Some Separation Axioms in Biminimal Structure Spaces

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Abstract

The purpose of this paper is to introduce and study some fundamental properties of $CT_0$-space, $CT_1$-space, $C$-Hausdorff space, $C$-regular space and $C$-normal space in a biminimal structure space.

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1 Introduction

The concept of minimal structure (briefly m-structure) was introduced by V. Popa and T. Noiri [3] in 2000. Also they introduced the notion of $m_X$-open set and $m_X$-closed set and characterize those sets using $m_X$-closure and $m_X$-interior operators respectively. Further they introduced $M$-continuous functions and studied some of its basic properties. C. Boonbok [1] in 2009 introduced the concept of biminimal structure space and studied $m^1_X m^2_X$-closed sets and $m^1_X m^2_X$-open sets in biminimal structure spaces. In this paper we introduce and study some separation axioms in a biminimal structure space.

2 Preliminaries

Definition 2.1. [2] Let $X$ be a nonempty set and $P(X)$ the power set of $X$. A subfamily $m_X$ of $P(X)$ is called a minimal structure (briefly m-structure) on $X$ if $\emptyset \in m_X$ and $X \in m_X$. 
By \((X, m_X)\), we denote a nonempty set \(X\) with an m-structure \(m_X\) on \(X\) and it is called an m-space. Each member of \(m_X\) is said to be \(m_X\)-open and the complement of an \(m_X\)-open set is said to be \(m_X\)-closed.

**Definition 2.2.** [1] Let \(X\) be a nonempty set and \(m_X^1, m_X^2\) be minimal structures on \(X\). A triple \((X, m_X^1, m_X^2)\) is called a **biminimal structure space** (briefly **bim-space**).

**Definition 2.3.** [1] Let \((X, m_X^1, m_X^2)\) be a biminimal structure space and \(Y\) be a subset of \(X\). Define minimal structures \(m_Y^1\) and \(m_Y^2\) on \(Y\) as follows:
\[
m_Y^1 = \{A \cap Y | A \in m_X^1\}
\]
\[
m_Y^2 = \{B \cap Y | B \in m_X^2\}.
\]
A triple \((Y, m_Y^1, m_Y^2)\) is called a **biminimal structure subspace** (briefly **bim-subspace**) of \((X, m_X^1, m_X^2)\).

## 3 Separation Axioms

In this section, we introduce the concepts of \(CT_1\)-space, \(C\)-Hausdorff space, \(C\)-normal space and \(C\)-regular space in biminimal structure spaces and study some of their fundamental properties.

**Definition 3.1.** A biminimal structure space \((X, m_X^1, m_X^2)\) is called a **\(CT_0\)-space** if for each point \(x\) different from \(y\) in \(X\), there exists an open set \(A\) of \(x\) in \(m_X^1\) and open set \(B\) of \(y\) in \(m_X^2\) in which \(x \notin B\) or \(y \notin A\).

**Proposition 3.2.** Let \((Y, m_Y^1, m_Y^2)\) be a biminimal structure subspace of \((X, m_X^1, m_X^2)\). If \((X, m_X^1, m_X^2)\) is a \(CT_0\)-space, so is \((Y, m_Y^1, m_Y^2)\).

**Proof.** Let \((X, m_X^1, m_X^2)\) be a \(CT_0\)-space and \((Y, m_Y^1, m_Y^2)\) a subspace of \((X, m_X^1, m_X^2)\). If \(x, y \in Y\) such that \(x \neq y\), there exists an open set \(A\) of \(x\) in \(m_X^1\) and open set \(B\) of \(y\) in \(m_X^2\) in which at least one of \(A\) and \(B\) not contain all of \(x\) and \(y\). We defined an open set \(U = A \cap Y \in m_Y^1\) and \(V = B \cap Y \in m_Y^2\), hence \(x \in U\), \(y \in V\) and one of \(U\) and \(V\) not contain all of \(x\) and \(y\).

**Definition 3.3.** A biminimal structure space \((X, m_X^1, m_X^2)\) is called a **\(CT_1\)-space** if for each point \(x\) different from \(y\) in \(X\), there exists an open set \(A\) in \(m_X^1\) and open set \(B\) in \(m_X^2\) in which \(x \in A\), \(y \in B\), \(x \notin B\) and \(y \notin A\).

**Proposition 3.4.** Let \((Y, m_Y^1, m_Y^2)\) be a biminimal structure subspace of \((X, m_X^1, m_X^2)\). If \((X, m_X^1, m_X^2)\) is a \(CT_1\)-space, so is \((Y, m_Y^1, m_Y^2)\).

**Proof.** Let \((X, m_X^1, m_X^2)\) be a \(CT_1\)-space and \((Y, m_Y^1, m_Y^2)\) a subspace of \((X, m_X^1, m_X^2)\). If \(x, y \in Y\) such that \(x \neq y\), there is an open set \(A \in m_X^1\) and \(B \in m_X^2\) such that \(x \in A\), \(y \in B\), \(x \notin B\) and \(y \notin A\). We defined an open set \(U = A \cap Y \in m_Y^1\) and \(V = B \cap Y \in m_Y^2\), and hence \(x \in U\), \(y \in V\), \(x \notin V\) and \(y \notin U\).
Definition 3.5. A biminimal structure space $(X, m^1_X, m^2_X)$ is called a $C$-Hausdorff space if for each point $x$ different from $y$ in $X$, there exists an open set $A$ in $m^1_X$ and open set $B$ in $m^2_X$ in which $x \in A$, $y \in B$ and $A \cap B = \emptyset$.

Proposition 3.6. Let $(Y, m^1_Y, m^2_Y)$ be a biminimal structure subspace of $(X, m^1_X, m^2_X)$. If $(X, m^1_X, m^2_X)$ is a $C$-Hausdorff space, so is $(Y, m^1_Y, m^2_Y)$.

Proof. Let $(X, m^1_X, m^2_X)$ be a $C$-Hausdorff space and $(Y, m^1_Y, m^2_Y)$ a subspace of $(X, m^1_X, m^2_X)$. If $x, y \in Y$ such that $x \neq y$, there is an open set $A \in m^1_X$ and $B \in m^2_X$ such that $x \in A$, $y \in B$ and $A \cap B = \emptyset$. We defined an open set $U = A \cap Y \in m^1_Y$ and $V = B \cap Y \in m^2_Y$, we have $x \in U$, $y \in V$ and $U \cap V = \emptyset$. \hfill \Box

Definition 3.7. A biminimal structure space $(X, m^1_X, m^2_X)$ is called a $C$-regular space if $x$ is any point in $X$ and $F$ any nonempty subset of $X$ which does not containing $x$, there exists an open set $A$ in $m^1_X$ and open set $B$ in $m^2_X$ such that $x \in A$, $F \subset B$ and $A \cap B = \emptyset$.

Proposition 3.8. A subspace of $C$-regular space is $C$-regular.

Proof. Let $(X, m^1_X, m^2_X)$ be a $C$-regular space and $(Y, m^1_Y, m^2_Y)$ a subspace of $(X, m^1_X, m^2_X)$. If $x \in Y$ and $F$ is any subset of $Y$, there exists an open set $A$ in $m^1_X$ and open set $B$ in $m^2_X$ such that $x \in A$, $F \subset B$ and $A \cap B = \emptyset$. We defined an open set $U = A \cap Y \in m^1_Y$ and $V = B \cap Y \in m^2_Y$, then $x \in U$, $F \subset V$ and $U \cap V = \emptyset$. \hfill \Box

Definition 3.9. A biminimal structure space $(X, m^1_X, m^2_X)$ is called a $C$-normal space if for every pair of nonempty disjoint subsets $A$ and $B$ of $X$ can be separated by open sets $U \in m^1_X$ and $V \in m^2_X$ such that $U \cap V = \emptyset$.

Proposition 3.10. A subspace of $C$-normal space is $C$-normal space.

Proof. Let $(X, m^1_X, m^2_X)$ be a $C$-normal space and $(Y, m^1_Y, m^2_Y)$ a subspace of $(X, m^1_X, m^2_X)$. If $A$ and $B$ are nonempty disjoint closed subsets of $X$, there exists an open set $C$ in $m^1_X$ and open set $D$ in $m^2_X$ such that $A \subset C$, $B \subset D$ and $C \cap D = \emptyset$. We defined an open set $U = C \cap Y \in m^1_Y$ and $V = D \cap Y \in m^2_Y$, then $A \subset U$, $B \subset V$ and $U \cap V = \emptyset$. \hfill \Box

Example 3.11. Let $X = \{1, 2, 3\}$. Define $m$-structures $m^1_X$ and $m^2_X$ on $X$ as follows: $m^1_X = \{\emptyset, \{1\}, \{2\}, \{1, 3\}, \{2, 3\}, X\}$ and $m^2_X = \{\emptyset, \{1\}, \{2\}, \{1, 3\}, \{2, 3\}, X\}$. Then $(X, m^1_X, m^2_X)$ is a $C_{T_0}$-space, $C_{T_1}$-space, $C$-Hausdorff space, and $C$-normal space, but not $C$-regular space by consider a point $x = 3$ in $X$ and a subset $F = \{1, 2\}$ of $X$.

Example 3.12. By example 3.9, If we defined an $m$-structure $m^2_X = \{\emptyset, \{1\}, \{2\}, \{1, 2\}\}\{1, 3\}, \{2, 3\}, X\}$, so we have $(X, m^1_X, m^2_X)$ is a $C$-regular space.
Example 3.13. By example 3.9, Let $Y = \{1, 2\}$, so $Y \subset X$ and we have an $m$-structures $m^1_Y$ and $m^2_Y$ on $Y$ as follows: $m^1_Y = \{\emptyset, \{1\}, \{2\}, \{1, 2\}, Y\}$ and $m^2_Y = \{\emptyset, \{1\}, \{2\}, \{1, 2\}, Y\}$. Then $(Y, m^1_Y, m^2_Y)$ is a $C\!T_0$-space, $C\!T_1$-space, $C$-Hausdorff space, and $C$-normal subspace of $(X, m^1_X, m^2_X)$.


Corollary 3.15. A $C$-Hausdorff space is a $C\!T_1$-space.

Example 3.16. Let $X = \{1, 2, 3\}$ and $Y = \{a, b, c\}$. Define an $m$-structures on $X$ and $Y$ as follows: $m^1_X = \{\emptyset, \{1\}, \{2, 3\}, \{1, 3\}, \{2\}, X\}$ and $m^1_Y = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}, Y\} = m^2_Y$. Then $(X, m^1_X, m^2_X)$ and $(Y, m^1_Y, m^2_Y)$ are $C$-Hausdorff spaces. Then $(X \times Y, m^1_{X\times Y}, m^2_{X\times Y})$ is a $C$-Hausdorff space.

Proposition 3.17. Let $(X, m^1_X, m^2_X)$ and $(Y, m^1_Y, m^2_Y)$ be a $C$-Hausdorff bi-minimal structure space. Then $(X \times Y, m^1_{X\times Y}, m^2_{X\times Y})$ is a $C$-Hausdorff space.

Proof. Let $(x_1, y_1), (x_2, y_2) \in X \times Y$ such that $(x_1, y_1) \neq (x_2, y_2)$. Then $x_1 \neq x_2$ and $y_1 \neq y_2$. Since $(X, m^1_X, m^2_X)$ is a $C$-Hausdorff space, there exists an open set $A$ in $m^1_X$ and open set $B$ in $m^2_X$ such that $x_1 \in A$, $x_2 \in B$ and $A \cap B = \emptyset$. Similarly, since $(Y, m^1_Y, m^2_Y)$ is a $C$-Hausdorff space, there exists an open set $C$ in $m^1_Y$ and open set $D$ in $m^2_Y$ such that $y_1 \in C$, $y_2 \in D$ and $C \cap D = \emptyset$. Consequently, $A \times C \in m^1_{X\times Y}$, $B \times D \in m^2_{X\times Y}$ such that $(x_1, y_1) \in A \times C$, $(x_2, y_2) \in B \times D$ and $(A \times C) \cap (B \times D) = \emptyset$. Hence, $(X \times Y, m^1_{X\times Y}, m^2_{X\times Y})$ is a $C$-Hausdorff space.

References


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