Chaos Synchronization and

Linear Feedback Control

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Abstract

Chaos synchronization of generalized non-linear dynamical systems has been analyzed. The considered synchronization criterion is consisting of identical drive and response systems coupled with linear state error variables. The optimized criteria are applied to explain the Lorenz system.

Keywords: Nonlinear dynamical system, chaos synchronization, positive definite matrix, negative definite matrix, Lyapunov function

1. Introduction

In the study of dynamical systems theory, chaos synchronization of generalized non-linear dynamical systems is interesting as well as important because of theoretical applications. The linear feedback control is a robust and can be easily used to control techniques available for chaos synchronization. Recently, this technique has been widely used to synchronize the identical classical Lorenz...
systems [2, 3, 4, 6 10], Chen systems [8], Lu systems [5, 7] and unified chaotic systems [5].

In this paper, we focus to study the chaos synchronization of generalized nonlinear dynamical systems which can exhibit a Chaotic attractor for different parameter values by using Linear Feedback control Laws. We propose a theorem for synchronization of an n-dimensional generalized nonlinear dynamical system. Numerical simulations are used to verify the theoretical results.

2. Formulation of nonlinear dynamical systems

Consider the drive system:

\[ \dot{x}_d[t] = F(t, x_d[t]), \]  \hspace{1cm} (2.1)

and response system:

\[ \dot{y}_r[t] = F(t, y_r[t]) + u[t] \]  \hspace{1cm} (2.2)

where the subscripts “d” and “r” stand for the drive and response systems respectively. If we denote

\[ x_d[t] = (x_{1d}[t], x_{2d}[t], x_{3d}[t], ..., x_{nd}[t], ..., x_{md}[t])^T \]

\[ y_r[t] = (y_{1r}[t], y_{2r}[t], y_{3r}[t], ..., y_{kr}[t], ..., y_{mr}[t])^T \]

as drive system variable and response system variables respectively and

\[ F(t, x_d[t]) = Ax_d[t] + f(t, x_d[t]) \]  \hspace{1cm} (2.3)

and

\[ F(t, y_r[t]) = Ay_r[t] + f(t, y_r[t]) \]  \hspace{1cm} (2.4)

where

\[ F : R_+ \times R^n \rightarrow R^n \]

is a function that consists of linear and non linear functions \( Ax_d[t] \), \( Ay_r[t] \) and \( f(t, x_d[t]) \), \( f(t, y_r[t]) \) respectively.

\[ u[t] = (u_1[t], u_2[t], u_3[t], ..., u_k[t], ..., u_n[t])^T \]

is a control function added such that response system remain bounded after time \( t \).

The error dynamical system on \( e[t] \) according to (2.1) is given by

\[ e[t] = F(t, x_d[t]) - F(t, y_r[t]) - U. \]  \hspace{1cm} (2.5)
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Where \( U = \{u[t]\} \).

is a generalized n-dimensional nonlinear dynamical system with \( k \) nonlinear functions:

\[
f_i(t, x_d[t]), f_2(t, x_d[t]), f_3(t, x_d[t]), \ldots, f_k(t, x_d[t]); 0 \leq k \leq n.
\]

We decide to add control functions \( u_1[t], u_2[t], \ldots, u_k[t] \) only to the respective states of nonlinearity. Now (2.1) can be restated as identified drive system:

\[
\dot{x}_d[t] = A x_d[t] + \sum_{i=1}^{k} \psi_i(x_d) \varphi_i(\chi) + \theta[I]
\] (2.6)

and (2.2) can also be restated as identified response system:

\[
\dot{y}_r[t] = A y_r[t] + \sum_{i=1}^{k} \psi_i(y_r) \varphi_i(\chi) + \theta[I] + U[I]
\] (2.7)

using (2.6) and (2.7) error system can be reformulated as:

\[
\dot{e}[t] = A e[I] + \sum_{i=1}^{k} \psi_i(e) \varphi_i(\chi) + U[I]
\] (2.8)

**Definition 2.1.** For arbitrary given initial points, \((x_{i_d}[t], x_{2_d}[t], x_{3_d}[t], \ldots, x_{n_d}[t])\) and \((y_{i_r}[t], y_{2_r}[t], y_{3_r}[t], \ldots, y_{n_r}[t]) \in \mathbb{R}^n\), of the drive system (2.6) and the response system (2.7), respectively, if the solution of the error system (2.8) has the estimation \(\sum_{i=1}^{n} e_i^2(t) \leq m(e[t_0]) \exp(-\alpha[t-t_0])\), where \(m(e[t_0]) > 0\), is a constant depending on the initial value \(e[t_0]\), while \(\alpha > 0\) is a constant independent of \(e[t_0]\), then the zero solution of the error system (2.8) is said to be globally, exponentially stable, and thus the drive-response system (2.6) and (2.7) are globally exponentially synchronized.

**Lemma 2.1.** The zero solution of the error dynamical system (2.8) is globally, exponentially stable, i.e. the drive-response systems (2.6) and (2.7) are globally exponentially synchronized if, there exists a positive definite quadratic polynomial \(V = (e_1 e_2 e_3 \ldots e_k \ldots e_n)^T P (e_1 e_2 e_3 \ldots e_k \ldots e_n)\) Such that

\[
\dot{V} = -(e_1 e_2 e_3 \ldots e_k \ldots e_n) Q (e_1 e_2 e_3 \ldots e_k \ldots e_n)^T.
\]

Moreover, the following negative Lyapunov exponent estimation for the error dynamical system (2.8) holds:

\[
\sum_{i=1}^{n} e_i^2(t) \leq \frac{\lambda_{\max}(P)}{\lambda_{\min}(P)} \sum_{i=1}^{n} e_i^2(t_0) \exp\left[-\frac{\lambda_{\min}(Q)}{\lambda_{\max}(P)} (t-t_0)\right].
\]
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Where \( P = P^T \in R^{n \times n} \) and \( Q = Q^T \in R^{n \times n} \) are both positive definite matrices, \( \lambda_{\text{max}}(P) \) and \( \lambda_{\text{min}}(Q) \) stand for the maximum and minimum Eigen values of the matrix \( P \) and \( Q \) respectively.

3. Linear Feedback control laws

**Theorem 3.1** For the given drive system (2.6) consists of \( k \) \((0 \leq k \leq n)\) non linear terms and the response system (2.7) with unknown controllers \( u_1[t], u_2[t], u_3[t], \ldots, u_k[t] \) together with the corresponding error dynamical system (2.8); if \( M_{x_{id}} \) \((i = 1, 2, 3, \ldots, k, \ldots, n)\) are the upper bounds of the state variables \( x_{id} \). Further choosing linear control laws for the response system (2.7); 

\[
\dot{u}_i[t] = h_i e_i[t] \quad \forall i = 1, 2, 3, \ldots, k \text{ and } u_i[t] = 0 \quad \forall i = k + 1, k + 2, k + 3, \ldots, n,
\]

where \( h_1, h_2, h_3, \ldots, h_k \) to be determined such that derivative of Lyapunov polynomial \( L(e[t]) \) is positive definite. Then zero solution of the error dynamical system (2.8) is globally, exponentially stable, and thus the drive system (2.6) and response system (2.7) are globally exponentially synchronized.

**Proof:** suppose (2.6) consists of \( k \) nonlinear terms, then response system (2.7) can be restated after using the control function \( u_i[t] = (u_1[t], u_2[t], \ldots, u_k[t], \ldots, u_n[t])^T \) taken as \( u_i[t] = h_i e_i[t] \quad \forall i = 1, 2, 3, \ldots, k \) and \( u_i[t] = 0 \quad \forall i = k + 1, k + 2, k + 3, \ldots, n \).

Therefore

\[
\dot{y}_i[t] = Ay_i[t] + \sum_{i=1}^{k} \psi_i(y_i) \phi_i(\chi) + Be[t] + \theta[t] \tag{3.1.1}
\]

where \( A = [a_{ij}]_{n \times n \times n}, B = [h_{ij}]_{k \times k} \) \((i \neq j)\).

Now error system (2.8) becomes

\[
\dot{e}[t] = Ae[t] + \sum_{i=1}^{k} \psi_i(e[t]) \phi_i(\chi) + Be[t] \tag{3.1.2}
\]

if we choose Lyapunov function \( L(e[t]) \) such that

\[
L(e[t]) = \frac{1}{2} (e_1^2[t] + e_2^2[t] + e_3^2[t] + \ldots + e_k^2[t] + \ldots + e_n^2[t]) \tag{3.1.3}
\]

or

\[
L(e[t]) = (e_1[t]e_2[t]e_3[t] \ldots e_k[t] \ldots e_n[t])P(e_1[t]e_2[t]e_3[t] \ldots e_k[t] \ldots e_n[t])^T \tag{3.1.4}
\]

where \( P = \text{diag}[0.5, 0.5, 0.5, \ldots, 0.5, \ldots, 0.5] \) and \( \lambda_{\text{max}} = \lambda_{\text{min}} = 0.5 \).

Differentiating (3.1.3) with respect to \( t \) we obtain
Let 

\[ L(e[t]) = e_1[t]\dot{e}_1[t] + e_2[t]\dot{e}_2[t] + e_3[t]\dot{e}_3[t] + \ldots + e_n[t]\dot{e}_n[t] \]


\begin{align*}
&= e_1[t](a_{11}e_1[t] + a_{12}e_2[t] + a_{13}e_3[t] + \ldots + a_{1k}e_k[t] + a_{1k+1}e_{k+1}[t] + \ldots + a_{1n}e_n[t] \\
&\quad + f_1(t, x_d) - f_1(t, y_d) - h_1 e_1[t]) \\
&+ e_2[t](a_{21}e_1[t] + a_{22}e_2[t] + a_{23}e_3[t] + \ldots + a_{2k}e_k[t] + a_{2k+1}e_{k+1}[t] + \ldots + a_{2n}e_n[t] \\
&\quad + f_2(t, x_d) - f_2(t, y_d) - h_2 e_2[t]) \\
&+ \ldots \\
&+ e_k[t](a_{k1}e_1[t] + a_{k2}e_2[t] + a_{k3}e_3[t] + \ldots + a_{kk}e_k[t] + a_{kk+1}e_{k+1}[t] + \ldots + a_{kn}e_n[t] \\
&\quad + f_k(t, x_d) - f_k(t, y_d) - h_k e_k[t]) \\
&+ \ldots \\
&+ e_n[t](a_{n1}e_1[t] + a_{n2}e_2[t] + a_{n3}e_3[t] + \ldots + a_{nk}e_k[t] + a_{nk+1}e_{k+1}[t] + \ldots + a_{nn}e_n[t]) 
\end{align*}

now, if we consider the nonlinear functions \( f_i(t, x_d) - f_i(t, y_d) = g_i(e[t], x_d) \) as follows

\[ f_i(t, x_d) - f_i(t, y_d) \leq g_i(e[t], M_{x_d}); (i = 1, 2, 3, \ldots, k) \] (3.1.6)

using (3.1.6) we obtain the following inequality
\[ L(e_i[t]) = \begin{cases} 
(a_{11} - h_k) e_1^2[t] + a_{12} e_1[t] e_2[t] + a_{13} e_1[t] e_3[t] + \ldots + a_{1k} e_1[t] e_k[t] \\
+ a_{k+1} e_1[t] e_{k+1}[t] + \ldots + a_{in} e_1[t] e_n[t] + e_1[t] (g_1(e_i[t], x_d)) \\
+ a_{21} e_2[t] e_2[t] + (a_{22} - h_k) e_2^2[t] e_2[t] + a_{23} e_2[t] e_3[t] + \ldots + a_{2k} e_2[t] e_k[t] \\
+ a_{2k+1} e_2[t] e_{k+1}[t] + \ldots + a_{n2} e_2[t] e_n[t] + e_2[t] (g_2(e_i[t], x_d)) \\
+ a_{31} e_3[t] e_3[t] + a_{32} e_3[t] e_2[t] + (a_{33} - h_k) e_3^2[t] + \ldots + a_{3k} e_3[t] e_k[t] \\
+ a_{3k+1} e_3[t] e_{k+1}[t] + \ldots + a_{n3} e_3[t] e_n[t] + e_3[t] (g_3(e_i[t], x_d)) \\
\vdots \\
+ a_{k1} e_k[t] e_k[t] + a_{k2} e_k[t] e_2[t] + a_{k3} e_k[t] e_3[t] + \ldots + (a_{k} - h_k) e_k^2[t] \\
+ a_{k+1} e_k[t] e_{k+1}[t] + \ldots + a_{nk} e_k[t] e_n[t] + e_k[t] (g_k(e_i[t], M_{x_k})) \\
\end{cases} \]
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since the terms corresponding to each \( g_1, g_2, g_3, \ldots, g_k \) can be arranged in such a way that polynomial \( \hat{L}(e[t]) \) is positive definite

\[
\hat{L}(e[t]) = (e_1 e_2 \ldots e_k \ldots e_n)Q(e_1 e_2 \ldots e_k \ldots e_n)^T
\]

where \( Q \) is positive definite matrix given by-

\[
Q = \begin{bmatrix}
    a_{11} - h_1 & \frac{1}{2}(a_{12} + M_{s_2}) & \ldots & \frac{1}{2}(a_{1k} + M_{s_k}) & \ldots & \frac{1}{2}(a_{1n} + M_{s_n}) \\
    \frac{1}{2}(a_{21} + M_{s_1}) & a_{22} - h_2 & \ldots & \frac{1}{2}(a_{2k} + M_{s_k}) & \ldots & \frac{1}{2}(a_{2n} + M_{s_n}) \\
    \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\
    \frac{1}{2}(a_{k1} + M_{s_1}) & \frac{1}{2}(a_{k2} + M_{s_2}) & \ldots & a_{kk} - h_k & \ldots & \frac{1}{2}(a_{kn} + M_{s_n}) \\
    \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\
    \frac{1}{2}(a_{n1} + M_{s_1}) & \frac{1}{2}(a_{n2} + M_{s_2}) & \ldots & \frac{1}{2}(a_{nk} + M_{s_k}) & \ldots & a_{nn}
\end{bmatrix}
\]

Hence by lemma (2.1) zero solution of the error system (3.1.2) is globally; exponentially stable. Proof of the Theorem (3.1) is complete.

4. Controlling Lorenz system

The Lorenz system described by the following system of non linear differential equations

\[
\begin{align*}
\dot{x}[t] &= \sigma(y[t] - x[t]) \\
\dot{y}[t] &= rx[t] - y[t] - x[t]z[t] \\
\dot{z}[t] &= -bz[t] + x[t]y[t]
\end{align*}
\]

for the parameter values \( \sigma = 10, r = 28, b = \frac{8}{3} \) has a chaotic attractor portrayed in fig. 4.1(a) and its projection in xy- plane is in fig. 4.1(b).
Now, consider the Lorenz chaotic system

\[
\begin{align*}
\dot{x}_1[t] &= \sigma(x_{2d}[t] - x_{id}[t]) \\
\dot{x}_2[t] &= r x_{id}[t] - x_{2d}[t] - x_{1d}[t] x_{3d}[t] \\
\dot{x}_3[t] &= -b x_{3d}[t] + x_{1d}[t] x_{2d}[t]
\end{align*}
\]

(4.1)
as a drive system and the response system given by

\[
\begin{align*}
\dot{y}_1[t] &= \sigma(y_2[t] - y_1[t]) + u_1[t] \\
\dot{y}_2[t] &= ry_1[t] - y_2[t] - y_3[t] + u_2[t] \\
\dot{y}_3[t] &= -by_3[t] + y_1[t]y_2[t] + u_3[t]
\end{align*}
\] (4.2)

where \( u_i[t] \) (\( i = 1, 2, 3 \)) are unknown feedback controlling functions of \((t, x_{id}, y_{id}) \) \( i = 1, 2, 3 \).

\[
e[t] = (e_1[t], e_2[t], e_3[t])^T = (x_{id}[t] - y_1[t], x_{id}[t] - y_2[t], x_{id}[t] - y_3[t])^T.
\]

Then from (4.1) and (4.2), we obtain the following four error dynamical systems:

\[
\begin{align*}
\dot{e}_1[t] &= \sigma(e_2[t] - e_1[t]) - u_1[t] \\
\dot{e}_2[t] &= re_1[t] - e_2[t] - x_{id}[t]e_3[t] - y_{id}e_1[t] - u_2[t] \\
\dot{e}_3[t] &= -be_3[t] + x_{id}[t]e_2[t] + y_{id}e_1[t] - u_3[t] \\
\dot{e}_4[t] &= \sigma(e_2[t] - e_1[t]) - u_4[t] \\
\dot{e}_5[t] &= re_1[t] - e_2[t] - x_{id}[t]e_3[t] - y_{id}e_1[t] - u_2[t] \\
\dot{e}_6[t] &= -be_3[t] + x_{id}[t]e_2[t] + y_{id}e_1[t] - u_3[t] \\
\dot{e}_7[t] &= \sigma(e_2[t] - e_1[t]) - u_7[t] \\
\dot{e}_8[t] &= re_1[t] - e_2[t] - x_{id}[t]e_3[t] - y_{id}e_1[t] - u_2[t] \\
\dot{e}_9[t] &= -be_3[t] + x_{id}[t]e_2[t] + y_{id}e_1[t] - u_3[t]
\end{align*}
\] (4.3)

using \( u_1 = h_1e_1, u_2 = h_2e_2, u_3 = h_3e_3 \), and \( y_1 = x_1 - e_1, y_2 = x_2 - e_2, y_3 = x_3 - e_3 \), and \( x_1 \leq M_1, x_2 \leq M_2, x_3 \leq M_3 \), these four error systems becomes-

\[
\begin{align*}
\dot{e}_1[t] &= \sigma e_2[t] - (\sigma + h_1)e_1[t] \\
\dot{e}_2[t] &= (r - M_2)e_1[t] - (1 + h_2)e_3[t] - M_1e_1[t] + e_1[t]e_3[t] \\
\dot{e}_3[t] &= M_1e_1[t] + M_2e_2[t] - (b + h_3)e_3[t] - e_1[t]e_3[t] \\
\dot{e}_4[t] &= \sigma e_2[t] - (\sigma + h_1)e_1[t] \\
\dot{e}_5[t] &= (r - M_2)e_1[t] - (1 + h_2)e_3[t] - M_1e_1[t] + e_1[t]e_3[t] \\
\dot{e}_6[t] &= M_1e_1[t] + M_2e_2[t] - (b + h_3)e_3[t] - e_1[t]e_3[t] \\
\dot{e}_7[t] &= \sigma e_2[t] - (\sigma + h_1)e_1[t] \\
\dot{e}_8[t] &= (r - M_2)e_1[t] - (1 + h_2)e_3[t] - M_1e_1[t] + e_1[t]e_3[t] \\
\dot{e}_9[t] &= M_1e_1[t] + M_2e_2[t] - (b + h_3)e_3[t] - e_1[t]e_3[t]
\end{align*}
\] (4.4)

Now theorem (3.1) for a particular case (Lorenz system) can be restated as follow
Theorem 4.1 If drive system (4.1) consists of two non-linear terms, the response system (4.2) with unknown controllers \( u_1[t], u_2[t], u_3[t] \) and corresponding error dynamical systems (4.3a)-(4.3d), \( M_{xy_i} \) \((i = 1, 2, 3)\) are the upper bounds of the state variables \( x_{id} \). The following linear control law is chosen; 
\[
u_i[t] = 0, u_2[t] = h_2e_2[t], u_3[t] = h_3e_3[t], \quad \text{where} \quad h_2, h_3 \text{ to be determined such that derivative of Lyapunov polynomial } L(e[t]) \text{ positive definite. Then zero solution of the error dynamical system (4.3a)-(4.3d) is globally, exponentially stable, and thus the drive system (4.1) and response system (4.2) are globally exponentially synchronized.}
\]

**Proof:** Since the drive system (4.1) consists of two nonlinear terms, therefore the response system (4.2) can be restated after taking control functions 
\[
u(t) = (u_1[t], u_2[t], u_3[t])^T \quad \text{as} \quad u_i[t] = 0, u_2[t] = h_2e_2[t], u_3[t] = h_3e_3[t],
\]
\[
\dot{y}_i[t] = Ay_i[t] + \sum_{i=1}^{2} \psi_i(y_i)\phi(\chi) + Be[t] + \theta[t]
\]

where
\[
A = \begin{bmatrix}
-\sigma & \sigma & 0 \\
r & -1 & 0 \\
0 & 0 & b
\end{bmatrix},
\]
\[
B = [0, h_2, h_3]^T.
\]

now error system (4.3a)-(4.3d) become
\[
\dot{e}[t] = Ae[e] + \sum_{i=1}^{2} \psi(e_i)\phi(\chi) + Be[t]
\]

if we choose Lyapunov function
\[
L(e[t]) = \frac{1}{2}(e_1^2[t] + e_2^2[t] + e_3^2[t])^T
\]

or
\[
L(e[t]) = (e_1[t]e_2[t]e_3[t]P(e_1[t]e_2[t]e_3[t]))^T
\]
where \( P = \text{diag}[0.5, 0.5, 0.5] \) and 
\[ \lambda_{\text{max}} = \lambda_{\text{min}} = 0.5. \]

Differentiating (4.1.3) with respect to \( t \), we obtain:
\[
\dot{L}(e[t]) = e_1[t]\dot{e}_1[t] + e_2[t]\dot{e}_2[t] + e_3[t]\dot{e}_3[t]
\]
\[= -\sigma e_1^2[t] - (1 + h_2) e_2^2[t] - (b + h_3) e_3^2[t] + (\sigma + r - M_3) e_1 e_2 + M_2 e_2 e_3\]
or 
\[
\dot{L}[e[t]] = [e_1[t] e_2[t] e_3[t]]Q[e_1[t] e_2[t] e_3[t]]^T
\]
(4.1.5)

where 
\[
Q = \begin{bmatrix}
-\sigma & \frac{1}{2}(\sigma + r - M_3) & \frac{1}{2}M_2 \\
\frac{1}{2}(\sigma + r - M_3) & -(1 + h_2) & 0 \\
\frac{1}{2}M_2 & 0 & -(b + h_3)
\end{bmatrix}
\]
is negative definite matrix if the following conditions holds-

1. \( -\sigma < 0 \)

2. \[\begin{vmatrix}
-\sigma & \frac{1}{2}(\sigma + r - M_3) \\
\frac{1}{2}(\sigma + r - M_3) & -(1 + h_2)
\end{vmatrix} < 0\]

3. \[\begin{vmatrix}
-\sigma & \frac{1}{2}(\sigma + r + M_3) & \frac{1}{2}M_2 \\
\frac{1}{2}(\sigma + r + M_3) & -(1 + h_2) & 0 \\
\frac{1}{2}M_2 & 0 & -(b + h_3)
\end{vmatrix} < 0.\]

This leads to

1. \( \sigma > 0 \)
2. \( h_2 < \frac{1}{4} (\sigma + r - M_3)^2 - 1 \)

3. \( h_3 < \frac{M_2^2(1 + h_2)}{4\sigma(1 + h_2) - (\sigma + r - M_3)^2} - b \);

Now by using lemma (2.1), we have the exponential estimation for \( Q \)

\[
\sum_{i=1}^{3} e_i^2(t) \leq \frac{\lambda_{\max}(P)}{\lambda_{\min}(P)} \sum_{i=1}^{3} e_i^2(t_0) \exp\left[-\frac{\lambda_{\min}(Q)}{\lambda_{\max}(P)}(t-t_0)\right]
\]

Proof of Theorem (4.1) is complete.

5. Numerical simulation results

In this section we verify the control laws presented in the previous sections via numerical simulations. Keeping \( M_1 = 12, M_2 = 15, M_3 = 48, r = 28, b = \frac{8}{3} \) fixed and choosing \( \sigma, h_2, h_3 \) according to proposed theorem, one can obtain a number of values for which Lorenz system synchronized. For example if we choose a particular approximation for which \( \sigma = 20, h_2 = -1, h_3 = -\frac{8}{3} \) and 
\( x_{d_{10}} = 10, x_{d_{20}} = -10, x_{d_{30}} = 20, \) and \( y_{r_{10}} = 7, y_{r_{20}} = -10, y_{r_{30}} = 10, \) for the chaotic drive and response Lorenz systems and 
\( x_{r_{10}} = 50, x_{r_{20}} = -10, x_{r_{30}} = 20, \) and 
\( y_{r_{10}} = 20, y_{r_{20}} = 10, y_{r_{30}} = 20, \) for the synchronized drive and response Lorenz systems than we can see an excellent agreement with the proposed theorem. Figure 5.1(a) - 5.1(d) are the trajectories of chaotic Lorenz system. Figure 5.1(e) – 5.1(h) are the synchronization between two identical Lorenz systems.
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Fig 5.1(a) $x = x[t]$ for $0 \leq t \leq 10$

Fig 5.1(b) $y = y[t]$ for $0 \leq t \leq 10$

Fig 5.1(c) $z = z[t]$ for $0 \leq t \leq 10$

Fig 5.1(e) $x = x[t]$ for $0 \leq t \leq 10$

Fig 5.1(f) $y = y[t]$ for $0 \leq t \leq 10$

Fig 5.1(g) $z = z[t]$ for $0 \leq t \leq 10$
6. Conclusions

In this paper, we focus to study the chaos synchronization of generalized nonlinear dynamical systems which can exhibit a Chaotic attractor for different parameter values by using Linear Feedback control Laws. We propose a theorem for synchronization of an n-dimensional generalized nonlinear dynamical system. Numerical simulations are used to verify the theoretical results.

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References


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