Some Results on Intuitionistic Fuzzy BCI-(Positive Implicative, Implicative, Commutative) Ideals in BCI-Algebras

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Abstract

In this paper, we interrelate the intuitionistic fuzzification of the concept of BCI-(positive implicative, implicative, commutative) ideals in BCI-algebras and investigate some of their properties.

Keywords: Intuitionistic fuzzy set, Intuitionistic fuzzy ideal, Intuitionistic fuzzy BCI-(positive implicative, implicative, commutative) ideal

1 Introduction and Preliminaries

In 1966, Y. Imai and K. Iseki introduced the notion of BCK-algebra 1. Until 1966, various algebras were introduced after considering the properties of the union and the intersection. The case of set difference and its properties remained unexplored. The consideration of these operations and its properties by K. Iseki gave birth to the notion of a BCK-algebra. In 1966, K. Iseki introduced another notion called BCI-algebra, which is in fact a generalization the notion of a BCK-algebra. After the introduction of the concept of fuzzy sets by Zadeh, several researches were conducted on the generalization of the notion of fuzzy sets. The idea of ”intuitionistic fuzzy set” was first published by Atanassov2, 3, as a generalization of the notion of fuzzy sets. After that many researchers considered the fuzzification of ideals and subalgebras in BCK(BCI)-algebras.
In this paper, we interrelate the concept of intuitionistic fuzzy BCI-(positive implicative, implicative, commutative) ideals in BCI-algebras and investigate some of its properties.

Let us recall that an algebra \((X, *, 0)\) of type \((2, 0)\) is called a BCI-algebra if it satisfies the following conditions:

1.1. \(((x * y) * (x * z)) * (z * y) = 0\)
1.2. \((x * (x * y)) * y = 0\)
1.3. \(x * x = 0\)
1.4. \(x * y = 0\) and \(y * x = 0\) imply \(x = y\) for all \(x, y, z \in X\).

In a BCI-algebra, we can define a partial ordering "\(\leq\)" by \(x \leq y\) if and only if \(x * y = 0\). In a BCI-algebra \(X\), the set \(M = \{x \in X \mid 0 * x = 0\}\) is a subalgebra and is called the BCK-part of \(X\). A BCI-algebra \(X\) is called proper if \(X - M \neq \emptyset\). Otherwise it is improper. Moreover, in a BCI-algebra the following conditions hold:

1.5. \((x * y) * z = (x * z) * y\)
1.6. \(x * 0 = x\)
1.7. \(x \leq y\) implies \(x * z \leq y * z\) and \(z * y \leq z * x\)
1.8. \(0 * (x * y) = (0 * x) * (0 * y)\)
1.9. \(0 * (0 * (x * y)) = 0 * (y * x)\)
1.10. \((x * z) * (y * z) \leq x * y\)

A mapping \(f : X \rightarrow Y\) of BCI-algebras is called a homomorphism if \(f(x * y) = f(x) * f(y)\) for all \(x, y \in X\).

By a fuzzy set \(\mu\) in a nonempty set \(X\) we mean a function \(\mu : X \rightarrow [0, 1]\) and the complement of \(\mu\), denoted by \(\bar{\mu}\), is the fuzzy set in \(X\) given by \(\bar{\mu}(x) = 1 - \mu(x)\) for all \(x \in X\).

**Basic definitions** Basic definitions related to fuzzy \(p\)-ideals, fuzzy \(q\)-(or \(H\))-ideals and fuzzy \(a\)-(or \(\alpha\))-ideals can be seen in 9. Also the basic definitions related to intuitionistic fuzzy sets, intuitionistic fuzzy \(\alpha\)-(or \(a\))-ideals and intuitionistic fuzzy \(H\)-(or \(q\))-ideals can be seen in 5, 3 and 4 respectively whereas the basic definitions related to fuzzy BCI-(positive implicative, implicative, commutative) ideals can be seen in 7. We have already submitted a paper on "intuitionistic fuzzy BCI-positive implicative ideals in BCI-algebras" in your journal. Here we define only intuitionistic fuzzy BCI-(positive implicative, implicative, commutative) ideals and give some results which interrelate them with each other and with intuitionistic fuzzy \((p, H, \alpha)\) ideals.

**2. Intuitionistic fuzzy BCI-positive implicative ideal** An IFSA =
(\alpha_A, \beta_A) in a BCI-algebra X is called an intuitionistic fuzzy BCI-positive implicative ideal of X if it satisfies:

(IFBCI-PI-1) \alpha_A(0) \geq \alpha_A(x) and \beta_A(0) \leq \beta_A(x)

(IFBCI-PI-2) \alpha_A(x * z) \geq \min\{\alpha_A(((x * z) * z) * (y * z)), \alpha_A(y)\}

(IFBCI-PI-3) \beta_A(x * z) \leq \max\{\beta_A(((x * z) * z) * (y * z)), \beta_A(y)\} for all x, y \in X.

Example 2.1 consider the BCI-algebra X = \{0, 1, 2, 3\} with the following cayley table:

\begin{center}
\begin{array}{cccc}
* & 0 & 1 & 2 & 3 \\
\hline
0 & 0 & 0 & 0 & 3 \\
1 & 1 & 0 & 0 & 3 \\
2 & 2 & 2 & 0 & 3 \\
3 & 3 & 3 & 3 & 0 \\
\end{array}
\end{center}

Define an IFSA = (\alpha_A, \beta_A) in X as follows:

\alpha_A(0) = \alpha_A(3) = 1, \alpha_A(1) = \alpha_A(2) = t

\beta_A(0) = \beta_A(3) = 0, \beta_A(1) = \beta_A(2) = s

where s, t \in (0, 1) and s + t \leq 1

By routine calculations it is easy to verify that IFSA = (\alpha_A, \beta_A) is an intuitionistic fuzzy BCI-positive implicative ideal of X.

An IFSA = (\alpha_A, \beta_A) in X is called an intuitionistic fuzzy closed BCI-positive implicative ideal of X if it satisfies

(IFBCI-PI-2) , (IFBCI-PI-3) and

(IFBCI-PI-4) \alpha_A(0 * x) \geq \alpha_A(x) and \beta_A(0 * x) \leq \beta_A(x)

for all x \in X.

2.2. Intuitionistic fuzzy BCI-implicative ideal An IFSA = (\alpha_A, \beta_A) in a BCI-algebra X is called an intuitionistic fuzzy BCI-implicative ideal of X if it satisfies:

(IFBCI-I-1) \alpha_A(0) \geq \alpha_A(x) and \beta_A(0) \leq \beta_A(x)

(IFBCI-I-2) \alpha_A(x * ((y * (y * x)) * (0 * (0 * (x * y))))) \geq \min\{\alpha_A(((x * y) * y) * (0 * y)) * z), \alpha_A(z)\}

(IFBCI-I-3) \beta_A(x * ((y * (y * x)) * (0 * (0 * (x * y))))) \leq \max\{\beta_A(((x * y) * y) * (0 * y)) * z), \beta_A(z)\} for all x, y \in X.

Example 2.3 consider the BCI-algebra X = \{0, 1, 2, 3\} with the following cayley table:
Define an IFSA = \((\alpha_A, \beta_A)\) in \(X\) as follows:
\[
\alpha_A(0) = \alpha_A(1) = 1, \alpha_A(2) = \alpha_A(3) = t
\]
\[
\beta_A(0) = \beta_A(1) = 0, \beta_A(2) = \beta_A(3) = s
\]
where \(s, t \in (0, 1)\) and \(s + t \leq 1\)

By routine calculations it is easy to verify that IFSA = \((\alpha_A, \beta_A)\) is an intuitionistic fuzzy BCI-implicative ideal of \(X\).

An IFSA = \((\alpha_A, \beta_A)\) in \(X\) is called an intuitionistic fuzzy closed BCI-implicative ideal of \(X\) if it satisfies
\[
(IFBCI-I-2), (IFBCI-I-3)\text{ and } (IFBCI-I-4)
\]
\[
\alpha_A(0 \ast x) \geq \alpha_A(x) \text{ and } \beta_A(0 \ast x) \leq \beta_A(x)
\]
for all \(x \in X\).

2.4. Intuitionistic fuzzy BCI-commutative ideal
An IFSA = \((\alpha_A, \beta_A)\) in a BCI-algebra \(X\) is called an intuitionistic fuzzy BCI-commutative ideal of \(X\) if it satisfies:
\[
(IFBCI-C-1) \quad \alpha_A(0) \geq \alpha_A(x) \text{ and } \beta_A(0) \leq \beta_A(x)
\]
\[
(IFBCI-C-2) \quad \alpha_A(x \ast (y \ast (y \ast x)) \ast (0 \ast (0 \ast (x \ast y)))) \geq \min\{\alpha_A((x \ast y) \ast z), \alpha_A(z)\}
\]
\[
(IFBCI-C-3) \quad \beta_A(x \ast (y \ast (y \ast x)) \ast (0 \ast (0 \ast (x \ast y)))) \leq \max\{\beta_A((x \ast y) \ast z), \beta_A(z)\}
\]
for all \(x, y \in X\).

Example 2.5 consider the BCI-algebra \(X = \{0, 1, 2, 3\}\) with the following cayley table:

\[
\begin{array}{c|cccc}
\ast & 0 & 1 & 2 & 3 \\
\hline
0 & 0 & 0 & 0 & 0 \\
1 & 1 & 0 & 0 & 1 \\
2 & 2 & 1 & 0 & 2 \\
3 & 3 & 3 & 3 & 0
\end{array}
\]

Define an IFSA = \((\alpha_A, \beta_A)\) in \(X\) as follows:
\(\alpha_A(0) = \alpha_A(3) = 1, \alpha_A(1) = \alpha_A(2) = t\)
\(\beta_A(0) = \beta_A(3) = 0, \beta_A(1) = \beta_A(2) = s\)
where \(s, t \in (0, 1)\) and \(s + t \leq 1\)

By routine calculations it is easy to verify that \(\text{IFSA} = (\alpha_A, \beta_A)\) is an intuitionistic fuzzy BCI-commutative ideal of \(X\).

An \(\text{IFSA} = (\alpha_A, \beta_A)\) in \(X\) is called an intuitionistic fuzzy closed BCI-commutative ideal of \(X\) if it satisfies (IFBCI - C - 2), (IFBCI - C - 3) and

(IFBCI-C-4) \(\alpha_A(0 * x) \geq \alpha_A(x)\) and \(\beta_A(0 * x) \leq \beta_A(x)\)
for all \(x \in X\).

3. **Some already known results**

Now here we give some already known results which will help us in our results. Here we are not giving the proof of these results. These results are taken from 3, 4, 5, 6, 7 and 9 whereas some of these results have been established in our recently submitted three papers on intuitionistic fuzzy BCI-(positive implicative, implicative, commutative) ideals.

**Lemma 3.1** Let \(\text{IFSA} = (\alpha_A, \beta_A)\) be an intuitionistic fuzzy ideal of \(X\). If the inequality \(x * y \leq z\) holds in \(X\), then \(\alpha_A(x) \geq \min\{\alpha_A(y), \alpha_A(z)\}\)
and \(\beta_A(x) \leq \max\{\beta_A(y), \beta_A(z)\}\)

**Lemma 3.2** Let \(\text{IFSA} = (\alpha_A, \beta_A)\) be an intuitionistic fuzzy ideal of \(X\). If the inequality \(x \leq y\) holds in \(X\), then \(\alpha_A(x) \geq \alpha_A(y)\) and \(\beta_A(x) \leq \beta_A(y)\), that is \(\alpha_A\) is order reversing while \(\beta_A\) is order preserving.

**Theorem 3.3** Every intuitionistic fuzzy BCI-positive implicative ideal of a BCI-algebra \(X\) is an intuitionistic fuzzy ideal of \(X\) but the converse is not true.

**Theorem 3.4** Every intuitionistic fuzzy BCI-implicative ideal of a BCI-algebra \(X\) is an intuitionistic fuzzy ideal of \(X\) but the converse is not true.

**Theorem 3.5** Every intuitionistic fuzzy BCI-commutative ideal of a BCI-algebra \(X\) is an intuitionistic fuzzy ideal of \(X\) but the converse is not true.

**Theorem 3.6** Let \(\text{IFSA} = (\alpha_A, \beta_A)\) be an intuitionistic fuzzy ideal of a BCI-algebra \(X\). Then the following conditions are equivalent:
1. \(\text{IFSA} = (\alpha_A, \beta_A)\) is an intuitionistic fuzzy BCI-positive implicative ideal of \(X\).
2. \(\alpha_A((x * y) * z) \geq \alpha_A(((x * z) * z) * (y * z)))\)
and
\(\beta_A((x * y) * z)) \leq \beta_A(((x * z) * z) * (y * z)))\)
3. \(\alpha_A((x * y) * z)) = \alpha_A(((x * z) * z) * (y * z)))\)
and
\[
\beta_A((x * y) * z)) = \beta_A(((x * z) * y) * (y * z))
\]

4. \(\alpha_A(x * y) = \alpha_A(((x * y) * y) * (0 * y))\)
and \(\beta_A(x * y) = \beta_A(((x * y) * y) * (0 * y))\)

5. \(\alpha_A(x * y) \geq \alpha_A(((x * y) * y) * (0 * y))\)
and \(\beta_A(x * y) \leq \beta_A(((x * y) * y) * (0 * y))\)

for all \(x, y, z \in X\).

**Theorem 3.7** Let IFSA = \((\alpha_A, \beta_A)\) be an intuitionistic fuzzy ideal of a BCI-algebra \(X\). Then the following conditions are equivalent:

1. IFSA = \((\alpha_A, \beta_A)\) is an intuitionistic fuzzy BCI-implicative ideal of \(X\).
2. \(\alpha_A(x * ((y * (y * x)) * (0 * (0 * (x * y)))))) \geq \alpha_A(((x * y) * y) * (0 * y))\)
and \(\beta_A((x * ((y * (y * x)) * (0 * (0 * (x * y)))))) \leq \beta_A(((x * y) * y) * (0 * y))\)
3. \(\alpha_A(x * ((y * (y * x)) * (0 * (0 * (x * y)))))) = \alpha_A(((x * y) * y) * (0 * y))\)
and \(\beta_A((x * ((y * (y * x)) * (0 * (0 * (x * y)))))) = \beta_A(((x * y) * y) * (0 * y))\)

for all \(x, y \in X\).

**Theorem 3.8** Let IFSA = \((\alpha_A, \beta_A)\) be an intuitionistic fuzzy ideal of a BCI-algebra \(X\). Then the following conditions are equivalent:

1. IFSA = \((\alpha_A, \beta_A)\) is an intuitionistic fuzzy BCI-commutative ideal of \(X\).
2. \(\alpha_A(x * ((y * (y * x)) * (0 * (0 * (x * y)))))) \geq \alpha_A(x * y)\)
and \(\beta_A((x * ((y * (y * x)) * (0 * (0 * (x * y)))))) \leq \beta_A(x * y)\)
3. \(\alpha_A(x * ((y * (y * x)) * (0 * (0 * (x * y)))))) = \alpha_A(x * y)\)
and \(\beta_A((x * ((y * (y * x)) * (0 * (0 * (x * y)))))) = \beta_A(x * y)\)

for all \(x, y \in X\).

**Theorem 3.9** Let IFSA = \((\alpha_A, \beta_A)\) be an intuitionistic fuzzy ideal of a BCI-algebra \(X\). Then the following conditions are equivalent:

1. IFSA = \((\alpha_A, \beta_A)\) is an intuitionistic fuzzy H-ideal of \(X\).
2. \(\alpha_A((x * y) * z) \geq \alpha_A(x * (y * z))\) and \(\beta_A((x * y) * z) \leq \beta_A(x * (y * z))\)
3. \(\alpha_A(x * y) \geq \alpha_A(x * (0 * y))\) and \(\beta_A(x * y) \leq \beta_A(x * (0 * y))\)

for all \(x, y, z \in X\).

**Theorem 3.10** Let IFSA = \((\alpha_A, \beta_A)\) be an intuitionistic fuzzy ideal of a BCI-algebra \(X\). Then the following conditions are equivalent:

1. IFSA = \((\alpha_A, \beta_A)\) is an intuitionistic fuzzy p-ideal of \(X\).
2. \(\alpha_A(x) \geq \alpha_A(0 * (0 * x))\) and \(\beta_A(x) \leq \beta_A(0 * (0 * x))\)

for all \(x \in X\).
Theorem 3.11 Let IFSA = (α_A, β_A) be an intuitionistic fuzzy ideal of a BCI-algebra X. Then the following conditions are equivalent:
1. IFSA = (α_A, β_A) is an intuitionistic fuzzy α-ideal of X.
2. α_A(y*(x*z)) ≥ α_A((x*z)*(y*0)) and β_A(y*(x*z)) ≤ β_A((x*z)*(y*0))
3. α_A(y*x) ≥ α_A(x*(y*0)) and β_A(y*x) ≤ β_A(x*(0*y))
for all x, y, z ∈ X.

Theorem 3.12 Every intuitionistic fuzzy α-ideal of a BCI-algebra X is an intuitionistic fuzzy p-ideal of X but the converse of this Theorem is not true.

Theorem 3.13 Every intuitionistic fuzzy α-ideal of a BCI-algebra X is an intuitionistic fuzzy H-ideal of X but the converse of this Theorem is not true.

Theorem 3.14 An IFSA = (α_A, β_A) in a BCI-algebra X is an intuitionistic fuzzy α-ideal of X if and only if IFSA = (α_A, β_A) is both an intuitionistic fuzzy p-ideal and an intuitionistic fuzzy H-ideal of X.

4. Some new results Now here we give some results interrelating intuitionistic fuzzy BCI-positive implicative, implicative, commutative) ideals with each other and with intuitionistic fuzzy p-ideal and intuitionistic fuzzy α-ideal.

Theorem 4.1 Every intuitionistic fuzzy BCI-implicative ideal of a BCI-algebra X is an intuitionistic fuzzy BCI-commutative ideal of X but the converse of this Theorem is not true.

Proof. Let IFSA = (α_A, β_A) be an intuitionistic fuzzy BCI-implicative ideal of a BCI-algebra X. Then by Theorem 3.7(2) we have
α_A(x*(y*(y*x))*(0*(0*(x*y)))) ≥ α_A(((x*y)*y)*(0*y)) and
β_A(x*(y*(y*x))*(0*(0*(x*y)))) ≤ β_A(((x*y)*y)*(0*y))
for all x, y ∈ X.

Consider ((x*y)*y)*(0*y) = ((x*y)*y)*(0*y) ≤ x*y (By 1.1)
⇒ α_A(((x*y)*y)*(0*y)) ≥ α_A(x*y) and β_A(((x*y)*y)*(0*y)) ≤ β_A(x*y)
Hence we get
α_A(x*((y*(y*x))*(0*(0*(x*y)))) ≥ α_A(x*y) and β_A(x*((y*(y*x))*(0*(0*(x*y)))) ≤ β_A(x*y)

Therefore by Theorem 3.8 IFSA = (α_A, β_A) is an intuitionistic fuzzy BCI-commutative ideal of X. Whereas the converse of this is not true in general.

To examine this consider the example 2.5. By routine calculations it can easily be verified that IFSA = (α_A, β_A) is an intuitionistic fuzzy BCI-commutative ideal of X but it is not an intuitionistic fuzzy BCI-implicative ideal of X because

α_A(2*(1*(1*2))*(0*(0*(2*1)))) = α_A(1) = t < 1 = α_A(0) = min{α_A(((2*1)*1)*(0*1))*, 0}, α_A(0)
Theorem 4.2 Every intuitionistic fuzzy BCI-implicative ideal of a BCI-algebra X is an intuitionistic fuzzy BCI-positive implicative ideal of X but the converse of this Theorem is not true.

**Proof.** Let IFSA = \((\alpha_A, \beta_A)\) be an intuitionistic fuzzy BCI-implicative ideal of BCI-algebra X. Then \(\alpha_A(x \ast ((y \ast (y \ast x)) \ast (0 \ast (0 \ast (x \ast y)))) \geq \alpha_A(((x \ast y) \ast y) \ast (0 \ast y))\) and 
\(\beta_A(x \ast ((y \ast (y \ast x)) \ast (0 \ast (0 \ast (x \ast y)))) \leq \beta_A(((x \ast y) \ast y) \ast (0 \ast y))\)
for all \(x, y \in X\).
Consider \((y \ast (y \ast x)) \ast (0 \ast (0 \ast (x \ast y))) = (y \ast (y \ast x)) \ast (0 \ast (y \ast x))\) (By 1.9)
\(\Rightarrow (y \ast (y \ast x)) \ast (0 \ast (0 \ast (x \ast y))) \leq y \ast 0 = y\) (By 1.10)
\(\Rightarrow x \ast y \leq x \ast ((y \ast (y \ast x)) \ast (0 \ast (0 \ast (x \ast y))))\)
\(\Rightarrow \alpha_A(x \ast y) \geq \alpha_A(x \ast ((y \ast (y \ast x)) \ast (0 \ast (0 \ast (x \ast y))))\) and \(\beta_A(x \ast y) \leq \beta_A(x \ast ((y \ast (y \ast x)) \ast (0 \ast (0 \ast (x \ast y))))\)
Hence we get
\(\alpha_A(x \ast y) \geq \alpha_A(((x \ast y) \ast y) \ast (0 \ast y))\) and \(\beta_A(x \ast y) \leq \beta_A(((x \ast y) \ast y) \ast (0 \ast y))\).
Therefore by Theorem 3.6 IFSA = \((\alpha_A, \beta_A)\) is an intuitionistic fuzzy BCI-positive implicative ideal of X. Whereas the converse of this Theorem is not true in general. To examine this consider the example 2.1. By routine calculations it can be easily verified that IFSA = \((\alpha_A, \beta_A)\) is an intuitionistic fuzzy BCI-positive implicative ideal of X but it is not an intuitionistic fuzzy BCI-implicative ideal of X because
\(\alpha_A(1 \ast ((2 \ast (2 \ast 1)) \ast (0 \ast (0 \ast (1 \ast 2)))) = \alpha_A(1) = t < 1 = \alpha_A(0) = min\{\alpha_A(((1 \ast 2) \ast 2) \ast (0 \ast 2)) \ast 0\}, \alpha_A(0)\}\)

**Theorem 4.3** An IFSA = \((\alpha_A, \beta_A)\) in a BCI-algebra X is an intuitionistic fuzzy BCI-implicative ideal of X if and only if IFSA = \((\alpha_A, \beta_A)\) is both an intuitionistic fuzzy BCI-positive implicative ideal and an intuitionistic fuzzy BCI-commutative ideal of X.

**Proof.** Suppose that IFSA = \((\alpha_A, \beta_A)\) is an intuitionistic fuzzy BCI-implicative ideal of X. Then by Theorem 4.1 and Theorem 4.2 IFSA = \((\alpha_A, \beta_A)\) is both an intuitionistic fuzzy BCI-commutative ideal and an intuitionistic fuzzy BCI-positive implicative ideal of X.
Conversely suppose that IFSA = \((\alpha_A, \beta_A)\) is both an intuitionistic fuzzy BCI-commutative ideal and an intuitionistic fuzzy BCI-positive implicative ideal of X. Then by Theorem 3.8 we have
\(\alpha_A(x \ast ((y \ast (y \ast x)) \ast (0 \ast (0 \ast (x \ast y)))) \geq \alpha_A(x \ast y)\) (a) and
\(\beta_A(x \ast ((y \ast (y \ast x)) \ast (0 \ast (0 \ast (x \ast y)))) \leq \beta_A(x \ast y)\) (b)
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Since is also an intuitionistic fuzzy BCI-positive implicative ideal of X. Therefore by Theorem 3.6(5) we have
\[ \alpha_A(x * y) \geq \alpha_A(((x * y) * y) * (0 * y)) \] (c)
and
\[ \beta_A(x * y) \leq \beta_A(((x * y) * y) * (0 * y)) \] (d)
From (a) and (c) and (b) and (d) we have
\[ \alpha_A(x * ((y * (y * x)) * (0 * (x * y)))) \geq \alpha_A(((x * y) * y) * (0 * y)) \]
and
\[ \beta_A(x * ((y * (y * x)) * (0 * (0 * (x * y)))))) \leq \beta_A(((x * y) * y) * (0 * y)) \]
Hence by Theorem 3.7 IFSA = (\( \alpha_A, \beta_A \)) is an intuitionistic fuzzy BCI-implicative ideal of X.

**Theorem 4.4** Any intuitionistic fuzzy p-ideal of a BCI-algebra X is an intuitionistic fuzzy BCI-implicative ideal of X but the converse is not true.

**Proof.** Let IFSA = (\( \alpha_A, \beta_A \)) be an intuitionistic fuzzy p-ideal of a BCI-algebra X. Then we know that IFSA = (\( \alpha_A, \beta_A \)) will also be an intuitionistic fuzzy ideal of X. In order to prove that IFSA = (\( \alpha_A, \beta_A \)) is an intuitionistic fuzzy BCI-implicative ideal of X, it is sufficient to prove that
\[ \alpha_A(x * ((y * (y * x)) * (0 * (0 * (x * y))))) \geq \alpha_A(((x * y) * y) * (0 * y)) \]
and
\[ \beta_A(x * ((y * (y * x)) * (0 * (0 * (x * y)))))) \leq \beta_A(((x * y) * y) * (0 * y)) \]
for all \( x, y \in X \). (From Theorem 3.7)
Since \((0 * (0 * (x * y))))*(((x * y) * y) * (0 * y)) = (0 * (0 * (x * y))))*(((x * y) * y) * (0 * y)) = (0 * (0 * (x * y))))*(((x * y) * y) * (0 * y)) = (0 * (0 * (x * y))))*(((x * y) * y) * (0 * y)) = (0 * (0 * (x * y))))*(((x * y) * y) * (0 * y)) = (0 * (0 * (x * y))))*(((x * y) * y) * (0 * y)) = 0 \Rightarrow (0 * (0 * (x * y))))*(((x * y) * y) * (0 * y)) = 0 \Rightarrow \alpha_A(0 * (0 * (x * y))))*(((x * y) * y) * (0 * y)) = \beta_A(0 * (0 * (x * y))))*(((x * y) * y) * (0 * y)) = (0 * (0 * (x * y))))*(((x * y) * y) * (0 * y)) = (0 * (0 * (x * y))))*(((x * y) * y) * (0 * y)) = (0 * (0 * (x * y))))*(((x * y) * y) * (0 * y)) (By lemma 3.2)
Now since \( 0 * ((0 * (x * (y * (y * x)) * (0 * (0 * (x * y)))))) = 0 * ((0 * (x * (y * (y * x)) * (0 * (0 * (x * y)))))) \)
\( \leq (0 * ((y * (y * x)) * (0 * (0 * (x * y)))))) * (0 * (x * x)) = (0 * ((0 * (y * (y * x)) * (0 * (0 * (x * y)))))) * (0 * (x * x)) \) (By 1.1)
\( = (((0 * (y * (y * x))) * (0 * (0 * (0 * (x * y)))))) * (0 * (x * x)) = (((0 * (y * (y * x))) * (0 * (0 * (x * x)))) * (0 * (x * x)) \) (By 1.8)
\( = 0 * (0 * (y * (y * x))) * (0 * (x * x)) \) (By 1.5)
\( \Rightarrow 0 * (0 * (x * (y * (y * x)) * (0 * (0 * (x * y)))))) \leq 0 * (0 * (x * y)) \)
Therefore by lemma 3.2
\[ \alpha_A(0 \ast (0 \ast (x \ast ((y \ast (y \ast x)) \ast (0 \ast (0 \ast (x \ast y))))))) \geq \alpha_A(0 \ast (0 \ast (x \ast y))) \]
and
\[ \beta_A(0 \ast (0 \ast (x \ast ((y \ast (y \ast x)) \ast (0 \ast (0 \ast (x \ast y))))))) \leq \beta_A(0 \ast (0 \ast (x \ast y))) \]
Now since IFSA = (\( \alpha_A, \beta_A \)) is an intuitionistic fuzzy p-ideal of X so by using Theorem 3.10 we get
\[ \alpha_A(x \ast ((y \ast (y \ast x)) \ast (0 \ast (0 \ast (x \ast y)))))) \geq \alpha_A(0 \ast (0 \ast (x \ast y))) \]
and
\[ \beta_A(x \ast ((y \ast (y \ast x)) \ast (0 \ast (0 \ast (x \ast y)))))) \leq \beta_A(0 \ast (0 \ast (x \ast y))) \]
Therefore by Theorem 3.7 IFSA = (\( \alpha_A, \beta_A \)) is an intuitionistic fuzzy BCI-implicative ideal of X.
Whereas the converse of this Theorem may not be true. For this consider the following example:
Consider the BCI-algebra X = \{0, 1, 2, 3\} with the following cayley table:

<table>
<thead>
<tr>
<th>*</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>2</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>3</td>
<td>2</td>
<td>0</td>
</tr>
</tbody>
</table>

Define an IFSA = (\( \alpha_A, \beta_A \)) in X as follows:
\[ \alpha_A(0) = \alpha_A(3) = 1, \alpha_A(1) = \alpha_A(2) = t \]
\[ \beta_A(0) = \beta_A(3) = 0, \beta_A(1) = \beta_A(2) = s \]
where \( s, t \in (0, 1) \) and \( s + t \leq 1 \)
By routine calculations it is easy to verify that IFSA = (\( \alpha_A, \beta_A \)) is an intuitionistic fuzzy BCI-implicative ideal of X but it is not an intuitionistic fuzzy p-ideal of X because:
\[ \alpha_A(1) = t < 1 = \alpha_A(0) = min\{\alpha_A((1 \ast 3) \ast (0 \ast 3), \alpha_A(0))\}. \]

**Corollary 4.5** Any intuitionistic fuzzy p-ideal of a BCI-algebra X is an intuitionistic fuzzy BCI-positive implicative ideal of X but the converse is not true.

**Proof.** It is obvious since every intuitionistic fuzzy BCI-implicative ideal is
an intuitionistic fuzzy BCI-positive implicative ideal.

**Corollary 4.6** Any intuitionistic fuzzy p-ideal of a BCI-algebra X is an intuitionistic fuzzy BCI-commutative ideal of X but the converse is not true.

**Proof.** It is obvious since every intuitionistic fuzzy BCI-implicative ideal is an intuitionistic fuzzy BCI-commutative ideal.

**Theorem 4.7** Any intuitionistic fuzzy \( \alpha \)-ideal of a BCI-algebra X is an intuitionistic fuzzy BCI-implicative ideal of X but the converse is not true.

**Proof.** By Theorem 3.12, Since every intuitionistic fuzzy \( \alpha \)-ideal of a BCI-algebra X is an intuitionistic fuzzy p-ideal of X and by the Theorem 4.4 every intuitionistic fuzzy p-ideal of X is an intuitionistic fuzzy BCI-implicative ideal of X. Therefore every intuitionistic fuzzy \( \alpha \)-ideal of X is an intuitionistic fuzzy BCI-implicative ideal of X. Whereas the converse of this does not hold in general and can be seen by the following example:

Consider the BCI-algebra X = \{0, 1, 2, 3\} with the following cayley table:

\[
\begin{array}{c|cccc}
* & 0 & 1 & 2 & 3 \\
\hline
0 & 0 & 0 & 3 & 2 \\
1 & 1 & 0 & 3 & 2 \\
2 & 2 & 2 & 0 & 3 \\
3 & 3 & 3 & 2 & 0 \\
\end{array}
\]

Define an IFSA = \((\alpha_A, \beta_A)\) in X as follows:
\[
\alpha_A(0) = \alpha_A(3) = 1, \quad \alpha_A(1) = \alpha_A(2) = t
\]
\[
\beta_A(0) = \beta_A(3) = 0, \quad \beta_A(1) = \beta_A(2) = s
\]
where \( s, t \in (0, 1) \) and \( s + t \leq 1 \)

By routine calculations it is easy to verify that IFSA = \((\alpha_A, \beta_A)\) is an intuitionistic fuzzy BCI-implicative ideal of X but it is not an intuitionistic fuzzy \( \alpha \)-ideal of X because:
\[
\alpha_A(1 \ast 3) = \alpha_A(2) = t < 1 = \alpha_A(3) = \min\{\alpha_A((3 \ast 0) \ast (0 \ast 1)), \alpha_A(0)\}
\]

**Corollary 4.8** Any intuitionistic fuzzy \( \alpha \)-ideal of a BCI-algebra X is an intuitionistic fuzzy BCI-positive implicative ideal of X but the converse is not true.

**Proof.** It is obvious since every intuitionistic fuzzy BCI-implicative ideal is an intuitionistic fuzzy BCI-positive implicative ideal.

**Corollary 4.9** Any intuitionistic fuzzy \( \alpha \)-ideal of a BCI-algebra X is an intuitionistic fuzzy BCI-commutative ideal of X but the converse is not true.
**Proof.** It is obvious since every intuitionistic fuzzy BCI-implicative ideal is an intuitionistic fuzzy BCI-commutative ideal.

**REFERENCES**

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