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Numerical Solution of Fredholm Integro-Differential Equation by Adomian's Decomposition Method

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Abstract

In this article, we use Adomian's decomposition method which is a well-known method for solving functional equations now-a-days to solve linear Fredholm integro-differential equations. We apply ADM, CAS wavelet method and the method presented in[2] to solve three examples given in [1] and compare the results with the exact solution. We show that ADM give more accurate approximation compared to two other method and is easier.

Keywords: Adomian's decomposition method; CAS Wavelets; Integro-differential equation

1 Introduction

The topic of the Adomian's decomposition method (ADM) has been rapidly growing in recent years. The concept of this method was first introduced by G. Adomian in the beginning of 1980 [3, 4]. This method can be used to solve all types of linear and non-linear equations such as differential and integral equations such as differential and integral equations so it is known as a powerful method. Another important advantage of this is that it can reduce the size of computation, while increases the accuracy of the solutions.

Wavelets theory is a relatively new and an emergine area in mathematical research. It has been applied in a wide range of engineering disciplines, particularly Wavelets are very successfully used in signal analysis for Wave form representations and segmentations, time-frequency analysis and fast algorithm for easy implementation. Han Danfu introduced CAS Wavelet method to solve integro-differential equations [1]. P. Darania introduced a method for solve integro-differential equations[2].

In this work we apply Adomian's decomposition method to solve linear Fredholm integro-differential equations and we will show that convergent rate of ADM is more accelerate than the methods presented in [1, 2]. It should be noted that we only report the numerical results of CAS Wavelets and the method presented in[2] and we do not explain this methods.

2 ADM to solving linear Fredholm integro-differential equations

Consider the linear Fredholm integro-differential equation as

$$y'(t) = f(t) + \int_a^b k(s,t)y(t)dt \qquad y(0) = y_0. \tag{1}$$

Denoting $\frac{d}{dt}$ by L ,we have L^{-1} as n-fold integration from 0 to t. Therefore (1) can be written as

$$Ly(t) = f(x) + \int_a^b k(s,t)y(t)dt \tag{2}$$

operating whit L^{-1},

$$L^{-1}Ly(t) = L^{-1}(f(t)) + L^{-1}(\int_a^b k(s,t)y(t))dt) \tag{3}$$

$$y(x) = y(0) + L^{-1}(f(t)) + L^{-1}(\int_a^b k(s,t)y(t)dt) \tag{4}$$

To use ADM, let $y(t) = \sum_{m=0}^{\infty} y_m$. Hence, from (4) we obtain

$$y_0 = y(0) + L^{-1}(f(t)) \tag{5}$$

$$y_1 = L^{-1}(g(x)\int_a^b h(t)y_0(t))dt) \tag{6}$$

$$\vdots$$

$$y_m + 1 = L^{-1}(\int_a^b k(s,t)y_m(t))dt) \tag{7}$$

$$\vdots$$

In practice, all the terms of the series $y(t) = \sum_{m=0}^{\infty} y_m(t)$, can not be determined and so we use an approximation of the solution by the following truncated series

$\phi_k(x) = \sum_{m=0}^{k-1} y_m(t)$, with $\lim \phi_k(x)$=y(t).

2.1 Numerical examples

Example 1. Consider the following linear fredholm integro -differential equation

$$y'(x) = xe^x + e^x - x + \int_0^1 xy(t)dt, y(0) = 0 \tag{8}$$

with the exact solution $y(x) = xe^x$. we solve this example by applying Adomian's decomposition method and CAS Wavelet method and the method presented in [2]. Results are shown in the following table. we show the error of the standard Adomian's method (ESA)in table 1.

Time	CAS Wavelet	ESA	Method in [2]
0.1	$1.34917637 \times 10^{-3}$	$1.00118319 \times 10^{-2}$	2.433330×10^{-5}
0.2	$1.15960044 \times 10^{-3}$	$2.78651355 \times 10^{-2}$	9.735080×10^{-5}
0.3	$5.67152531 \times 10^{-3}$	$5.08730892 \times 10^{-2}$	2.193150×10^{-4}
0.4	$5.93105645 \times 10^{-2}$	$7.55356316 \times 10^{-2}$	3.917420×10^{-4}
0.5	$1.32330751 \times 10^{-2}$	$9.71888592 \times 10^{-2}$	6.200050×10^{-4}
0.6	$4.39287720 \times 10^{-2}$	$1.09551714 \times 10^{-2}$	9.184720×10^{-4}
0.7	$1.41201624 \times 10^{-2}$	$1.04133232 \times 10^{-2}$	1.319230×10^{-3}
0.8	$1.34514117 \times 10^{-2}$	$6.94512700 \times 10^{-2}$	1.885530×10^{-3}
0.9	$1.32045209 \times 10^{-2}$	$1.00034260 \times 10^{-2}$	2.731360×10^{-3}

Table 1. Numerical results of applied methods for example 1

Example 2. Consider the following linear fredholm integro-differential equation

$$y'(x) = 1 - \frac{1}{3}x + \int_0^1 xty(t)dt, y(0) = 0 \tag{9}$$

with the exact solution $y(x) = x$. We solve this problem by applying ADM and CAS Wavelet method and the method presented in [2]. Numerical results are shown in table 2.

Time	CAS Wavelet	ESA	Method in [2]
0.1	$2.17942375 \times 10^{-4}$	$1.66666667 \times 10^{-3}$	2.06509×10^{-4}
0.2	$6.38548213 \times 10^{-4}$	$6.09388620 \times 10^{-3}$	8.04069×10^{-4}
0.3	$7.91370487 \times 10^{-4}$	$1.32017875 \times 10^{-2}$	1.72624×10^{-3}
0.4	$2.15586005 \times 10^{-2}$	$2.29140636 \times 10^{-2}$	2.86044×10^{-3}
0.5	$4.99358429 \times 10^{-3}$	$3.51578404 \times 10^{-2}$	4.04527×10^{-3}
0.6	$2.21728810 \times 10^{-2}$	$6.69648304 \times 10^{-2}$	5.06663×10^{-3}
0.7	$1.05645449 \times 10^{-4}$	$7.12430514 \times 10^{-2}$	5.65279×10^{-3}
0.8	$1.43233681 \times 10^{-3}$	$8.63983845 \times 10^{-2}$	5.46844×10^{-3}
0.9	$2.07747461 \times 10^{-2}$	$1.08103910 \times 10^{-1}$	4.10753×10^{-3}

Table 2. Numerical results of applied methods for example 2

Example 3. Consider the following linear fredholm integro-differential equation

$$y'(x) = \int_0^1 sin(4\pi x + 2\pi t)y(t)dt + y(x) - cos(2\pi x) - 2\pi sin(2\pi x) - \frac{1}{2}sin4\pi x, \tag{10}$$

with initial condition $y(0) = 1$ and exact solution $y(x) = cos(2\pi x)$. This problem is solved by the same methods applied in example 2. Results are shown in table 3.

Time	CAS Wavelat k=2, M=1	ESA two iterations	CAS Wavelat k=4, M=1	ESA four iterations
0.1	1.19453×10^{-1}	2.40432×10^{-3}	1.18278×10^{-2}	6.77227×10^{-4}
0.2	3.13357×10^{-2}	5.07123×10^{-3}	2.73216×10^{-2}	3.57926×10^{-4}
0.3	2.26948×10^{-2}	6.25477×10^{-3}	4.32333×10^{-2}	7.20389×10^{-4}
0.4	3.28912×10^{-3}	3.87315×10^{-3}	5.87271×10^{-2}	1.65557×10^{-3}
0.5	3.80337×10^{-1}	1.74601×10^{-2}	7.05550×10^{-2}	2.33402×10^{-3}
0.6	3.22315×10^{-2}	1.58482×10^{-2}	7.27075×10^{-2}	3.76522×10^{-3}
0.7	3.15119×10^{-2}	8.41721×10^{-3}	6.28710×10^{-2}	6.78844×10^{-2}
0.8	6.38539×10^{-2}	9.48709×10^{-3}	4.74727×10^{-2}	1.09211×10^{-2}
0.9	2.32893×10^{-2}	9.65467×10^{-3}	3.97887×10^{-2}	1.49581×10^{-2}

Table 3. Numerical results of applied methods for example 3.

3 Conclusion

In this work, we used Adomian's decomposition method, CAS Wavelet and the method presented in[2] and compared the results with the exact solution. The study shows that ADM calculates better approximations to the exact solution of linear fredholm integro-differential equations in compare with CAS Wavelet method and the method presented in [2] and we introduce Adomian's method as simple and efficient method.

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