An Inventory Model for Weibull Time-Dependence Demand Rate with Completely Backlogged Shortages

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Abstract

This paper deals with in developing an inventory model with time-dependent Weibull demand rate where shortages are allowed. Here we have considered shortages are completely backlogged. The production rate is assumed to be finite and proportional to the demand rate. The analytical solution of the model has been done to obtain the optimal solution of the problem. Numerical examples are given to illustrate the model developed. Sensitivity analysis of the decision variables has been done to examine the effect of changes in the values of the parameters and affect of these changes on the optimal policy of inventory.

Mathematics Subject Classification: 90B05

Keywords: Inventory; Economic order quantity; demand; Shortage; Weibull distribution

1. Introduction

Demand is a major factor in inventory management. In inventory models, four types of demand are basically assumed i.e. constant demand, time-dependent
demand, probabilistic demand and stock-dependent demand. Inventory models dealing with stock-dependent demand have more relevance in present situation. Therefore, many authors have studied such models. Initially the demand rate of the item was assumed to be constant. However, in real market situations, demand varies with time. Silver and Meal [8], were the first to suggest a simple modification of the EOQ formula for the case of varying demand. Later, Silver and Meal [7] developed an approximate solution procedure, known as the Silver-Meal Heuristic for general case of a deterministic, time dependent demand pattern. However, Donaldson [23] discussed, for the first time, the classical no-shortage inventory policy for the case of a linear, time-dependent demand. Following Donaldson [23], significant contributions in this direction came from researchers like Silver [9], Ritchie [6] and Sachan [18], etc. The question of inventory shortages and backlogging was not considered by the above researchers. Deb and Chaudhuri [14] were the first to incorporate shortages into the inventory lot sizing problem with a linearly increasing time-varying demand. Subsequent contributions in this type of modeling came from Goyal [21], Hariga [16] and many others. EOQ models for deteriorating items with trended demand were considered by several researchers like Bahari-Kashani [10], Goswami and Chaudhuri [1] Chung and Ting [13], Hariga [15], Jalan et al. [3], Giri and Chaudhuri [4], Jalan and Chaudhuri [2] and Lin et al. [5].

Some of the recent works on time-varying demand for deteriorating items are due to Skouri and Papachristos [12], Teng et al. [11], Khanra and Chaudhuri [19], Sana et al. [20], etc. Further a group of researchers have devoted their attention to inventory replenishment problems with exponentially time-varying demand patterns and works in this direction are due to Aggarwal and Bahari-Kashani [22], Hollier and Mak [17], etc.

In the present paper, we have developed an inventory model assuming that time-dependence of demand follows a three-parameter Weibull distribution. Here the production rate is assumed to be finite and proportional to the demand rate. In this model shortages are allowed and are completely backlogged. An analytical solution of the mode is discussed and illustrated with the help of numerical examples. We have also done the sensitivity analysis of the optimal solution with respect to changes in various parametric values.

2. Assumption and Notation

The following assumptions & notations are used to develop the proposed model.

Assumption

- The demand rate at any time ‘t’ is: \( R(t) = \alpha (t - \gamma)^{\beta - 1} \), where \( \alpha > 0 \), \( \beta > 0 \) and \( \gamma > 0 \) are scale parameter, shape parameter and location parameter, respectively.
• The production rate is $K(t) = \lambda R(t)$ where $(\lambda > 1)$ a constant is also. Therefore $K(t) > R(t)$.

• The on-hand inventory does not deteriorate with time.
• Lead time is zero.
• Shortages are allowed & are completely backlogged.

Notations

$c_1$  Carrying cost per unit per unit time.
$c_2$  Shortage cost per unit per unit time.
$c_3$  Setup cost per production run.

$c_1, c_2, c_3$ are all assumed to be known & fixed during production cycle.

‘$C$’ the total average cost for a production cycle.

3. Formulation & Solution

The inventory level is initially (i.e. at time $t = 0$) zero. The shortage starts at $t = 0$ which accumulates up to the level $P$ at time $t = t_1$. The production starts at $t = t_1$ and the backlog is cleared at $t = t_2$. The stock level attains a level $S$ at $t = t_3$. The production is stopped at this point of time i.e. at $t = t_3$. Therefore the inventory level gradually decreases due to demand and becomes zero at $t = t_4$ after which the cycle is completed and it repeats itself. We are interested in determining the optimum values of ‘$S$’, ‘$P$’ and ‘$C$’. $Q(t)$ be the instantaneous inventory level at any time $t (0 \leq t \leq t_4)$. The differential equations describing the instantaneous states of $Q(t)$ in the interval $(0, t_4)$ are

$$\frac{dQ(t)}{dt} = -R(t) \quad 0 \leq t \leq t_1 \quad (1)$$

$$\frac{dQ(t)}{dt} = K(t) - R(t) \quad t_1 \leq t \leq t_2 \quad (2)$$

$$\frac{dQ(t)}{dt} = K(t) - R(t) \quad t_2 \leq t \leq t_3 \quad (3)$$

$$\frac{dQ(t)}{dt} = -R(t) \quad t_3 \leq t \leq t_4 \quad (4)$$
with the boundary conditions
\[ Q(0) = 0, \quad Q(t_1) = -P, \quad Q(t_2) = 0, \quad Q(t_3) = S, \quad Q(t_4) = 0. \] (5)

Substituting \( R(t) = \alpha(t - \gamma)^{\beta-1} \) and \( K(t) = \lambda R(t) \) in the equations (1)-(4) and solving them using the boundary conditions (5), we get the following solutions:

\[ Q(t) = -\frac{\alpha}{\beta} \left[(t - \gamma)^{\beta} - (t - \gamma)^{\beta}\right] \quad 0 \leq t \leq t_1 \] (6)

\[ Q(t) = (\lambda - 1) \frac{\alpha}{\beta} \left[(t - \gamma)^{\beta} - (t_2 - \gamma)^{\beta}\right] \quad t_1 \leq t \leq t_2 \] (7)

\[ Q(t) = (\lambda - 1) \frac{\alpha}{\beta} \left[(t - \gamma)^{\beta} - (t_2 - \gamma)^{\beta}\right] \quad t_2 \leq t \leq t_3 \] (8)

\[ Q(t) = \frac{\alpha}{\beta} \left[(t_4 - \gamma)^{\beta} - (t - \gamma)^{\beta}\right] \quad t_3 \leq t \leq t_4 \] (9)

Using the condition in \( Q(t) = -P \) in equation (6), we get

\[ P = \frac{\alpha}{\beta} \left[(t_1 - \gamma)^{\beta} - (t_1 - \gamma)^{\beta}\right] \] (10)

Similarly using the condition \( Q(t_1) = -P \) in equation (7), we get

\[ P = -(\lambda - 1) \frac{\alpha}{\beta} \left[(t_1 - \gamma)^{\beta} - (t_2 - \gamma)^{\beta}\right] \] (11)

Equating these two values of \( P \), we get

\[ t_1 = \left[\left(t_2 - \gamma\right)^{\beta} \left(\frac{\lambda - 1}{\lambda} + \frac{1}{\lambda} (-\gamma)^{\beta}\right)\right]^{\frac{1}{\beta}} + \gamma \] (12)

Again, using the condition \( Q(t_3) = S \) in equation (8) and equation (9) we get

\[ S = (\lambda - 1) \frac{\alpha}{\beta} \left[(t_3 - \gamma)^{\beta} - (t_2 - \gamma)^{\beta}\right] \] (13)

and

\[ S = \frac{\alpha}{\beta} \left[(t_4 - \gamma)^{\beta} - (t_3 - \gamma)^{\beta}\right] \] (14)
Equating these two values of ‘$S$’ we get

$$t_3 = \left[ \frac{(t_4 - \gamma)^\beta + (t_2 - \gamma)^\beta (\lambda - 1)}{\lambda} \right]^{-\frac{1}{\beta}} + \gamma \quad (15)$$

Now we shall try to find different costs involved in the system.

The total shortage cost in the system is

$$SC = c_2 \left[ \int_0^{t_1} -Q(t) \, dt + \int_{t_1}^{t_2} Q(t) \, dt \right]$$

$$= c_2 \left[ \alpha \frac{\beta}{\beta + 1} \frac{(t_1 - \gamma)^{\beta+1} - (\gamma)^{\beta+1}}{\beta+1} + (t_2 - \gamma)^{\beta+1} (t_2 - t_1) \right] \quad (16)$$

The total inventory holding cost in the system is

$$HC = c_1 \left[ \int_{t_1}^{t_2} Q(t) \, dt + \int_{t_2}^{t_3} Q(t) \, dt \right]$$

$$= c_1 \left[ (\lambda-1) \frac{\alpha}{\beta} \frac{(t_2 - \gamma)^{\beta+1} - (t_1 - \gamma)^{\beta+1}}{\beta+1} + (t_4 - \gamma)^{\beta+1} (t_4 - t_3) \right] \quad (17)$$

Therefore the average cost of the system is

$$C = \frac{1}{t_4} (SC + HC + c_3)$$

$$= \frac{1}{t_4} \left[ c_2 \left[ \frac{\alpha}{\beta} \frac{(t_1 - \gamma)^{\beta+1} - (\gamma)^{\beta+1}}{\beta+1} - (\gamma)^\beta t_1 \right] + \frac{\alpha(\lambda-1)}{\beta} \left[ \frac{(t_2 - \gamma)^{\beta+1} - (t_1 - \gamma)^{\beta+1}}{\beta+1} - (t_2 - \gamma)^\beta (t_2 - t_1) \right] ight]$$

$$+ c_1 \left[ (\lambda-1) \frac{\alpha}{\beta} \frac{(t_2 - \gamma)^{\beta+1} - (t_1 - \gamma)^{\beta+1}}{\beta+1} - (t_2 - \gamma)^\beta (t_3 - t_2) \right] + \frac{\alpha}{\beta} \left[ \frac{(t_4 - \gamma)^{\beta+1} - (t_3 - \gamma)^{\beta+1}}{\beta+1} + (t_4 - \gamma)^\beta (t_4 - t_3) \right] + c_3 \right] \quad (18)$$

Putting the values of $t_1$ and $t_3$ in equation (18), ‘C’ becomes a function of variables $t_2$ and $t_4$. Therefore the average cost of the system is
Here 'C' will be minimum if
\[ \frac{\partial C}{\partial t_2} = 0, \quad \frac{\partial C}{\partial t_4} = 0 \] (20)

Provided
\[ \frac{\partial^2 C}{\partial t_2^2} \times \frac{\partial^2 C}{\partial t_4^2} - \left( \frac{\partial^2 C}{\partial t_2 \partial t_4} \right)^2 > 0 \] (21)

From equation (19) and equation (20), we get the equations
\[
\frac{1}{t_4} \left[ c_1 \left\{ \frac{\alpha}{\beta} \left( (t_2 - \gamma)^\beta \left( \frac{\lambda - 1}{\lambda} \right) + \frac{1}{\lambda} (\gamma)^\beta \right) \left( \frac{\lambda - 1}{\lambda} \right) (t_2 - \gamma)^{\beta - 1} - (\gamma)^\beta \left( (t_2 - \gamma)^\beta \left( \frac{\lambda - 1}{\lambda} \right) + \frac{1}{\lambda} (\gamma)^\beta \right) \right\} \right.
\]
\[
\times \left( \frac{\lambda - 1}{\lambda} \right) (t_2 - \gamma)^{\beta - 1} \right\} - (\lambda - 1) \frac{\alpha}{\beta} \left\{ \left( t_2 - \gamma \right)^\beta \left( \frac{\lambda - 1}{\lambda} \right) + \frac{1}{\lambda} (\gamma)^\beta \right\} \left( \frac{\lambda - 1}{\lambda} \right) (t_2 - \gamma)^{\beta - 1} - t_2 \beta (t_2 - \gamma)^{\beta - 1} \right. 
\]
\[
\left. + \left( t_2 - \gamma \right)^\beta \left( \frac{\lambda - 1}{\lambda} \right) + \frac{1}{\lambda} (\gamma)^\beta \right\} \left( \frac{\lambda - 1}{\lambda} \right) (t_2 - \gamma)^{\beta - 1} (t_2 - \gamma) + \beta (t_2 - \gamma) \right. 
\]
\[
\gamma \beta (t_2 - \gamma)^{\beta - 1} \right\} + c_1 \left\{ (\lambda - 1) \frac{\alpha}{\beta} \left\{ \left( t_2 - \gamma \right)^\beta + (t_2 - \gamma)^\beta \left( \lambda - 1 \right) \right\} \frac{\beta}{\lambda} \right. 
\]
\[
\left. \left( t_2 - \gamma \right)^{\beta - 1} (\lambda - 1) - \beta (t_2 - \gamma)^{\beta - 1} \right. 
\]
\[
\left. - (t_2 - \gamma)^\beta \left( \frac{\left( t_2 - \gamma \right)^\beta + (t_2 - \gamma)^\beta \left( \lambda - 1 \right)}{\lambda} \right) \frac{\beta}{\lambda} \left( t_2 - \gamma \right)^{\beta - 1} (\lambda - 1) - \beta (t_2 - \gamma)^{\beta - 1} \right. 
\]
\[
\left. + \frac{\alpha}{\beta} \left\{ - (t_4 - \gamma)^\beta \left( \frac{\left( t_4 - \gamma \right)^\beta + (t_4 - \gamma)^\beta \left( \lambda - 1 \right)}{\lambda} \right) \frac{\beta}{\lambda} \right. \right. 
\]
\[
\left. \left( t_4 - \gamma \right)^{\beta - 1} (\lambda - 1) \right. 
\]
\[
\left. + \left( t_4 - \gamma \right)^\beta \left( t_4 - \gamma \right)^\beta \left( \lambda - 1 \right) \right. \right. 
\]
\[
\left. + \frac{\beta}{\lambda} \left( t_4 - \gamma \right)^{\beta - 1} (\lambda - 1) \right. 
\]
\[
\left. - (t_4 - \gamma)^\beta \right) \right\} \left. \right\} \right\} = 0 \tag{22} \]

and

\[
\frac{1}{t_4} \left[ c_1 \left\{ (\lambda - 1) \frac{\alpha}{\beta} \right\} \left( \frac{\left( t_4 - \gamma \right)^\beta + (t_4 - \gamma)^\beta \left( \lambda - 1 \right)}{\lambda} \right) \frac{\beta}{\lambda} \right. 
\]
\[
\left. \left( t_4 - \gamma \right)^{\beta - 1} (\lambda - 1) \right. 
\]
\[
\left. + \alpha \right\} \left. \right\} \left( t_4 - \gamma \right)^{\beta - 1} - \beta (t_4 - \gamma)^{\beta - 1} \right. 
\]
\[
\left. - (t_4 - \gamma)^\beta \right) \right\} \left. \right\} \]
\begin{equation}
\frac{(t_4 - \gamma)\beta + (t_3 - \gamma)\beta(\lambda - 1)}{\lambda} - \frac{1}{\lambda} \left( (t_4 - \gamma)\beta \alpha t_3 - \gamma + (t_3 - \gamma)\beta \right) - \frac{1}{\lambda} \left( (t_4 - \gamma)\beta + (t_2 - \gamma)\beta(\lambda - 1) \right) - \frac{1}{\lambda} (t_4 - \gamma)\beta \right)
\end{equation}

\begin{equation}
\frac{1}{\beta + 1} \left( (t_2 - \gamma)\beta \left( \frac{\lambda - 1}{\lambda} \right) \right) + \frac{1}{\beta + 1} \left( (t_2 - \gamma)\beta \left( \frac{\lambda - 1}{\lambda} \right) \right) - \frac{1}{\beta + 1} (t_2 - \gamma)\beta
\end{equation}

\begin{equation}
\frac{1}{\beta + 1} \left( (t_2 - \gamma)\beta \left( \frac{\lambda - 1}{\lambda} \right) \right) - \frac{1}{\beta + 1} (t_2 - \gamma)\beta
\end{equation}

\begin{equation}
\frac{1}{\beta + 1} \left( (t_2 - \gamma)\beta \left( \frac{\lambda - 1}{\lambda} \right) \right) - \frac{1}{\beta + 1} (t_2 - \gamma)\beta
\end{equation}

\begin{equation}
\frac{1}{\beta + 1} \left( (t_2 - \gamma)\beta \left( \frac{\lambda - 1}{\lambda} \right) \right) - \frac{1}{\beta + 1} (t_2 - \gamma)\beta
\end{equation}

\begin{equation}
\frac{1}{\beta + 1} \left( (t_2 - \gamma)\beta \left( \frac{\lambda - 1}{\lambda} \right) \right) - \frac{1}{\beta + 1} (t_2 - \gamma)\beta
\end{equation}

\begin{equation}
\frac{1}{\beta + 1} \left( (t_2 - \gamma)\beta \left( \frac{\lambda - 1}{\lambda} \right) \right) - \frac{1}{\beta + 1} (t_2 - \gamma)\beta
\end{equation}

\begin{equation}
\frac{1}{\beta + 1} \left( (t_2 - \gamma)\beta \left( \frac{\lambda - 1}{\lambda} \right) \right) - \frac{1}{\beta + 1} (t_2 - \gamma)\beta
\end{equation}

\begin{equation}
\frac{1}{\beta + 1} \left( (t_2 - \gamma)\beta \left( \frac{\lambda - 1}{\lambda} \right) \right) - \frac{1}{\beta + 1} (t_2 - \gamma)\beta
\end{equation}

4. Numerical examples

Let $\alpha = 200$, $\lambda = 1.05$, $c_1 = 20$, $c_2 = 30$, $c_3 = 40$, $\gamma = 0.2$ in appropriate units. From (22) and (23), we obtain the optimum values of $t_3$ and $t_4$. Putting the optimum values of $t_2$ and $t_4$ in equation (12) and (15) we obtain the optimum values of $t_1$ and $t_2$ respectively.

Example-1: Suppose $\beta = 10$, the optimum values of $t_i$ ($i = 1, 2, 3, 4$) are $t_1^* = 0.835609$, $t_2^* = 0.835609$, $t_3^* = 1.17059$, $t_4^* = 1.17391$. Substitute these optimum values of $t_1$, $t_2$, $t_3$ and $t_4$ in equation (18), we get the optimum average.
cost $C^* = 748.454$. The optimum values of $P$ and $S$ have been obtained from equation (10) and equation (14) respectively and the values are $P^* = 0.215242$ and $S^* = 0.515918$.

Example-2: Suppose $\beta = 12$, the optimum values of $t_i (i = 1, 2, 3, 4)$ are $t_1^* = 0.903648$, $t_2^* = 1.10686$, $t_3^* = 1.19339$, $t_4^* = 1.1961$. Substitute these optimum values of $t_1$, $t_2$, $t_3$ and $t_4$ in equation (18), we get the optimum average cost $C^* = 761.443$. The optimum values of $P$ and $S$ obtained from equation (10) and equation (14) respectively and the values are $P^* = 0.245535$ and $S^* = 0.511773$.

5. Sensitivity Analysis

To study the effects of changes in the system of parameters, $c_1, c_2, c_3, \alpha, \beta, \gamma$ and $\lambda$ on the optimal cost derived by the proposed method, sensitivity analysis has been performed by changing (increasing or decreasing) the parameters by 20% and 50% and changing one parameter at a time and keeping the remaining parameters at their original values. Increase in the value of the parameters $\beta$ then cycle length $t_1^*, t_2^*, t_3^*, t_4^*$ and $C^*$ is increased. The results are shown in Table -1 given below.
Table -1  

<table>
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<tr>
<th>Parameter</th>
<th>% Change</th>
<th>Change in $I_1^*$</th>
<th>Change in $I_2^*$</th>
<th>Change in $I_3^*$</th>
<th>Change in $I_4^*$</th>
<th>Change in $C^*$</th>
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<td>0.9998130</td>
<td>1.10078</td>
<td>1.10386</td>
<td>377.309</td>
</tr>
</tbody>
</table>
On the basis of the results of Table - 1, the following observations can be made:

(i) Decrease in the values of either of the parameters $\beta, c_3$ will result in decrease of $t_1^*, t_2^*, t_3^*, t_4^*$ and $C^*$.

(ii) Decrease in the value of the parameters $c_1$ will result in increase in $t_1^*, t_2^*, t_3^*, t_4^*$ and decrease in $C^*$.

(iii) Decrease in the value of the parameters $\gamma$ will result in decrease in $t_1^*, t_2^*, t_3^*, t_4^*$ and increase in $C^*$.

(iv) Decrease in the value of the parameters $\lambda, c_1$ will result in decrease in $t_1^*, t_2^*$ and increase in $t_3^*, t_4^*, C^*$.

(v) Decrease in the value of the parameter $c_2, c_1$ will result in decrease in $t_1^*, t_2^*, t_3^*$ and increase in $t_4^*, C^*$.

6. Conclusion

Time-dependence of demand is usually non-linear in nature and the degree of non-linearity determines the intensity of its increase or decrease. Power law functional form in time is more appropriate for demand rate of a product. Considering these facts we have assumed that the time-dependence of demand $R(t)$ follows a three parameters Weibull distribution in this paper. Here the production rate has been assumed to be finite and proportional to the demand where shortages are allowed, which is completely backlogged. Analytical solutions of the model are illustrated with the help of suitable numerical examples. Sensitivity analysis has also been made in the present paper.

References


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