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Conservation Laws for the Calogero-Degasperis Family of Equations which Describe Pseudo-Spherical Surfaces

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Abstract

A generalized inverse scattering method and the fundamental equations of pseudo-spherical surfaces are given by extending the results of Konno, Wadati [14] and Sasaki [5] respectively. It is characterized by a one-form which can be put in the form of a Riccati system. It is shown how this system can be linearized. Based on this, a procedure for generating an infinite number of conservation laws is given.

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1. Introduction

The development of the study of nonlinear evolution equations (NLEEs), their integrability and associated soliton solutions has produced many fascinating results. Such equations have Bäcklund transformations (BTs) [1, 14-17] and moreover their solutions can be associated with the generation of such geometrical objects as surfaces. This is the case for both constant mean curvature surfaces as well as surfaces which have constant Gaussian curvature [2,3].

It has been shown that constant mean curvature surfaces play an important role in soliton theory by means of the generalized Weierstrass representation of Konopelchenko [4]. Moreover, Sasaki [5] established a geometrical interpretation for the inverse-scattering problem which was originally formulated by

Ablowitz et al. [6] and associates in terms of pseudo-spherical surfaces (pss) [7-9]. This encompasses a large class of NLEEs.

The BT was originally introduced as a transformation which maps one pss into another. These ideas were developed and extended by Chern and Tenenblat [1] who obtained a systematic procedure to determine a linear problem for which a given equation is the integrability condition. They also realized how the geometrical properties of a surface can provide analytic input for such equations. Subsequently, Cavalcante and Tenenblat [10] gave a method to derive conservation laws for evolution equations that describe pss. it is to be understood here that in the Chern and Tenenblat [1] approach to integrability, a type of geometric integrability is implied. To formulate this more precisely, we call an evolution equation $f(x, t, u, u_x, u_t, ...) = 0$ geometrically integrable if it describes a nontrivial, one-parameter family of pss. There are, in addition to this, other formulations of integrability such as formal integrability and the existence of a Lax pair [11] for the system. It is hoped that more insight into the relationship between geometric integrability and other types of integrability can be achieved by investigating the types of conservation laws which can be obtained.

A different procedure from that of Cavalcante and Tenenblat [10] is applied to yield a more standard form for these conservation laws. How these different formulations of integrability correspond can be seen by studying the conservation laws which can be produced. Reyes [4] has studied these different types of integrability and generalized the approach to conservation laws, but in a different form from that here.

Once this is done, some further properties of the system will be developed which lead to two conserved one-forms. It will be shown how these Riccati equations can be transformed, or put in correspondence with, a linear system of one-forms as well. Finally, it will be shown how these results can be used to generate infinite classes of conservation laws. By introducing an appropriate expansion for the quantity in the relevant Riccati equation in terms of a parameter that appears in the one-forms for the surface, an infinite number of conservation laws can be produced. The form of the conservation equations which are generated by this procedure are seen to correspond to those given by other approaches to integrability. As an example, the conservation laws for a specific nonlinear equation, the Calogero-Degasperis family of equations, will be determined at the end.

2. Inverse scattering problem

It is well-known [1-5] that a differential equation (DE) for a real-valued function $u(x, t)$ or a differential system for a two-vector valued function $u(x, t)$, is said to describe a pss if it is the necessary and sufficient condition for the existence of smooth real functions $f_{ij}, \quad 1 \leq i \leq 3, \quad 1 \leq j \leq 2$, depending only on u and a finite number of its derivatives, such that the one-forms

$$\omega_i = f_{i1}dx + f_{i2}dt, \quad 1 \leq i \leq 3, \tag{1}$$

satisfying the structure equations of a surface of constant Gaussian curvature $K = -1$,

$$d\omega_1 = \omega_3 \wedge \omega_2, \quad d\omega_2 = \omega_1 \wedge \omega_3, \quad d\omega_3 = \omega_1 \wedge \omega_2. \tag{2}$$

It is a straightforward computation to verify that (2) is equivalent to saying that

$$d\nu = \Omega\nu, \tag{3}$$

where d denotes exterior differentiation, $\nu = \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$ and $\Omega = 1/2\begin{pmatrix} \omega_2 & \omega_1 - \omega_3 \\ \omega_1 + \omega_3 & -\omega_2 \end{pmatrix}$ is $sl(2, R)$ valued one-form matrix for the cases of equations describing pss. The associated integrability conditions for (3), which are obtained by cross differentiation, then take the matrix form

$$d\Omega - \Omega \wedge \Omega = 0, \tag{4}$$

that is, imply that the connection one-form Ω is flat.

Chern and Tenenblat [1] introduced the notion of a DE for a function $u(x,t)$ that describes a pss and they obtained a classification for such equations of type

$$u_t = F(u, u_x, ..., u_{x^k}), \qquad u_{x^k} = \partial^k u/\partial x^k, \tag{5}$$

which describe pss, under the assumption that $f_{21} = \eta$. They also gave a geometrical method for constructing BTs and conservation laws for these equations.

They observed that most of the NLEEs solvable by the inverse scattering method (ISM) [6-8], such as the Korteweg-de Vries equation (KdVE) and modified Korteweg-de Vries equation (mKdVE), have the property of describing pss. They also showed that if f_{21} is equal to a constant parameter η and the functions f_{11} and f_{31} do not depend on η, then the linear system (3) reduces to the inverse scattering problem (ISP) considered by Ablowitz et al. in [6] with η corresponding to the spectral parameter.

The classification and solution by ISM for equations of more general type than (5) which also describe pss were considered in subsequent papers [9-11], still under the assumption that f_{21} is a constant parameter. Independent of the connections between DEs describing pss and ISM, there are other important aspects of these equations that need to be further investigated. Indeed, while the assumption that f_{21} is a constant parameter is natural in the context of ISP, the problem of classifying DEs describing pss without that restrictive assumption is a question of genuine geometrical interest.

The ISM was introduced first for the KdVE [12]. Later it was extended by Zakharov and Shabat [8] to a 2×2 scattering problem for the NLSE and that was subsequently generalized by Ablowitz, Kaup, Newell and Segur (AKNS) [13] to include a variety of NLEEs. In this section, we generalize the results of Konno and Wadati [14] by considering ν as a three component vector and Ω as a traceless 3×3 matrix one-form. The above definition of a DE is equivalent to saying that the DE for u is the integrability condition for the problem [15-21]

$$d\nu = \Omega\nu, \quad \nu = \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}, \tag{6}$$

where ν is a vector and the 3×3 matrix Ω $(\Omega_{ij}, i, j = 1, 2, 3)$ is traceless

$$tr\Omega = 0, \tag{7}$$

and consists of a one-paramter (η), family of one-forms in the independent variables (x, t), the dependent variable u and its derivatives.

Khater et al. [22] introduced the ISP:

$$\nu_{1x} = f_{31}\nu_2 - f_{11}\nu_3, \quad \nu_{2x} = -f_{31}\nu_1 - \eta\nu_3, \quad \nu_{3x} = -f_{11}\nu_1 - \eta\nu_2, \tag{8}$$

$$\nu_{1t} = f_{32}\nu_2 - f_{12}\nu_3, \quad \nu_{2t} = -f_{32}\nu_1 - f_{22}\nu_3, \quad \nu_{3t} = -f_{12}\nu_1 - f_{22}\nu_2. \tag{9}$$

The integrability condition for Eq. (6) is given by

$$d\Omega - \Omega\wedge\Omega = 0, \tag{10}$$

or in component form

$$f_{12,x} - f_{11,t} = f_{31}f_{22} - \eta f_{32}, \tag{11}$$

$$f_{22,x} = f_{11}f_{32} - f_{12}f_{31}, \tag{12}$$

$$f_{32,x} - f_{31,t} = f_{11}f_{22} - \eta f_{12}. \tag{13}$$

By various choices of the coefficients f_{ij}, it can be shown that the conditions (11)-(13) are equivalent to a large class of NLEEs. The procedure is clarified in the following example:

The Calogero-Degasperis family of equations

$$u_t = \alpha_1(t)u_{xxx} - 6\alpha_1(t)uu_x + (\alpha_0(t) - 4x\alpha_1(t))u_x - 8\alpha_1(t)u, \tag{14}$$

where

$$\Omega = \begin{pmatrix} 0 & C_1 & -A_1 \\ -C_1 & 0 & -B_1 \\ -A_1 & -B_1 & 0 \end{pmatrix}, \tag{15}$$

such that

$$A_1 = budx + [b\alpha_1(t)u_{xx} - \eta b\alpha_1(t)u_x - 2b\alpha_1(t)u^2 + b(\alpha_0(t) - \alpha_1(t)(4x + a^2 - \eta^2))u + \frac{4\eta\alpha_1(t)}{a}]dt,$$

$$B_1 = \eta dx + [2\alpha_1(t)u_x - 2\eta\alpha_1(t)u + \frac{a_t}{a} + \eta\alpha_0(t) + 4\alpha_1(t) - \eta\alpha_1(t)(4x + a^2 - \eta^2)]dt,$$

$$C_1 = (bu+a)dx + [b\alpha_1(t)u_{xx} - \eta b\alpha_1(t)u_x - 2b\alpha_1(t)u^2 + b(\alpha_0(t) - \alpha_1(t)(4x - \eta^2))u +$$

$$\frac{4\eta\alpha_1(t)}{a} - a\alpha_1(t)(4x + a^2 - \eta^2) + a\alpha_0(t)]dt, \tag{16}$$

in which

$$ab = -2, \quad a_x = 0 \qquad \text{and} \qquad \frac{(a^2)_t}{4} + 2\alpha_1(t)a^2 - 2\eta\alpha_1(t) = 0.$$

The essence of the first step of the ISM is summarized as follows [22]. Find nine one - forms ω_i^j, $i = 1, 2, 3$, $j = 1, 2, 3$ consisting of independent and dependent variables and their derivatives, such that the NLEE is given by

$$\Theta \equiv d\Omega - \Omega \wedge \Omega = 0, \quad \Omega = \begin{pmatrix} \omega_1^1 & \omega_1^2 & \omega_1^3 \\ \omega_2^1 & \omega_2^2 & \omega_2^3 \\ \omega_3^1 & \omega_3^2 & \omega_3^3 \end{pmatrix}, \quad Tr\Omega = 0. \tag{17}$$

It should be noted that the solution of these equations are of very special kind. In general, Eq. (17) gives three different equations, which cannot be satisfied simultaneously by one dependent variable u. It has been pointed out [5,22] that Ω can be interpreted as a connection one-form for the principle $SL(3, R)$ bundle on R^3 and Θ as its curvature two form. The geometrical explanation of the $SL(3, R)$ structure is given in section 3.

3. On equations describing pss

In this section we shall show that the fundamental equations of pss, can be written in the form of Eq. (17). Let us start with the general description of a three-dimensional Riemannian manifold S following reference [23]. An orthonormal basis is

$$\{e_i\}, \quad i = 1, 2, 3, \quad e_i.e_j = \delta_{ij}, \tag{18}$$

with respect to the Riemannian metric introduced on the tangent plane T_p at each point $p \in S$. Then the structure equations for S read

$$dp = \omega_i e_i, \quad i = 1, 2, 3 \tag{19}$$

$$de_i = \sum_{j=1}^{3} \omega_i^j e_j, \tag{20}$$

where ω_i are one forms dual to $\{e_i\}$ and $\omega_i^j, \quad (i, j = 1, 2, 3)$ are called the connection one form. The integrability conditions are obtained by differentiating Eq. (19) and using (20). These conditions are

$$d\omega_i = \sum_{j=1}^{3} \omega_i \wedge \omega_i^j, \tag{21}$$

$$d\omega_i^j = \sum_{k=1}^{3} \omega_i^k \wedge \omega_k^j. \tag{22}$$

To sum up, a set of one forms $(\omega_i, \ \omega_i^j)$ satisfying Eqs. (21) and (22) describes a pss locally through the structure Eqs. (19) and (20). It is easy to show that Eqs. (21) and (22) can be written in the form of Eq. (17) by choosing

$$\Omega = \begin{pmatrix} 0 & \omega_3 & -\omega_1 \\ -\omega_3 & 0 & -\omega_2 \\ -\omega_1 & -\omega_2 & 0 \end{pmatrix}, \tag{23}$$

from Eqs. (17) and (23) we obtain this relations

$$\omega_1^1 = \omega_2^2 = \omega_3^3 = 0, \quad \omega_1 = -\omega_1^3 = -\omega_3^1, \ \omega_2 = -\omega_2^3 = -\omega_3^2, \quad \omega_3 = \omega_1^2 = -\omega_2^1. \tag{24}$$

Let M^2 be a two - dimensional differentiable manifold parametrized by coordinates x, t. We consider a metric on M^2 defined by ω_1, ω_2. The first two equations in (2) are the structure equations which determine the connection form ω_3, and the last equation in (2), the Gauss equation, determines that the Gaussian curvature of M^2 is -1, i.e. M^2 is a pss. Moreover, an evolution equation must be satisfied for the existence of forms (1) satisfying (2). This justifies the definition of a DE which describes a pss that we considered in the introduction.

It has been known, for a long time, that the sine-Gordon equation describes a pss. KdVE and mKdVE, were also shown to describe such surfaces [13]. Here we show that other equations such as the Calogero-Degasperis family of equations also describe pss as well. The latter equations proved to be of great importance in many physical applications [1,15-22]. The procedure is clarified in the following example: Let M^2 be a differentiable surface, parametrized by coordinates x, t.

The Calogero-Degasperis family of equations
Consider

$$\omega_1^3 = \omega_3^1 = budx + [b\alpha_1(t)u_{xx} - \eta b\alpha_1(t)u_x - 2b\alpha_1(t)u^2 + b(\alpha_0(t) - \alpha_1(t)(4x + a^2 - \eta^2))u + \frac{4\eta\alpha_1(t)}{a}]dt,$$

$$\omega_2^3 = \omega_3^2 = \eta dx + [2\alpha_1(t)u_x - 2\eta\alpha_1(t)u + \frac{a_t}{a} + \eta\alpha_0(t) + 4\alpha_1(t) - \eta\alpha_1(t)(4x + a^2 - \eta^2)]dt,$$

$$\omega_1^2 = -\omega_2^1 = (bu + a)dx + [b\alpha_1(t)u_{xx} - \eta b\alpha_1(t)u_x - 2b\alpha_1(t)u^2 + b(\alpha_0(t) - \alpha_1(t)(4x - \eta^2))u +$$

$$\frac{4\eta\alpha_1(t)}{a} - a\alpha_1(t)(4x + a^2 - \eta^2) + a\alpha_0(t)]dt. \tag{25}$$

Then M^2 is a pss iff u satisfies the Calogero-Degasperis family of equations (14).

4. Conservation laws for the Calogero-Degasperis family of equations which describe pss

In this section, the Riccati system of equations derived there can be put in explicit conservation form, and to do so, we will write the one-forms ω_i in terms of coordinates in a different way from that of (1). In doing so, the Riccati system of equations will take a form which is very convenient from the point of view of obtaining and expressing the conservation laws. The Riccati equations take the form [24-33]:

$$\frac{\partial\Gamma}{\partial x} = q + \eta\Gamma - r\Gamma^2, \quad \frac{\partial\Gamma}{\partial t} = B + 2A\Gamma - C\Gamma^2, \tag{26}$$

where

$$\Gamma = \frac{\nu_1}{\nu_2}$$

$$f_{11} - f_{31} = 2q, \quad f_{11} + f_{31} = 2r,$$

$$2A = f_{22},\ 2B = f_{12} - f_{32},\ 2C = f_{12} + f_{32},$$

such that the functions r, q, A, B and C satisfy the equations [22]

$$A_x = qC - rB,$$

$$q_t - 2Aq - B_x + \eta B = 0, \tag{27}$$

$$C_x = r_t + 2Ar - \eta C.$$

Equations (26) imply that

$$C\Gamma_x - r\Gamma_t = (Cq - rB) + (\eta C - 2Ar)\Gamma, \tag{28}$$

Adding $-r_t\Gamma$ to both sides, we obtain

$$C\Gamma_x - (r\Gamma)_t = (Cq - rB) + (\eta C - 2Ar - r_t)\Gamma, \tag{29}$$

we using eq. (27), eq. (29) takes the form

$$(r\Gamma)_t = (A + C\Gamma)_x. \tag{30}$$

The Riccati equations for Γ in the $x-$ variable can be rearranged to take the form

$$\eta(r\Gamma) = -rq + (r\Gamma)^2 + r[\frac{\partial}{\partial x}(\frac{r\Gamma}{r})]. \tag{31}$$

A similar pair of equations can be obtained for the t derivatives. Expand $(r\Gamma)$ into a power series in inverse powers of η so that

$$r\Gamma(x, t, \eta) = \sum_{n=1}^{\infty} \phi_n(x, t)\eta^{-n}. \tag{32}$$

The $\phi_n(x, t)$ are unknown at this point, however a recursion relation can be obtained for the $\phi_n(x, t)$ by using (31). Substituting (32) into Γ equation in (31), we find that

$$\sum_{n=1}^{\infty} \phi_n(x, t)\eta^{-n+1} = -rq + (\sum_{n=1}^{\infty} \phi_n(x, t)\eta^{-n})^2 + r\sum_{n=1}^{\infty}(\frac{\phi_n(x, t)}{r})_x\eta^{-n}. \tag{33}$$

Applying the Cauchy product formula to the square in (33), it then takes the form

$$\phi_1+\phi_2\eta^{-1}+\sum_{n=2}^{\infty}\phi_{n+1}(x,t)\eta^{-n}$$

$$=-rq+\sum_{n=2}^{\infty}(\sum_{j=1}^{n-1}\phi_j\phi_{n-j})\eta^{-n}+r\sum_{n=2}^{\infty}(\frac{\phi_n(x,t)}{r})_x\eta^{-n}+r(\frac{\phi_1(x,t)}{r})_x\eta^{-1}. \quad (34)$$

Now equate powers of η on both sides of this expression to produce the set of recursions

$$\phi_1=-rq, \qquad \phi_2=-rq_x,$$

$$\phi_{n+1}=\sum_{k=1}^{n-1}\phi_k(x,t)\phi_{n-k}(x,t)+r(\frac{\phi_n(x,t)}{r})_x, \quad n\geq 2. \quad (35)$$

Substituting (32) int(30), the following system of conservation laws appears

$$\sum_{n=1}^{\infty}\frac{\partial\phi_n(x,t)}{\partial t}\eta^{-n}=\frac{\partial}{\partial x}(A+C\sum_{n=1}^{\infty}\frac{\phi_n(x,t)}{r}\eta^{-n}). \quad (36)$$

In general, A and C will depend on parameter η, the function r and higher derivatives of r. Substituting A and C into (36) a particular case, eq. (36) will simplify under relations (35), and then like powers of η can be equated on both sides of (36). This procedure generates an infinite number of conservation laws for the equation under examination.

To obtain conservation laws using (36) in a particular example using this procedure, let us consider the Calogero-Degasperis family of equations which describe pss.

For the Calogero-Degasperis family of equations

$$q=-\frac{a}{2} \qquad r=bu+\frac{a}{2}$$

$$A=\frac{1}{2}[2\alpha_1(t)u_x-2\eta\alpha_1(t)u+\frac{a_t}{a}+\eta\alpha_0(t)+4\alpha_1(t)-\eta\alpha_1(t)(4x+a^2-\eta^2)],$$

$$B=-a^2\alpha_1(t)\frac{u}{2}+\frac{a}{2}[\alpha_1(t)(4x+a^2-\eta)-\alpha_0(t)], \quad (37)$$

$$C=[b\alpha_1(t)u_{xx}-\eta b\alpha_1(t)u_x-2b\alpha_1(t)u^2+b(\alpha_0(t)-\alpha_1(t)(4x+\frac{a^2}{2}-\eta^2))u+$$
$$\frac{4\eta\alpha_1(t)}{a}-\frac{a}{2}\alpha_1(t)(4x+a^2-\eta^2)+\frac{a}{2}\alpha_0(t).$$

Substituting (37) into (27), the first equation (27) reduces to an identity, and the remaining two hold modulo the Calogero-Degasperis family of equations (14).

Putting (37) into (36), it is found that

$$\sum_{n=1}^{\infty}\frac{\partial\phi_n(x,t)}{\partial t}\eta^{-n}=\alpha_1(t)[u_{xx}-\eta u_x-2\eta]+$$
$$\frac{\partial}{\partial x}[(b\alpha_1(t)u_{xx}-\eta b\alpha_1(t)u_x-2b\alpha_1(t)u^2+b(\alpha_0(t)-\alpha_1(t)(4x+\frac{a^2}{2}-\eta^2))u+$$
$$\frac{4\eta\alpha_1(t)}{a}-\frac{a}{2}\alpha_1(t)(4x+a^2-\eta^2)+\frac{a}{2}\alpha_0(t))\sum_{n=1}^{\infty}\frac{\phi_n(x,t)}{r}\eta^{-n}]. \tag{38}$$

However, from the recursions in (35), it follows that $\phi_1=u+\frac{a^2}{2}$ and $\phi_2=0$. Using these to simplify this, the remaining coefficients of η^{-n} can be equated on both sides, and the following set of conservation laws are obtained for $n\geq 1$,

$$\frac{\partial\phi_n(x,t)}{\partial t}=\frac{\partial}{\partial x}\{b\alpha_1(t)u_{xx}-2b\alpha_1(t)u^2+b(\alpha_0(t)-\alpha_1(t)(4x+\frac{a^2}{2}))u-$$
$$\frac{a}{2}\alpha_1(t)(4x+a^2)+\frac{a}{2}\alpha_0(t)]\frac{\phi_n(x,t)}{r}+[-b\alpha_1(t)u_x+\frac{4\alpha_1(t)}{a}]\frac{\phi_{n+1}(x,t)}{r} \tag{39}$$
$$[b\alpha_1(t)u+\frac{a}{2}\alpha_1(t)]\frac{\phi_{n++2}(x,t)}{r}\}.$$

5. Conclusion

The ISM [14,27,28] may be rewritten by considering ν as a three component vector and Ω as a traceless $3\times\ 3$ matrix one-form [22]. The latter yields directly the curvature condition (Gaussian curvature equal to -1, corresponding to pseudo-spherical surfaces). This geometrical method is considered for the Calogero-Degasperis family of equations which describe pss. This geometrical method allows some further generalization of the work on conserved charges given by Wadati, Sanuki and Konno [28]. An infinite number of conserved charges for the Calogero-Degasperis family of equations are derived in this way.

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