The Number of Same Order Elements and Solvability of Finite Groups

Rulin Shen

Department of Mathematics
Hubei University for Nationalities
Enshi, Hubei, P.R. China, 445000
shenrulin@hotmail.com

Abstract

In 1980’s, J.G.Thompson posed a problem that whether two same order type finite groups had same solvability. In this short paper we prove that if every number of elements of the same order of finite groups has no divisor 4, then Thompson problem is true.

Mathematics Subject Classification: 20D60, 20D06

Keywords: same order elements, Thompson problem, solvable groups

1. Introduction

Given a finite group $G$, how to judge the solvability of $G$? The well known Burnside Theorem asserted that groups with order $p^aq^b$ are solvable. The famous Odd Order Theorem proved that all finite groups of odd order are solvable. In view of the theorem of classification of finite simple groups, it is not hard to prove if $15 \nmid G$, then $G$ is solvable. In paper [18], Zhang proved that if the number of Sylow 2-subgroups is coprime to every one of Sylow $p$-subgroups for $p \neq 2$, then $G$ is solvable. Using some quantities of finite groups to judge the solvability, it is an interesting subject. In 1980’s, J.G.Thompson put forward a problem about the solvability of finite groups. For each finite group $G$ and each integer $d \geq 1$, let $G(d) = \{x \in G | x^d = 1 \}$. Called that $G_1$ and $G_2$ are of the same order type if and only if $|G_1(d)| = |G_2(d)|$, $d = 1, 2, \cdots$.

Problem (J.G. Thompson, [13]) Suppose $G_1$ and $G_2$ are groups of the same
order type. Suppose that $G_1$ is solvable. Is it true that $G_2$ is also necessarily solvable?

That is, for the groups $G$ of even order, we can not judge the solvability of $G$ only using the order of $G$, but we may judge it using the order type of $G$ if the answer of above Problem is in the affirmative. Thompson problem is true for the following cases:

- $G_1$ is super solvable (see [14]).
- the number of the maximal order elements of $G_1$ is $2p$, $2p^2$ or 30 (see [3], [8], [6]).
- the cardinality of the set of numbers of the same order elements of $G_1$ is not more than 2 (see [13]).

In this paper, we prove that if every number of elements of the same order of finite groups $G_1$ has no divisor 4, then Thompson problem is true. In fact, we have proved the following theorem.

**Theorem.** If every number of elements of the same order of finite groups $G$ has no divisor 4, then $G$ is solvable.

2. Preliminaries Let $G$ be a finite group. Denote by $\pi(n)$ the set of prime divisors of the integer $n$. Let $\pi(G)$ be $\pi(|G|)$. Denote by $\pi_e(G)$ the set of orders of elements of $G$. The prime graph of $G$ (denoted by $\Gamma(G)$) is defined as a graph with vertices $\pi(G)$, and two vertices $p$ and $q$ are connected by an edge if and only if $G$ contains an element of order $pq$. We denote the number of prime graph components of $G$ (that is, connected components of $\Gamma(G)$) by $t(\Gamma(G))$ and the set of vertices of prime graph components of $G$ as $\pi_i(G)$, where $1 \leq i \leq t(\Gamma(G))$. And if $G$ is of even order, we always assume that $2 \in \pi_1(G)$ (see [15]). Let $S$ be a subset of $G$, and denote by $f_T(n)$ the number of elements of order $n$ in $T$. Denote by $\phi(n)$ the Euler function.

A graph $\Gamma$ is said to a tree if it is a connected graph without loops. We call that a graph is a forest if any connected component of the graph is a tree. In the paper [9], Lucido determined the structure of groups with the prime graph $\Gamma(G)$ a tree. The following lemmas refer to [9].

**Lemma 1.** Let $G$ be a finite non-abelian simple group. Then $\Gamma(G)$ is a forest if and only if $G$ is one of the following groups. Therefore, in any case, $\Gamma(G)$ is not a tree.

- (a) $A_5$, $A_6$, $A_7$, $A_8$, $M_{11}$, $M_{22}$, $L_4(3)$, $S_4(3)$, $G_2(3)$, $U_4(3)$, $U_5(2)$, $2F_4(2)'$.
- (b) $L_2(q)$ with $|\pi(\frac{q-1}{2(q-1)})| \leq 2$ and $|\pi(\frac{q+1}{2(q-1)})| \leq 2$.
- (c) $L_3(q)$ with $|\pi(\frac{q^2+q+1}{3(q-1)})| \leq 2$ and $|\pi(\frac{q^2-1}{3(q-1)})| \leq 2$. 
(c) $U_3(q)$ with $|\pi(\frac{q^2-1}{3q+1})| \leq 2$ and $|\pi(\frac{q^2-1}{3q+1})| \leq 2$.

(d) $2B_2(q^2)$ with $|\pi(q^2 \pm \sqrt{2q+1})| \leq 2$ and $|\pi(q^2 - 1)| \leq 2$, where $q^2 = 2^f$ or $q = 2^f$ with $f$ an odd prime.

(e) $2G_2(q)$ with $|\pi(q \pm \sqrt{3q + 1})| \leq 2$ and $|\pi(q \pm 1)| \leq 2$, where $q = 3^f$, with $f$ an odd prime.

**Lemma 2.** Let $G$ be an almost simple group such that $\Gamma(G)$ is a tree. Then $G$ is one of the following:

(a) $\text{Aut}(A_6)$, $\text{Aut}(B_2(3))$.

(b) $\text{PGL}(4, 3) \leq G \leq \text{Aut}(\text{PSL}(4, 3))$.

(c) $\text{PGU}(3, 8) \leq G \leq \text{Aut}(\text{PSU}(3, 8))$.

(d) $L_2(p^f)(\alpha)$, where $p$ is a prime greater than 3, $f$ is an odd prime and $\alpha$ is a field automorphism of order $f$.

(e) $U_4(3)\langle \delta \rangle \leq G \leq \text{Aut}(U_4(3))$, where $\delta$ is a diagonal automorphism of order 2.

(f) $\text{PGL}(3, 4) \leq G \leq \text{PGL}(3, 4)\langle \alpha \rangle$ with $\alpha$ a graph-field automorphism of order 2.

(g) $L_3(q)\langle \alpha \rangle$ with $q = 9, 25, 49$ and $\alpha$ a field automorphism of order 2.

(h) $2B_2(2^{f^2})\langle \alpha \rangle$ with $\alpha$ a field automorphism of order $f$.

**Lemma 3.** Let $G$ be a finite non-solvable group such that $\Gamma(G)$ is a tree. If $N$ is the soluble radical of $G$, then

(i) $\overline{G} = G/N$ is an almost simple group, that is $S \leq \overline{G} \leq \text{Aut}(S)$, with $S$ a finite simple non-abelian group of items (a) – (e) listed in the lemma 1.

(ii) $|\pi(N)| \leq 3$. Moreover, if $\pi(N) = \{p_1, p_2, p_3\}$, then $\pi(N) \notin \pi(\overline{G})$.

Next we describe a relation of the same order element of cosets and quotient groups.

**Lemma 4.** Let $G$ be a finite group with a normal subgroup $N$. If $x \in G \setminus N$ has order $m$, then $f_{N_x}(m) = f_{Ny}(m)$ for all cosets $Ny$ which are $G/N$-conjugate to $Nx$.

**Proof.** Suppose $Ny$ is $G/N$-conjugate to $Nx$, so $Ny = N \gamma^{-1} x \gamma$ for some $\gamma \in G$. Then the map $\phi : Nx \mapsto Ny$, defined by $nx \mapsto g^{-1} nx g$, induces a bijection between the subset of elements of order $m$ in $Nx$ and the corresponding subset of $Ny$. \hfill $\square$

3. **Proof of the theorem** Let $m = p_1^{t_1} p_2^{t_2} \cdots p_k^{t_k}$. Suppose that $G$ has an element of order $m$. Clearly, $\prod_{i=1}^k (p_i - 1) \mid f_G(m) \mid f_G(m)$. Since $4 \nmid f_G(m)$, we have $\pi(m) \subseteq \{2, p\}$ with $p$ an odd prime. Moreover, $8 \notin \pi_u(G)$ and $f_G(pr) = 0$ for every pair distinct odd primes $p$ and $r$. Hence the prime graph $\Gamma(G)$ of $G$
is a forest. In addition, it is easy to see that \( p \equiv -1 \pmod{4} \) and \( f_G(4p) = 0 \) for every odd prime \( p \in \pi(G) \). In particular, 5 and 13 don’t divide the order of \( G \). In view of the lemma 3, we have \( \overline{G} = G/N \) is an almost simple group, i.e., \( S \leq \overline{G} \leq Aut(S) \), where \( N \) is the solvable radical of \( G \) and \( S \) is one of listed groups of the lemma 1. Next we will prove that the possible group \( \overline{G} \) is one of the following.

1. \( L_2(7), L_2(3^5), L_2(8) \) and \( L_2(8)\langle \alpha \rangle \) with \( \alpha \) a field automorphism of order 3.
2. \( L_2(p) \), where \( p \equiv 3 \pmod{8} \), \( 16 \nmid p^2 - 1 \), \( |\pi(p - 1)| = 1 \) and \( |\pi(p + 1)| = 1 \).
3. \( 2G_2(q) \), where \( q = 3^f \), \( f > 3 \) a prime, \( |\pi(q \pm \sqrt{3q} + 1)| = 1 \) and \( |\pi(q \pm 1)| = 2 \).

In fact, since \( 8 \notin \pi_e(G) \), we have \( 8 \notin \pi_e(S) \), and hence \( S \) is one of following groups by the theorem 2 of [4].
(a) \( A_7, A_8, A_9, J_1, A_6, S_4(3), U_4(2) \).
(b) \( L_2(q), q > 3, q \neq 9, q^2 - 1 \) is not divisible by 32.
(c) \( L_3(2^m), U_3(2^m) \), where \( m \geq 2 \).
(d) \( L_4(2^m), U_4(2^m) \), where \( m \geq 2 \).
(e) \( 2G_2(3^{2m+1}) \), or \( 2B_2(2^{2m+1}) \), where \( m \geq 1 \).
(f) \( S_4(3^m), m > 1 \), or \( S_4(q), q > 3, q \neq 3^m \) and \( q^2 - 1 \) is not divisible by 16.

Since \( 5, 13 \notin \pi(G) \), we have \( 5, 13 \notin \pi(S) \), and then it is easy to see that \( S \) is one of above (b), (c) and \( 2G_2(3^{2m+1}) \) by [5]. Moreover, if \( S = L_2(p^f) \) with \( p \) an odd prime, then \( \frac{p^f - 1}{2} \) is odd since \( p \equiv -1 \pmod{4} \). Also since there exists an element of order \( \frac{p^f - 1}{2} \), we have \( |\pi(p^f - 1)| = 1 \). Suppose that \( \frac{p^f - 1}{2} = r^t \) with \( r \) prime. If \( f \geq 3 \), there is a primitive prime divisor \( r_f \) by Zsigmondy theorem [17]. Hence \( (\frac{p^f - 1}{2}, r_f) = 1 \), then \( p = 3 \). We can get a diphphantine equation \( \frac{3^f - 1}{2} = r^t \). This equation has only one solution \( (f, r, t) = (5, 11, 2) \) when \( f > 2 \) and \( t > 1 \) (see [1]), then \( S = L_2(3^5) \) or \( t = 1 \). On the other hand, since \( r \equiv -1 \pmod{4} \), if \( t = 1 \), then we have \( 3^f + 1 \equiv 0 \pmod{8} \), which contradicts that \( f \) is odd. If \( f = 2 \), then \( \frac{3^2 - 1}{2} = (\frac{p+1}{2})^2 \). Since \( (\frac{p+1}{2}, p+1) = 1 \), we have \( p = 3 \), and then \( S = L_2(9) \). But \( 5 | |L_2(9)| \), a contradiction. Thus \( S = L_2(p) \) or \( L_2(3^5) \). In addition, there exists an element order \( \frac{p^f + 1}{2} \). If \( 4 \in \pi_e(L_2(p)) \), then \( \frac{p^f + 1}{8} = 1 \) (otherwise \( 4 \cdot \frac{p^f + 1}{8} \in \pi_e(S) \), a contradiction), and hence \( S = L_2(7) \) or \( exp(P_2) = 2 \) with \( P_2 \) the Sylow 2-subgroup of \( S \). In view of well known Walter’s result about simple groups with ableian Sylow 2-sbubgroup (see [16]), we can get that \( p \equiv 3, 5 \pmod{8} \). Since \( p \equiv -1 \pmod{4} \), so we have \( p \equiv 3 \pmod{8} \).

Next we consider the case of \( L_2(2^f) \). Similarly, we have both \( 2^f + 1 \) and \( 2^f - 1 \) have only one prime divisor. If \( t = 1 \), then \( f = 2 \), and hence \( S = L_2(4) \).
which contradicts that $5 \notin \pi(S)$. Assume that $2^f + 1 = r^a$ or $2^f - 1 = s^b$ with $a, b > 1$. These equations are special cases of the Catalan equation $x^p - y^q = 1$. Catalan conjectures that this equation has only solution $(x, y, p, q) = (3, 2, 2, 3)$ if the unknowns $x, y, p$ and $q$ take integer values all $\geq 2$. Catalan’s conjecture was finally substantiated by P. Mihăilescu (see [12]). Thus $S$ is $L_2(8)$.

Now we consider $\overline{G}$. Since $8 \in \pi_e(PGL_2(7))$ and $5 \in \pi(Aut(L_2(3^5)))$, we have $\overline{G}$ is one of $PGL_2(3^5)$, $L_2(8)\langle \alpha \rangle$ with $\alpha$ a field automorphism of order 3, or $PGL_2(p)$. Moreover, by the theorem 2 of [2], we know that $PGL_2(q)$ has a cyclic subgroup of order $q + 1$. So $4 \cdot \frac{q + 1}{3} \in \pi_e(PGL_2(q)) = \pi_e(\overline{G})$, and then $\frac{q + 1}{3} = 1$, that is $PGL_2(3)$, which is solvable.

If $S$ is one of $L_3(2^m), U_3(2^m)$, which are denoted by $L_3^{-1}(2^m)$ and $L_3^{-1}(2^m)$, respectively. It is not hard to see $L_3^{-1}(2^m)$ has an element of order $\frac{4^{m+1}}{(3, 2^m - 1)}$, where $\epsilon = \pm 1$. Since $(2^m + 1, 2^m - 1) = 1$, we have $(2^m, 2^m - 1 - \epsilon) = 3 = 2^m - \epsilon$, that is, $2^m = \epsilon + 3$. So we have $S$ is $L_3(4)$ or $U_3(2)$. But it is clear that $5 \in \pi(L_3(4))$ and $U_3(2)$ is not a simple group, and hence it is impossible in this case.

Finally, if $S$ is a Ree group $^2G_2(3^f)$, by the lemma 1, then we have $|\pi(q \pm \sqrt{3}^2 + 1)| = 1$ and $|\pi(q \pm 1)| = 2$, where $q = 3^f$, with $f$ an odd prime. If $\overline{G}$ is a almost simple group, then $\Gamma(\overline{G})$ is disconnected by the lemma 2 above (otherwise, $\Gamma(\overline{G})$ is a tree), and hence $\overline{G}$ is $^2G_2(q)\langle \alpha \rangle$, where $\alpha$ is a field automorphism of order $3^f$ with $t \geq 1$. Since $f$ is prime, we have $f = 3$. That is $S = ^2G_2(27)\langle \alpha \rangle$ with $\alpha$ is a field automorphism of order 3. By [5], we know that $13 \in \pi(^2G_2(27))$, which is a contradiction.

In the sequel we consider the number of elements of the same order of $G$. We divide into two cases.

**Case 1.** $2 \notin \pi(N)$. Then $\Gamma(\overline{G})$ is also a tree. It is easy to see that above the prime graphs of groups (1), (2) and (3) are all disconnected, a contradiction.

**Case 2.** $2 \in \pi(N)$. If $\Gamma(\overline{G})$ is a tree, similarly, then it is impossible. If $\Gamma(\overline{G})$ is not a tree, then $\Gamma(\overline{G})$ is disconnected and $\Gamma_i(\overline{G})$ has only one prime for $i \geq 2$. Suppose that $\pi_i(\overline{G}) = \{p_i\}$. Since $|\pi(N)| \leq 3$, there exists a prime $r \in \pi(\overline{G})$ such that $r \notin \pi(N)$. In fact, otherwise $\pi(\overline{G}) \subseteq \pi(N)$, then $\pi(\overline{G}) = \pi(N)$ since $|\pi(\overline{G})| \geq 3$, which contradicts that $\pi(N) \nsubseteq \pi(\overline{G})$ in the lemma 3, (ii).

If $S$ is $L_2(8)\langle \alpha \rangle$ with $\alpha$ a field automorphism of order 3, then $\pi(S) = \{2, 3, 7\}$. By [5], there exist two conjugacy classes of order 3, whose sizes are $c_{31} = 168$ and $c_{32} = 252$. There exist three conjugacy classes of order 7, whose sizes are $c_{71} = c_{72} = c_{73} = 72$. If $3 \notin \pi(N)$, in view of the lemma 4, then $f_G(3) = c_{31}f_{N_{x_1}}(3) + c_{32}f_{N_{x_2}}(3) = 168f_{N_{x_1}}(3) + 252f_{N_{x_2}}(3)$, and hence $4 \nmid f_G(3)$, a contradiction. If $7 \notin \pi(N)$, then we have $f_G(7) = c_{71}f_{N_{y_1}}(7) + c_{72}f_{N_{y_2}}(7) + c_{73}f_{N_{y_3}}(7) = 72(f_{N_{y_1}}(7) + f_{N_{y_2}}(7) + f_{N_{y_3}}(7))$, which also contradicts that $4 \nmid f_G(7)$.

For remaining groups $\overline{G}$ of above (1), (2) and (3), we know that the number
t(\Gamma(G)) of prime graph components of \(G\) is 3 (see [5],[15]). Then there exists a prime \(p_i \in \pi_i(G)\) with \(i \geq 2\) such that \(p_i \not\in \pi(N)\). Indeed, for every \(p_i \in \pi_i(G)\) with \(i \geq 2\), if \(p_i\) is in \(\pi(N)\), then \(\pi(N) = \{2, p_2, p_3\}\), and hence \(\pi(N) \subset \pi(G)\), which also contradicts the lemma 3, (ii).

If \(S = L_2(7)\), then \(\tau_2(S) = \{3\}\) and \(\tau_3(S) = \{7\}\). In view of [5], there exist one conjugacy class of order 3 whose size is \(c_{31} = 56\), and two classes of order 7 whose sizes are \(c_{71} = c_{72} = 24\). If \(3 \not\in \pi(N)\), by the lemma 4, then \(f_G(3) = c_{31}f_{N_{x_1}}(3) = 56f_{N_{x_1}}(3)\), and hence \(4 \mid f_G(3)\), a contradiction. If \(7 \not\in \pi(N)\), then we have \(f_G(7) = c_{71}f_{N_{y_1}}(7) + c_{72}f_{N_{y_2}}(7) = 24(f_{N_{y_1}}(7) + f_{N_{y_2}}(7))\), which also contradicts that \(4 \nmid f_G(7)\).

If \(S = L_2(8)\), then \(\tau_2(S) = \{3\}\) and \(\tau_3(S) = \{7\}\). It is easy to see that there exist one conjugacy class of order 3 whose size is \(c_{31} = 56\), and three classes of order 7 whose sizes are \(c_{71} = c_{72} = c_{73} = 72\). If \(3 \not\in \pi(N)\), then \(f_G(3) = c_{31}f_{N_{x_1}}(3) = 56f_{N_{x_1}}(3)\), and hence \(4 \mid f_G(3)\), a contradiction. If \(7 \not\in \pi(N)\), then we have \(f_G(7) = c_{71}f_{N_{y_1}}(7) + c_{72}f_{N_{y_2}}(7) + c_{73}f_{N_{y_3}}(7) = 72(f_{N_{y_1}}(7) + f_{N_{y_2}}(7) + f_{N_{y_3}}(7))\), which also contradicts that \(4 \nmid f_G(7)\).

If \(S = L_2(3^5)\), then \(\tau_2(S) = \{11\}\) and \(\tau_3(S) = \{61\}\). Similarly, there exist two conjugacy classes of order 11 whose size is \(c_{1,1,1} = c_{1,1,2} = 2^23^5 \cdot 61\), and two classes of order 61 whose sizes are \(c_{61,1} = c_{61,2} = 2^23^511^2\). If \(11 \not\in \pi(N)\), then \(f_G(11) = c_{1,1,1}f_{N_{x_1}}(11) = 2^23^5 \cdot 61f_{N_{x_1}}(11)\), and hence \(4 \mid f_G(11)\), a contradiction. If \(61 \not\in \pi(N)\), then we have \(f_G(61) = c_{61,1}f_{N_{y_1}}(61) + c_{61,2}f_{N_{y_2}}(61) = 2^23^511^2(f_{N_{y_1}}(61) + f_{N_{y_2}}(61))\), which also contradicts that \(4 \nmid f_G(61)\).

If \(S = L_2(p)\) with \(p = 3 (\text{mod} 8)\), let \(p_2 \in \pi(p-1)\), then \(\tau_2(S) = \{p_2\}\) and \(\tau_3(S) = \{p\}\). By the Chap.2, Theorem 8.2, 8.3, 8.4 and 8.5 of [7], there exist two conjugacy classes of order \(p_2\) whose sizes are \(c_{p_2,1} = p(p+1)\), and two classes of order \(p\) whose sizes are \(c_{p,1} = c_{p,2} = \frac{p^2-1}{2}\). If \(p \not\in \pi(N)\), then \(f_G(p_2) = c_{p_2,1}f_{N_{x_1}}(p_2) + c_{p_2,2}f_{N_{x_2}}(p_2) = p(p+1)(f_{N_{x_1}}(p_2) + f_{N_{x_2}}(p_2))\), and hence \(4 \mid f_G(p_2)\), a contradiction. If \(p \not\in \pi(N)\), then we have \(f_G(p) = c_{p_1}f_{N_{y_1}}(p) + c_{p_2}f_{N_{y_2}}(p) = \frac{p^2-1}{2}(f_{N_{y_1}}(p) + f_{N_{y_2}}(p))\), which also contradicts that \(4 \nmid f_G(p)\).

If \(S = G_2(q)\), where \(q = 3^f\), let \(p_2 \in \pi(q^\sqrt{3q+1})\) and \(p_3 \in \pi(q-\sqrt{3q+1})\), then \(\tau_2(S) = \{p_2\}\) and \(\tau_3(S) = \{p_3\}\). By the results of [10], there exist six conjugacy classes of order \(p_2\) whose sizes are \(c_{p_2,1} = c_{p_2,2} = c_{p_2,3} = c_{p_2,4} = c_{p_2,5} = c_{p_2,6} = \frac{q^3(q^2+1)(q-1)}{q+\sqrt{3q+1}}\), and six conjugacy classes of order \(p_3\) whose sizes are \(c_{p_3,1} = c_{p_3,2} = c_{p_3,3} = c_{p_3,4} = c_{p_3,5} = c_{p_3,6} = \frac{q^3(q^2+1)(q-1)}{q-\sqrt{3q+1}}\). If \(p_2 \not\in \pi(N)\), then \(f_G(p_2) = c_{p_2,1}(f_{N_{x_1}}(p_2) + f_{N_{x_2}}(p_2)) + f_{N_{x_3}}(p_2) + f_{N_{x_4}}(p_2) + f_{N_{x_5}}(p_2) + f_{N_{x_6}}(p_2) = \frac{q^3(q^2+1)(q-1)}{q+\sqrt{3q+1}}(f_{N_{x_1}}(p_2) + f_{N_{x_2}}(p_2) + f_{N_{x_3}}(p_2) + f_{N_{x_4}}(p_2) + f_{N_{x_5}}(p_2) + f_{N_{x_6}}(p_2))\).

Since \(S\) has a cyclic Hall subgroup of order \(q \pm \sqrt{3q+1}\) (see also [10]), we have \(4 \mid f_G(p_2)\), a contradiction. If \(p_3 \not\in \pi(N)\), then \(f_G(p_3) = c_{p_3,1}(f_{N_{y_1}}(p_3) + f_{N_{y_2}}(p_3) + f_{N_{y_3}}(p_3) + f_{N_{y_4}}(p_3) + f_{N_{y_5}}(p_3) + f_{N_{y_6}}(p_3)) = \frac{q^3(q^2+1)(q-1)}{q-\sqrt{3q+1}}(f_{N_{y_1}}(p_3) + f_{N_{y_2}}(p_3) + f_{N_{y_3}}(p_3) + f_{N_{y_4}}(p_3) + f_{N_{y_5}}(p_3) + f_{N_{y_6}}(p_3))\) which also contradicts
that $4 \nmid f_G(p)$.

\begin{flushright}$\square$\end{flushright}

\section*{References}


Received: June, 2009