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Comparing performance of educational system of world's continents: A fuzzy three-stage DEA model

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Abstract

Data envelopment analysis (DEA) models treat the Decision making unit (DMU) as a black box. Thus, without any consideration about the intermediate steps inputs enter and outputs exit. Hence, it is difficult to detect the source of inefficiency within a DMU. Two-stage DEA model shows how to use DEA to look inside the DMU. Therefore, one can gain greater insight about the locations of organizational inefficiency. In this paper we expand this model into a model with three stages while the input-output data of each stage are fuzzy. We apply the model to efficiency assessment of educational systems of Asia, Africa, N-America, S-America and Europe.

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1 Introduction

Data Envelopment Analysis (DEA) is a linear programming-based methodology for evaluating the relative efficiency of each member of a set of organizational units. These units, which are called decision-making units (DMUs), consume various levels of each specified input and produce various levels of each specified output. In DEA, the relative comparison is examined within a Production Possibility Set (PPS) determined by all DMUs under appropriate assumptions regarding returns to scale and orientation. DEA deals with each DMU as a black box by taking into account only the inputs consumed and outputs produced by each DMU and makes no assumptions regarding the internal operations of a DMU. When the aim is to assess DMUs and identifying inefficient units the black box approach is adequate. However, such an approach cannot provide process guidance to managers to help them improve the DMUs efficiency and provides no insight considering the locations of inefficiency. many contributions have been made to this field such as following. Yang [**12**] presented a model which allows integration of the production performance and investment performance for the insurance companies and provides management overall performance evaluation and how to achieve efficiency systematically for the insurers involved. Cakravastia et al.[**2**] provides a research which aims to develop an analytical model of the supplier selection process in designing a supply chain network. Li et al. [**8**] proposed a two-stage approach for solving multi-objective system reliability optimization problems. Chen et al. [**4**] provided a paper examines relations and equivalence between two existing DEA approaches that address measuring the performance of two-stage processes. Chen et al. [**3**] developed an additive efficiency decomposition approach wherein the overall efficiency is expressed as a (weighted) sum of the efficiencies of the individual stages.
Usually a production process involves complicated inputs and outputs in which many factors are very difficult to measure precisely especially when a set of DMUs contains judgment data, or predictive data. Thus an approach is necessary is able to deal with inexact numbers, or numbers in ranges. Therefore one way is to represent the uncertain values by membership functions of the fuzzy set theory, Zadeh [**13**] and Zimmermann [**14**]. Generally speaking, uncertain information or imprecise data can be characterized by fuzzy numbers. Therefore, how to evaluate the efficiencies of a group of DMUs with fuzzy input and output data is a problem which have been under consideration of many researchers. Kao and Liu [**7**] developed a procedure to measure the efficiencies of DMUs with fuzzy observations. They formulated a pair of parametric programs to describe that family of crisp DEA models, via which the membership functions of the efficiency measures are derived. Since the efficiency measures are expressed by membership functions rather than by crisp values, more infor-

mation is provided for management. Wang et al. [**10**] proposed two new fuzzy DEA models constructed from the perspective of fuzzy arithmetic to deal with fuzziness in input and output data in DEA. These fuzzy DEA models are formulated as linear programming models and can be solved to determine fuzzy efficiencies of a group of DMUs. Wena et al. [**11**] defined a fuzzy comparison of fuzzy variables and extended the CCR model to be a fuzzy DEA model based on credibility measure. They also proposed a full ranking method in order to rank all the DMUs. Tlig and Rebai [**9**] developed DEA models using imprecise data represented by LR fuzzy numbers with different shapes. The resulting FDEA models take the form of fuzzy linear programming and can be solved by the use of some approaches to rank fuzzy numbers.
The paper unfolds as follows: in section 2, Preliminaries will be briefly reviewed. In section 3, the two-stage DEA model with fuzzy data will be presented. A Three-stage DEA model with fuzzy data with an application is documented in section 4, and section 5 concludes the paper.

2 Preliminaries

In this section we review two-stage DEA model and Fuzzy background and Metric for fuzzy numbers.

2.1 Data envelopment analysis

In this subsection we review how DEA models DMUs which produce in two stages, with output from the first stage becoming input to the second stage.
Generally, the objective of efficiency assessment is to find the weak domain so that proper effort can be devoted to improve performance. When a production system can be separated into two-processes, several studies pointed out that in addition to calculating the efficiency of the whole system by using the conventional DEA model, the efficiencies of the two sub-processes can also be calculated to identify the source that causes the inefficiencies of the whole system. The important issue of these studies, without taking into account the fact that the outputs of the first sub-process are the inputs of the second sub-process, is that the efficiencies of the whole process and the two processes are independently calculated. Two-stage DEA models are an extension of the DEA models in which each DMU as two sub-processes connected in series. The stage 1 sub-process consumes inputs to produce intermediate products. These, in turn, are the inputs to the stage 2 sub-process, which uses them to produce the DMUs outputs. This situation is portrayed in Figure 1. The objective of the two-stage DEA model is to evaluate the relative efficiencies of each DMU and each of its sub-processes. The important key feature of this model is that conventional DEA model usually misses inefficiencies but the two-stage DEA

model finds inefficiencies. Also, it distinguishes inefficiency in the first stage from that in the second one and allow managers to find inefficient stages of the production process. This model may be inefficient at either stage, or at both stages. However, for such inefficiency within each stage the managerial remedies will be different, and thus within each stage it is crucial for the teams ownership to identify the extent of inefficiency.

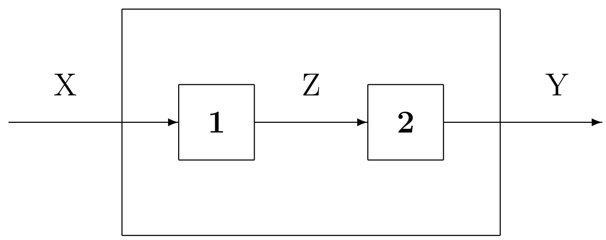

Figure 1: Each DMU has two sub-processes connected in series.

For the stage 1 sub-process at DMU_k, we solve the following model:

$$\begin{array}{lll} E_k^1 = \max & \sum_{l=1}^{f} p_l z_{lo} / \sum_{i=1}^{m} v_i x_{io} & \\ s.t. & \sum_{l=1}^{f} p_l z_{lj} / \sum_{i=1}^{m} v_i x_{ij} \leq 1, \quad j = 1, ..., n, & (1) \\ & p \geq 0, \quad v \geq 0. & \end{array}$$

and for the stage 2 sub-process at DMU_k, we solve the following model:

$$\begin{array}{lll} E_k^2 = \max & \sum_{r=1}^{a} u_r y_{ro} / \sum_{l=1}^{f} p_l z_{lo} & \\ s.t. & \sum_{r=1}^{s} u_r y_{rj} / \sum_{l=1}^{f} p_l z_{lj} \leq 1, \quad j = 1, ..., n, & (2) \\ & u \geq 0, \quad p \geq 0. & \end{array}$$

With these two models one can look inside the DMU_k, thus you can gain greater insight as to the locations of organizational inefficiency. Under DEA assumptions, the stage 1 sub-process at DMU_k could have increased its production of the intermediate product to level Z_k^*. Similarly, given the actual performance of the stage 1 sub-process at DMU_k, the stage 2 sub-process at DMU_k could have increased its production of the output to level Y_k^*.
Finally, to obtain DMU_ks organizational efficiency, we solve the following model:

$$
\begin{array}{lll}
E_k = \max & \sum_{r=1}^{s} u_r y_{ro} / \sum_{i=1}^{m} v_i x_{io} & \\
s.t. & \sum_{l=1}^{f} p_l z_{lj} / \sum_{i=1}^{m} v_i x_{ij} \leq 1, \quad j = 1, ..., n, & \\
& \sum_{r=1}^{s} u_r y_{rj} / \sum_{l=1}^{f} p_l z_{lj} \leq 1, \quad j = 1, ..., n, & (3) \\
& \sum_{r=1}^{s} u_r y_{rj} / \sum_{i=1}^{m} v_i x_{ij} \leq 1, \quad j = 1, ..., n, & \\
& u \geq 0, \quad v \geq 0, \quad p \geq 0. &
\end{array}
$$

It should be notice that the efficiencies of the whole process and the two sub-processes are independently calculated. Kao and Hwang [**6**] believe that the two sub-processes and the whole processes should link with each other and a model must describe this series relationship. Clearly, the overall efficiency is the product of the efficiencies of the two processes: $E_k = E_K^1 \times E_K 2$ where E_k, E_K^1 and E_K^2 are the optimal values of models (1), (2) and (3) respectively.The above mentioned models are fractional programming that can be easily convert into their linear counterparts. As it is evident from model (3) in constraints the third fraction is redundant thus be eliminating it the linear form of this model will be as following:

$$
\begin{array}{lll}
E_k = \max & \sum_{r=1}^{s} u_r y_{ro} & \\
s.t. & \sum_{i=1}^{m} v_i x_{io} = 1, & \\
& \sum_{l=1}^{f} p_l z_{lj} - \sum_{i=1}^{m} v_i x_{ij} \leq 0, \quad j = 1, ..., n, & (4) \\
& \sum_{r=1}^{s} u_r y_{rj} - \sum_{l=1}^{f} p_l z_{lj} \leq 0, \quad j = 1, ..., n, & \\
& u \geq 0, \quad v \geq 0, \quad p \geq 0. &
\end{array}
$$

2.2 Fuzzy background and Metric for fuzzy numbers

In this section give a brief review of essential notions of fuzzy set theory which will be used throughout this paper. Below, we give definitions and notations taken from Bezdek [**1**], Zimmermann [**14**], Dubois and Prade [**5**] and Zadeh [**13**].

Definition 1. *Let X be the universal set. $\tilde{A}$ is called a fuzzy set in X if $\tilde{A}$ is a set of ordered pairs*

$$\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) | x \in X\},$$

where $\mu_{\tilde{A}}(x)$ is the membership value of x in $\tilde{A}$.

Definition 2. *A convex fuzzy set $\tilde{A}$ on $\Re$ is a fuzzy number if the following conditions hold:*

(a) Its membership function is piecewise continuous.
(b) There exist only one x_0 that $\mu_A(x_0) = 1$.

Definition 3. *The support of a fuzzy set $\tilde{A}$ is a set of elements in X for which $\mu_{\tilde{A}}(x)$ is positive, that is,*

$$\text{supp}\tilde{A} = \overline{\{x \in X | \mu_{\tilde{A}}(x) > 0\}}.$$

Definition 4. *A fuzzy number $\tilde{A}$ is called positive , if $\;inf\;supp(A) \geq 0$.*

Definition 5. *(Generakized Left Right fuzzy number) A GLRFN fuzzy number is of L-R type fuzzy number if there exists reference function L (L for left), (R for right) and $a_1 \leq a_2 \leq a_3 \leq a_4$ with*

$$\mu_{\tilde{A}}(x) = \begin{cases} L(\frac{a_2-x}{a_2-a_1}), a_1 \leq x \leq a_2 \\ 1, \qquad\qquad a_2 \leq x \leq a_3 \\ R(\frac{x-a_3}{a_4-a_3}), a_3 \leq x \leq a_4 \\ 0, \qquad\qquad \mathit{Otherwise} \end{cases}$$

$\tilde{A}$ is denoted by $(a_1, a_2, a_3, a_4)_{LR}$.

Where L and R are strictly decreasing functions defined on $[0, 1]$ and satisfying the conditions:

$$L(x) = R(x) = 1 \quad if \quad x \leq 0$$
$$L(x) = R(x) = 0 \quad if \quad x \geq 1$$

For $a_2 = a_3$, we have the classical definition of left right fuzzy numbers (LRFN) of Dubois and Prade [**13**], a LRFN $\tilde{B}$ is denoted as $\tilde{B} = (b_1, b_2, b_3)_{LR}$. Trapezoidal fuzzy numbers (TrFN) are special cases of GLRFN with $L(x) = R(x) = 1 - x$. Triangular fuzzy numbers (TFN) are also special cases of GLRFN with $L(x) = R(x) = 1 - x$ and $a_2 = a_3$. It should be noted that L_A^{-1} and R_A^{-1} are the inverse of L_A and R_A functions.
A GLRFN $\tilde{A}$ is denoted as $\tilde{A} = (a_1, a_2, a_3, a_4)_{LR}$ and an $\alpha-$level interval of fuzzy number $\tilde{A}$ as:

$$[\tilde{A}]^\alpha = [A_l(\alpha), A_r(\alpha)] = [a_2 - (a_2 - a_1)L_A^{-1}(\alpha), a_3 + (a_4 - a_3)R_A^{-1}(\alpha)]$$

Definition 6. *Parametric form of a fuzzy number has been introduced and represented by $\tilde{A} = (\underline{A}(r), \overline{A}(r))$, where $\underline{A}(r)$ and $\overline{A}(r)$, $0 \leq r \leq 1$, satisfying the following requirements:*
1. *$\underline{A}(r)$ is monotonically increasing left continuous function.*
2. *$\overline{A}(r)$ is monotonically decreasing right continuous function.*
3. *$\underline{A}(r) \leq \overline{A}(r)$, $0 \leq r \leq 1$.*

Definition 7. *Let $f(x) = (a-b)x+b$ and $g(x) = (c-d)x+d$. The distance of two interval $[a,b]$ and $[c,d]$, $(a \leq b, c \leq d)$ is denoted by $d^{(p)}_{TMI}([a,b],[c,d])$ such that:*

$$d^{(p)}_{TMI}([a,b],[c,d]) = (D^{(p)}_{TMI}([a,b],[c,d]))^{\frac{1}{p}} \tag{5}$$

and

$$D^{(p)}_{TMI}([a,b],[c,d]) = \parallel f(x) - g(x) \parallel^p_{L_p} \tag{6}$$

Where $\parallel . \parallel$ is the usual norm in the L_p space on the $[0,1]$ $(p > 1)$.

Definition 8. *A distance between two GLRFNs $\tilde{A}$ and $\tilde{B}$ can be defined as:*

$$d^{(p)}_{TMF}(\tilde{A},\tilde{B},s) = (D^{(p)}_{TMF}(\tilde{A},\tilde{B},s))^{\frac{1}{p}} \tag{7}$$

Such that

$$D^{(p)}_{TMF}(\tilde{A},\tilde{B},s) = \frac{\int_0^1 s(\alpha) D^{(p)}_{TMI}([\tilde{A}]^\alpha,[\tilde{B}]^\alpha)d\alpha}{\int_0^1 s(\alpha)d\alpha} \tag{8}$$

Here s, is a weight function such that continuous positive function defined on $[0,1]$. It can be proved that $d^{(p)}_{TMF}(\tilde{A},\tilde{B},s)$ is a metric on GLRFNs.

Definition 9. *If $\tilde{E} = (a_1,a_2,a_3,a_4)$ and $\tilde{E} = (b_1,b_2,b_3,b_4)$ are two fuzzy numbers and p=1 with $s(\alpha) = 1$*

$$\begin{aligned} d(\tilde{A},\tilde{B}) =: d^{(1)}_{TMF}(\tilde{A},\tilde{B},1) &= \int_0^1 D^{(1)}_{TMI}([{A_l}^\alpha, {A_u}^\alpha],[{B_l}^\alpha,{B_u}^\alpha])d\alpha \\ = \int_0^1 (\int_0^1 &|(1-x){A_u}^\alpha + x{A_l}^\alpha - ((1-x){B_u}^\alpha + x{B_l}^\alpha)|dx)d\alpha. \end{aligned} \tag{9}$$

Also, the distance between $\tilde{A}$ and the origin is defined as follows:

$$\begin{aligned} d(\tilde{A}) =: d^{(1)}_{TMF}(\tilde{A},1) &= \int_0^1 D^{(1)}_{TMI}([{A_l}^\alpha,{A_u}^\alpha])d\alpha \\ &= \int_0^1 (\int_0^1 |(1-x){A_u}^\alpha + x{A_l}^\alpha|dx)d\alpha. \end{aligned} \tag{10}$$

Definition 10. *The ranking method for two positive fuzzy numbers is as follows:*

$$\tilde{A} \precsim \tilde{B} \quad \Leftrightarrow \quad d^{(1)}_{TMF}(\tilde{A}) \leq d^{(1)}_{TMF}(\tilde{B}) \tag{11}$$

$$\tilde{A} \cong \tilde{B} \quad \Leftrightarrow \quad d^{(1)}_{TMF}(\tilde{A}) = d^{(1)}_{TMF}(\tilde{B}) \tag{12}$$

$$\tilde{A} \precsim \tilde{B} \quad \Leftrightarrow \quad d^{(1)}_{TMF}(\tilde{A}) \geq d^{(1)}_{TMF}(\tilde{B}) \tag{13}$$

3 Fuzzy two-stage DEA model

Considering the aforesaid two-stage DEA model and the ranking method in this section we present two-stage DEA model with fuzzy data. Also, by applying a fuzzy metric and a ranking function, obtained from it, the multiplier fuzzy CCR model converts to its crisp counterpart.
Consider the following model with fuzzy data.

$$\begin{array}{ll} E_k = \max & \sum_{r=1}^{s} u_r \tilde{y}_{ro} \\ s.t. & \sum_{i=1}^{m} v_i \tilde{x}_{io} \cong \tilde{1}, \\ & \sum_{l=1}^{f} p_l \tilde{z}_{lj} - \sum_{i=1}^{m} v_i \tilde{x}_{ij} \precsim \tilde{0}, \quad j = 1, ..., n, \\ & \sum_{r=1}^{s} u_r \tilde{y}_{rj} - \sum_{l=1}^{f} p_l \tilde{z}_{lj} \precsim \tilde{0}, \quad j = 1, ..., n, \\ & u \geq 0, \quad v \geq 0, \quad p \geq 0. \end{array} \tag{14}$$

Taking into account the proposed ranking method in previous section and the definition (9) model (14) will be converted into the following one:

$$\begin{array}{ll} E_k = \max & \sum_{r=1}^{s} u_r d(\tilde{y}_{ro}) \\ s.t. & \sum_{i=1}^{m} v_i d(\tilde{x}_{io}) = d(\tilde{1}), \\ & \sum_{l=1}^{f} p_l d(\tilde{z}_{lj}) - \sum_{i=1}^{m} v_i (\tilde{x}_{ij}) \leq d(\tilde{0}), \quad j = 1, ..., n, \\ & \sum_{r=1}^{s} u_r d(\tilde{y}_{rj}) - \sum_{l=1}^{f} p_l d(\tilde{z}_{lj}) \leq d(\tilde{0}), \quad j = 1, ..., n, \\ & u \geq 0, \quad v \geq 0, \quad p \geq 0. \end{array} \tag{15}$$

Considering definition (10) this model will be converted into the following model:

$$\begin{array}{ll} E_k = \max & \sum_{r=1}^{s} u_r \bar{y}_{ro} \\ s.t. & \sum_{i=1}^{m} v_i \bar{x}_{io} = 1, \\ & \sum_{l=1}^{f} p_l \bar{z}_{lj} - \sum_{i=1}^{m} v_i \bar{x}_{ij} \leq 0, \quad j = 1, ..., n, \\ & \sum_{r=1}^{s} u_r \bar{y}_{rj} - \sum_{l=1}^{f} p_l \bar{z}_{lj} \leq 0, \quad j = 1, ..., n, \\ & u \geq 0, \quad v \geq 0, \quad p \geq 0. \end{array} \tag{16}$$

where:

$$\bar{y}_{rj} = \int_0^1 \int_0^1 |y_{rj}^l x + y_{rj}^u (1-x)| dx d\alpha, \quad r = 1, ..., s, \quad j = 1, ..., n,$$
$$\bar{x}_{ij} = \int_0^1 \int_0^1 |x_{ij}^l x + x_{ij}^u (1-x)| dx d\alpha, \quad i = 1, ..., m,, \quad j = 1, ..., n.$$
$$\bar{z}_{lj} = \int_0^1 \int_0^1 |z_{lj}^l x + z_{lj}^u (1-x)| dx d\alpha, \quad l = 1, ..., h,, \quad j = 1, ..., n.$$

4 Application

In this section, we generalize the two-stage DEA model with fuzzy data into the three-stage one and apply it to educational system of world's countries.

4.1 Data

Nowadays different countries in the world have various educational structures. By nature, these structures in each countries differ pursuant to plans, aims and politics of that country. Meanwhile, an attempt is being made to collaborate the educational system of each country to the proportional social evolution and cultural progresses which is proportional to the $21th$ century. Furthermore, the necessary harmony between educational plans and business in various domains of services,industries,agriculture and business is also being made.
In Asia we encountered with the variant sets of educational systems. Moreover, some of them due to the more conformity of the situation of system with the pre-assigned aims and benefiting from possibilities, equipment and educated manpower derive advantage from more desirable situations. Furthermore, some other ones by continuing sovereignty of traditional systems over school educational system, do not benefit from efficiency and due to the lack of procurement of necessary financial resources, lack of benefiting from possibilities, equipment and necessary circumstances are under privilege from dynamic education.
In Africa due to the existence of various countries we witness great diversity in educational systems. Definitely, most of the countries in Africa are among the category of undeveloped countries and some other ones are among the class of developing countries. Consequences of this situation, in general, cause such settings that except some limited instances, educational systems of this continent due to the lack of benefiting from possibilities and needed equipment, specialized manpower and lack of investment and necessary financial supports, do not benefit from considerable quality. Nonetheless, whole of these instances do not prevent that the positive view points do not exist as an example or

in some limited examples of prosperous system there exist no useful experiences. Generally, among key features in investigations of educational systems of Africa performing the planes and universal schedules of education can be indicated, which is performed for removing deprivation, increasing educational covering, advancement of educational quality and etc. In course of investigations that have been produced, it has been recognized that the examinations and researches for the aforesaid strategies , to a great extent, are effective and trailblazer.

America itself contains two parts, North America and South America. Comparatively, such a division, to some extent, can be generalized to the educational perspective of this continent since in North America educational quality of two developed countries, America and Canada which have constituted the most part of it, in comparison to other countries have observable desirability. The reasons of this desirability can be investigated through many factors such as paying attention to the multilateral investment in domain of pedagogy in the position of one of the most important factor of society development, continued investigation and examination for improving qualitative and quantitative procedure of education, benefiting from educational assistant tool and update technology, new approaches for teacher training and educational methods and etc. An other significant factor which has an undesirable effect on quality of education of the mentioned countries is benefiting from non centralized educational system in a way that each of these states has a separate educational ministry. These ministries in their own states limits can benefit from independence and making necessary decisions for governing events. This cause a constructive competition among different states and finally with the contribution of other factors in comparison with the South American countries more desirable feature of the educational system is portrayed. Of course, it is noteworthy of attention that mentioning the aforesaid features do not convey the idea that educational situations are inappropriate since the performed educational improvements in last two decades in some countries has made plenty of evolution in raising educational level. Investigating and examining each of those factors can be trailblaizer for experts and specialists.

From among world continents, Europe can be indicated as a place which contains a set of developed and plentiful educational systems. Reasons of the development of such systems can be poked trough the following factors:

1-settlement and conjunction of the most developed countries in this continent.

2-Cultural antiquity, successive and long background of education and knowledge in each country.

3-Being aware of the importance and the position of pedagogy in development of society and necessary investment in this field.

4-Recognition and useful exploitation of these countries from their own existing recourses, capabilities and potentials.

5-Planning, Scheduling and necessary foresight.

6-Suitable managerial structure controlling over educational system.

7-Taking advantages from effective humane participation.

8-Applying and benefiting from the developed technology in the educational framework.

It is noteworthy of attention that the efficiency of educational systems and benefiting from the mentioned factors in the different countries of this continent is not equivalent and due to this perspective various countries can be divided into variant levels. The most evident instances can be seen in Germany, England and Scandinavia. In the same way, as mentioned earlier here the aim is to cast a descriptive look upon the structure of educational systems and proposing the above issues is just for drawing the attention of the researchers and people who are interested in this domain to examine and investigate in the field of management and governing the education, educational innovations, connecting education to business and employment, effective attendance of people and humane participation in education and etc, which in the next steps it can put the suitable research domain forward to the people who have great interest in educational circumstances.

4.2 Formulating fuzzy three-stage model and the results

According to the above discussion we envision the continents as the DMUs. We consider equipment and providing financial resources as inputs for stage 1, which lead to educated manpower as an output of this stage. According to the series structure educated manpower will be an input for stage 2 that produce dynamic education. In stage 3 dynamic education is considered as an input which results performance as an output. These stages are schematically portrayed in Fig. 2.

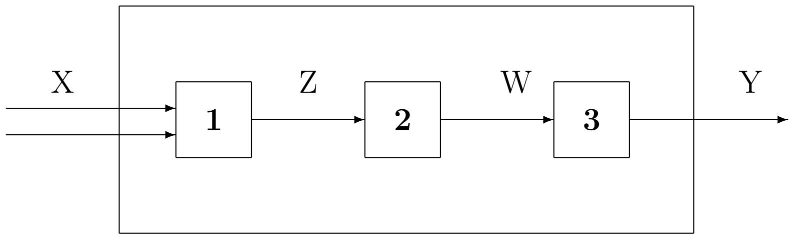

Figure 2: Each DMU has three sub-processes connected in series.

According to the mentioned information for stage 1 we have the following model:

$$\begin{array}{lll} E_k^1 = \max & \sum_{l=1}^{f} p_l \tilde{z}_{lo} & \\ s.t. & \sum_{i=1}^{m} v_i \tilde{x}_{io} = 1, & \\ & \sum_{l=1}^{f} p_l \tilde{z}_{lj} - \sum_{i=1}^{m} v_i \tilde{x}_{ij} \leq 0, \quad j = 1, ..., n, & \\ & v \geq 0, \quad p \geq 0. & \end{array} \tag{17}$$

and for stage 2 we solve this model:

$$\begin{array}{lll} E_k^2 = \max & \sum_{h=1}^{q} t_h \tilde{w}_{ho} & \\ s.t. & \sum_{l=1}^{f} p_l \tilde{z}_{lo} = 1, & \\ & \sum_{h=1}^{q} t_h \tilde{w}_{hj} - \sum_{l=1}^{f} p_l \tilde{z}_{lj} \leq 0, \quad j = 1, ..., n, & \\ & t \geq 0, \quad p \geq 0. & \end{array} \tag{18}$$

for stage 3 the succeeding model will be solved:

$$\begin{array}{lll} E_k^3 = \max & \sum_{r=1}^{s} u_r \tilde{y}_{ro} & \\ s.t. & \sum_{h=1}^{q} t_h \tilde{w}_{ho} = 1, & \\ & \sum_{r=1}^{s} u_r \tilde{y}_{rj} - \sum_{h=1}^{q} t_h \tilde{w}_{hj} \leq 0, \quad j = 1, ..., n, & \\ & u \geq 0, \quad t \geq 0. & \end{array} \tag{19}$$

Finally we have the subsequent model which links the three sub-processes with the whole processes:

$$\begin{array}{ll} E_k = \max & \sum_{r=1}^{s} u_r \tilde{y}_{ro} \\ s.t. & \sum_{i=1}^{m} v_i \tilde{x}_{io} = 1, \\ & \sum_{r=1}^{s} u_r \tilde{y}_{rj} - \sum_{h=1}^{q} t_h \tilde{w}_{hj} \leq 0, \quad j = 1, ..., n, \\ & \sum_{l=1}^{f} p_l \tilde{z}_{lj} - \sum_{i=1}^{m} v_i \tilde{x}_{ij} \leq 0, \quad j = 1, ..., n, \\ & \sum_{h=1}^{q} t_h \tilde{w}_{hj} - \sum_{l=1}^{f} p_l \tilde{z}_{lj} \leq 0, \quad j = 1, ..., n, \\ & u \geq 0, \quad v \geq 0, \quad p \geq 0, \quad t \geq 0. \end{array} \tag{20}$$

Here the input output data are linguistic variables whose values are words or sentences in a natural or artificial language. The possible values for these variables could be: very good, good, average, poor, and very poor. Each linguistic variable can be indicated by a triangular fuzzy number (TFN) within the scale range of 01. The membership functions of five levels of linguistic variables are respectively as follows; (0.00, 0.00, 0.25), (0.00, 0.25, 0.50), (0.25, 0.50, 0.75), (0.50, 0.75, 1.00),(0.75, 0.1, 0.00).Table 1 shows the qualitative data.

Table.1 *Data*

DMUs	I1	I2	Z1	W2	O1
Asia	*good*	*verypoor*	*good*	*poor*	*verypoor*
Africa	*very poor*	*very poor*	*very poor*	*poor*	*good*
N.America	*good*	*very good*	*good*	*good*	*very good*
S.America	*poor*	*poor*	*average*	*average*	*good*
Europe	*good*	*good*	*good*	*good*	*very good*

Table.2 *Results*

DMUs	E_k^1	E_k^2	E_k^3	E_k	black-box Eff
Asia	1.0000000	0.202492	0.2063492	0.4258000	0.2055594
Africa	0.5000000	1.000000	1.0000000	0.5000000	1.0000000
N.America	0.4999981	0.520000	0.6298816	0.1637686	0.3275372
S.America	1.0000000	0.520000	0.5000000	0.2600000	0.5200000
Europe	0.5000000	0.520000	0.6298815	0.1637692	0.3275384

These information are acquired from the website iranculture.org. Applying mentioned models the efficiency of each stage are gathered in Table 2.
Considering model (17) which assess the first stage of DMU_k Asia and S-America are efficient and Africa, N-America and Europe are some how the same. In the second and the third stage, where the educated manpower is utilized to produce dynamic education which also is used as an input of the next stage, just Africa is efficient. As mentioned in subsection (2.2) E_k is the product of E_k^1, E_k^2 and E_k^3 which can be easily calculated or even can be obtained thorough solving model (20). In Table 2 under the column of black-box efficiency the efficiency scores are obtained through solving a model with one stage for which equipment and providing financial resources are considered as inputs and performance is considered as an output, is listed.

5 Conclusion

As discussed in previous sections the two-stage DEA model is more preferable over a one-stage DEA model since this model finds inefficient stages of the production process. The important key feature of the two-stage DEA model is that it allows managers to focus efficiency enhancing strategies on the individual stages of the production process. While considering each DMU as a black box without any consideration about the intermediate steps inputs enter and outputs exit. Hence, it is difficult to detect the source of inefficiency within a DMU. But by the contribution of the two-stage DEA model it can be shown how to use DEA to look inside the DMU. Therefore, one can gain greater insight about the locations of organizational inefficiency. Thus, here the extension of two-stage DEA model is considered which includes three stages. The input-output data of each stage are fuzzy. It is noteworthy to mention that nowadays the use of linguistic variables is applied widely. Therefore, linguistic variables which are expressed by TFN are adopted to determine the efficiency score of DMUs each of which has three stages. In this paper the aim is to cast a descriptive look upon the structure of educational systems. Thus, while considering fuzzy data a three-stage DEA model for efficiency assessment of educational systems in Asia, Africa, N-America, S-America and Europe is used. As an instance a three-stage process is show in figure 2. The obtained results through solving the mentioned models, while tree-stage DEA model is used for efficiency assessment, with fuzzy input-output data, which are indicated in Table 1, are gathered in Table 2.

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