following conditions:

(a) $F$ is increasing on $[0,1]$

(b) $F(t) > t$, $\forall \ t \in (0,1]$ and $F(1) = 1$.

$M(Ax, By, t) > F[min \{M(Sx, Sy, t), M(Sx, By, t), M(Sy, By, t), M(Ax, Sx, t), M(Sy, Ax, t)\}]$ ...(3.8.1)

and pair $(A, S)$ or $(B, S)$ is owc pair. ...(3.8.2)

Then $A$, $B$ and $S$ have a unique common fixed point.
Theorem 3.9 Let \( A, B, S \) and \( T \) be four self mappings of a fuzzy metric space \((X, M, \ast)\) with \( t \ast t \geq t \) such that for each \( x \neq y \) in \( X \), \( t > 0 \) and function \( F : [0,1] \to [0,1] \) satisfying the following conditions:

(a) \( F \) is increasing on \([0,1]\)

(b) \( F(t) > t, \ \forall \ t \in (0,1] \) and \( F(1) = 1 \).

\[
M(Ax, By, t) > F\left[\min\{M(Sx, Ty, t), M(Sx, By, t), M(Ty, By, t), M(Sx, Ax, t), M(Ty, Ax, t)\}\right] \quad \text{...(3.9.1)}
\]

and pairs \((A, S)\) and \((B, T)\) are owc. \quad \text{...(3.9.2)}

Then \( A, B, S \) and \( T \) have a unique common fixed point.

References


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