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Optimal Planning of Failure-Step Stress Partially Accelerated Life Tests under Type II Censoring

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Abstract

This paper considers optimum plans for failure-step stress partially accelerated life tests with two stress levels under type-II censoring assuming Weibull distribution as a lifetime model. The optimum test plans determine the optimum proportion of test units failing at each stress according to a certain optimality criterion. The D-optimality criterion is considered. Some numerical illustrations are provided to illustrate the proposed procedure.

Keywords: Partially accelerated life test, Failure-step stress test, Weibull lifetime distribution, type-II censoring, Maximum likelihood method, Fisher-information matrix, Generalized asymptotic variance, Optimal design, D-optimality

Acronyms & Notations

ALT accelerated life test.

$PALT$ partially accelerated life test.

$FSPALT$ failure step stress partially accelerated life test.

MLE maximum likelihood estimate/estimator.

AB absolute bias.

MSE mean square error.

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LCB lower confidence bound.

UCB upper confidence bound.

GAV generalized asymptotic variance.

pdf probability density function.

x_1 design (normal) stress level.

x_2 high stress level.

n number of test units (total sample size).

n_i number of failed units at stress $x_i, i = 1, 2$.

n_c number of censored units.

Y_{ij} failure time j of test units at stress $x_i, i = 1, 2, j = 1, 2, ..., n_i$.

y_{ij} observed value of failure time j of test units at stress $x_i, i = 1, 2, j = 1, 2, ..., n_i$.

β acceleration factor ($\beta > 1$).

α the shape parameter ($\alpha > 0$).

θ the scale parameter ($\theta > 0$).

π_i Proportion of test units to be observed at stress $x_i, i = 1, 2$.

π_1^* optimal sample proportion of units failed at stress x_1 when switching to x_2.

π_c Proportion of test units to be censored.

ν confidence level.

$\wedge$ implies a maximum likelihood estimator.

$\downarrow (.)$ evaluated at (.).

1 Introduction

ALT is often used for reliability prediction. Specimens are tested at high stress levels to induce early failures; then the failure information is related to that at an operational stress level through a given stress-dependent model. When such model is unknown, the ALT can not be conducted and instead the PALT becomes suitable test. The PALT combines both ordinary and accelerated life tests. Thus, PALT is used for reliability analysis to save more time and money over the ordinary or traditional life tests.

According to Bai et al. [9] and Nelson [17], one way of applying stress to the test is a step-stress scheme which allows the stress setting of a unit to be changed at pre-specified times or upon the occurrence of a fixed number of failures. The former is called time-step stress test and the later failure-step stress test; which is considered in this paper. Generally, in step-stress scheme, a test unit is subjected to successively higher levels of stress. Thus, the stress is increased step by step until the test unit fails. Specifically, in FSPALT scheme, test units start to run at a design (normal) stress until the occurrence of a fixed number of failures. Then, stress on them is raised and fixed over a specified time to obtain the predetermined number of failures.

This paper presents the failure-step stress PALT. In the literature, there were no studies had been performed about FSPALT scheme. On the other hand, many authors have studied the time-step stress PALT, for example, see Goel [12], DeGroot and Goel [11], Bhattacharyya and Soejoeti [10], Bai and Chung [7], Bai et al. [8], Attia et al. [6], Abdel-Ghaly et al. [1], Madi [15], Abdel-Ghani [4], Abdel-Ghaly et al. [2], Abdel-Ghaly et al. [3], Ismail [13], Ismail [14], Aly and Ismail [5].

Specifically, in this paper, we will consider the problem of optimally designing failure-step stress PALT using type-II censoring under Weibull distribution. The reminder of this paper is organized as follows: In Section 2, we will introduce the model, some necessary assumptions will be given and the test procedure will be described. The maximum likelihood method will be used to obtain the point and interval estimation of the model parameters in Section 3. The problem of choosing the optimum proportion of units that should be failed at the design stress, π_1^* will be addressed using D-optimality criterion in Section 4. Simulation studies will be presented in Section 5 to demonstrate the theoretical results given in this paper. Finally, in Section 6 a conclusion will be made.

2 The Model

In this Section, the main assumptions are given. Moreover, the method of how to run the experiment in the case of FSPALT under type-II censoring using

Weibull distribution is explained.

Weibull Distribution as a Lifetime Model

The probability density function (pdf) of a two-parameter Weibull distribution is given by

$$f(t;\theta,\alpha) = \frac{\alpha}{\theta}(\frac{t}{\theta})^{\alpha-1}e^{-(t/\theta)^{\alpha}}, t \geq 0, \theta > 0, \alpha > 0, \tag{2.1}$$

with reliability function in the form

$$R(t) = e^{-(t/\theta)^{\alpha}}. \tag{2.2}$$

This distribution is used as a lifetime distribution of test units.

Basic Assumptions

1. The failure times Y_{ij}; $i = 1, 2; j = 1, 2, \ldots, n_i$ are independent and identically distributed (*i.i.d.*) random variables.

2. Two stress levels x_1 and x_2 (design and high) are used.

3. The total lifetime Y of an item in a FSPALT is as follows

$$Y = \begin{cases} T, & \text{if } T \leq y_{1n_1} \\ y_{1n_1} + \beta^{-1}(T - y_{1n_1}), & \text{if } T > y_{1n_1}, \end{cases} \tag{2.3}$$

   where T denotes the lifetime under the stress level x_1.

4. For any level of stress, the lifetime of test unit follows Weibull distribution.

Test Procedure

1. Suppose that n test units are initially placed on design stress x_1 and run until time y_{1n_1} when exactly n_1 failures are observed while testing at the stress level x_1.

2. The $(n - n_1)$ units are put on high stress x_2 and run until time y_{2n_2} when exactly n_2 failures are observed. The remaining $n_c = n - n_1 - n_2$ units are then censored. Let Y_{1n_1} and Y_{2n_2} be random variables and let $\pi_1 = n_1/n$, $\pi_2 = n_2/n$ and $\pi_c = n_c/n$. We consider the problem of determining π_1^* when π_c is pre-specified.

3 Point and Interval Maximum Likelihood Estimations

In this Section, the point and interval estimations of the model parameters are introduced using the maximum likelihood method.

3.1 Point Estimation

The likelihood function of the observations Y_{ij}, $i = 1, 2$, $j = 1, 2, \ldots, n_i$, and n_c censored data is as follows

$$\begin{aligned} L(\beta, \alpha, \theta) \ &\propto \ \prod_{j=1}^{n_1} \frac{\alpha}{\theta} (\frac{y_{1j}}{\theta})^{\alpha-1} e^{-(\frac{y_{1j}}{\theta})^{\alpha}} \\ &\times \ \prod_{j=1}^{n_2} \frac{\beta\alpha}{\theta} (\frac{\beta(y_{2j} - y_{1n_1}) + y_{1n_1}}{\theta})^{\alpha-1} e^{-[(\beta(y_{2j}-y_{1n_1})+y_{1n_1})/\theta]^{\alpha}} \\ &\times \ \prod_{j=1}^{n_c} e^{-[(\beta(y_{2n_2}-y_{1n_1})+y_{1n_1})/\theta]^{\alpha}}. \end{aligned} \tag{3.1}$$

Taking the natural logarithm of the likelihood function, we get

$$\begin{aligned} lnL \ &= \ n\pi_1(ln\alpha - \alpha ln\theta) + n\pi_2(ln\alpha - \alpha ln\theta + ln\beta) \\ &+ \ (\alpha - 1)\sum_{j=1}^{n_1} lny_{1j} - \frac{1}{\theta^{\alpha}} \sum_{j=1}^{n_1} y_{1j}^{\alpha} + (\alpha - 1)\sum_{j=1}^{n_2} ln(\beta(y_{2j} - y_{1n_1}) + y_{1n_1}) \\ &- \ \frac{1}{\theta^{\alpha}} \sum_{j=1}^{n_2} (\beta(y_{2j} - y_{1n_1}) + y_{1n_1})^{\alpha} - \frac{n_c}{\theta^{\alpha}} (\beta(y_{2n_2} - y_{1n_1}) + y_{1n_1})^{\alpha}. \end{aligned} \tag{3.2}$$

The MLEs of β, α and θ are obtained by solving the following equations

$$\frac{\partial lnL}{\partial \beta} = \frac{n\pi_2}{\beta} + (\alpha - 1)\sum_{j=1}^{n_2} \frac{y_{2j} - y_{1n_1}}{\beta(y_{2j} - y_{1n_1}) + y_{1n_1}} - \frac{\alpha\varphi}{\theta^{\alpha}}, \tag{3.3}$$

$$\begin{aligned} \frac{\partial lnL}{\partial \alpha} \ &= \ \frac{n(\pi_1 + \pi_2)}{\alpha} - n(\pi_1 + \pi_2)ln\theta + \sum_{j=1}^{n_1} lny_{1j} \\ &+ \ \sum_{j=1}^{n_2} ln(\beta(y_{2j} - y_{1n_1}) + y_{1n_1}) - \frac{\zeta}{\theta^{\alpha}} + \frac{\psi ln\theta}{\theta^{\alpha}}, \end{aligned} \tag{3.4}$$

and

$$\frac{\partial lnL}{\partial\theta} = -\frac{\alpha n(\pi_1+\pi_2)}{\theta} + \frac{\alpha\psi}{\theta^{\alpha+1}}, \tag{3.5}$$

where

$$\begin{aligned}\varphi &= \sum_{j=1}^{n_2}(\beta(y_{2j}-y_{1n_1})+y_{1n_1})^{\alpha-1}(y_{2j}-y_{1n_1}) \\ &+ n_c(\beta(y_{2n_2}-y_{1n_1})+y_{1n_1})^{\alpha-1}(y_{2n_2}-y_{1n_1}),\end{aligned} \tag{3.6}$$

$$\begin{aligned}\zeta &= \sum_{j=1}^{n_1} y_{1j}^{\alpha} lny_{1j} + \sum_{j=1}^{n_2}(\beta(y_{2j}-y_{1n_1})+y_{1n_1})^{\alpha} ln(\beta(y_{2j}-y_{1n_1})+y_{1n_1}) \\ &+ n_c(\beta(y_{2n_2}-y_{1n_1})+y_{1n_1})^{\alpha} ln(\beta(y_{2n_2}-y_{1n_1})+y_{1n_1}),\end{aligned} \tag{3.7}$$

and

$$\psi = \sum_{j=1}^{n_1} y_{1j}^{\alpha} + \sum_{j=1}^{n_2}(\beta(y_{2j}-y_{1n_1})+y_{1n_1})^{\alpha} + n_c(\beta(y_{2n_2}-y_{1n_1})+y_{1n_1})^{\alpha}. \tag{3.8}$$

When equating (3.5) with zero, the MLE of θ can be expressed as follows

$$\hat{\theta} = (\frac{\psi}{n(\pi_1+\pi_2)})^{1/\alpha}. \tag{3.9}$$

Replacing θ in equations (3.3) and (3.4) by $\hat{\theta}$ and equating them with zero, result in the following two equations, respectively

$$\frac{n\pi_2}{\beta} + (\alpha-1)\sum_{j=1}^{n_2}\frac{y_{2j}-y_{1n_1}}{(\beta(y_{2j}-y_{1n_1})+y_{1n_1})} = n\alpha(\pi_1+\pi_2)\frac{\varphi}{\psi} \tag{3.10}$$

and

$$\frac{n(\pi_1+\pi_2)}{\alpha} + \sum_{j=1}^{n_1} lny_{1j} + \sum_{j=1}^{n_2} ln(\beta(y_{2j}-y_{1n_1})+y_{1n_1}) = \frac{n(\pi_1+\pi_2)\zeta}{\psi}. \tag{3.11}$$

These equations do not admit explicit solutions. They will be solved simultaneously to obtain the MLE of the parameters β and α and then substituting in (3.9) to get the MLE of θ. This will be shown in Section 5.

The asymptotic (large-sample) Fisher's information matrix F has the form

$$F = \begin{pmatrix} f_{11} & f_{12} & f_{13} \\ f_{12} & f_{22} & f_{23} \\ f_{13} & f_{23} & f_{33} \end{pmatrix} = - \begin{pmatrix} \frac{\partial^2 lnL}{\partial \beta^2} & \frac{\partial^2 lnL}{\partial \beta \partial \alpha} & \frac{\partial^2 lnL}{\partial \beta \partial \theta} \\ \\ \frac{\partial^2 lnL}{\partial \beta \partial \alpha} & \frac{\partial^2 lnL}{\partial \alpha^2} & \frac{\partial^2 lnL}{\partial \alpha \partial \theta} \\ \\ \frac{\partial^2 lnL}{\partial \beta \partial \theta} & \frac{\partial^2 lnL}{\partial \alpha \partial \theta} & \frac{\partial^2 lnL}{\partial \theta^2} \end{pmatrix} \downarrow (\hat{\beta}, \hat{\alpha}, \hat{\theta}). \quad (3.12)$$

3.2 Interval Estimation

It is known that the MLE are consistent and asymptotically normally distributed. Therefore, the two-sided approximate $\nu 100$ percent confidence limits for a certain population parameter λ can be obtained such that

$$P[-z \leq \frac{\hat{\lambda} - \lambda}{\sigma(\hat{\lambda})} \leq z] \cong \nu, \quad (3.13)$$

where z is the $[100(1 - \nu/2)]^{th}$ standard normal percentile and $\sigma(\hat{\lambda})$ is the standard deviation of the MLE of λ. Therefore, the two-sided approximate $\nu 100$ percent confidence limits for β, α and θ are given, respectively, as follows:

$$L_\beta = \hat{\beta} - z\sigma(\hat{\beta}), U_\beta = \hat{\beta} + z\sigma(\hat{\beta}), \quad (3.14)$$

$$L_\alpha = \hat{\alpha} - z\sigma(\hat{\alpha}), U_\alpha = \hat{\alpha} + z\sigma(\hat{\alpha}), \quad (3.15)$$

and

$$L_\theta = \hat{\theta} - z\sigma(\hat{\theta}), U_\theta = \hat{\theta} + z\sigma(\hat{\theta}). \quad (3.16)$$

4 Optimum Failure-Step Stress Test Plan

In this Section, the problem of optimally designing failure-step stress PALT is considered. The optimal design of life test is important for improving precision in estimating the life distribution at design stress and thus for improving the quality of the statistical inference. The optimum failure-step stress test plan specifies the fraction of units that must fail at low (design) stress level according to a certain optimality criterion.

Optimization Criterion

An optimum plan provides the best estimate of a quantity. In this Section, the optimum choice of π_1, the sample fraction failing at x_1 when the stress is switched to x_2 is determined. The D-optimality criterion is considered. This criterion is based on the minimization of the generalized asymptotic variance of the MLEs of the model parameters at the design stress, x_1. Specifically, for failure-step stress test the variance is minimized by the optimum choice of π_1. According to Miller and Nelson [16], this choice also minimizes the asymptotic coefficient of the variation of any percentile estimator, that is, minimizes the relative error.

D-optimality

This criterion is based on the determinant of the Fisher's information matrix, $|F|$. Maximizing this determinant is equivalent to minimizing the generalized asymptotic variance (GAV) of the MLE of the model parameters. The GAV is the reciprocal of $|F|$; see Bai et al. [8]. That is,

$$GAV(\hat{\beta}, \hat{\alpha}, \hat{\theta}) = \frac{1}{|F|}. \tag{4.1}$$

Therefore, the optimal value of π_1 is chosen such that $|F|$ is maximized and then the GAV is minimized.

Now, the optimum failure-step stress test plan for products having Weibull lifetime distribution is to find the optimum stress-change point π_1^* by solving the equation

$$\frac{\partial |F|}{\partial \pi_1} = 0. \tag{4.2}$$

The solution of (4.2) is not in a closed form. It requires an iterative method to obtain a numerical solution.

5 Simulation Studies

Simulation studies have been conducted using the international mathematical and statistical library (IMSL) to get the MLEs and to study their properties through the absolute bias (AB) and the mean square error (MSE). Furthermore, the asymptotic confidence bounds of the model parameters are obtained. Besides, in this Section we illustrate the theoretical results of the optimal design problem numerically. The properties of the optimum test plans obtained by the proposed optimality criterion are investigated.

Getting the MLE

The steps of the simulation procedure can be described as follows:

- Random samples of sizes 60, 100, 200, 500, 700 and 1000 are generated from Weibull lifetime distribution under type II censoring using different combinations of the parameter values for β, α and θ.
- The data are used to get the estimates of θ in equation (3.9).
- Substituting in the normal equations, (3.10) and (3.11) and solving them simultaneously using IMSL, the estimates of β and α are obtained.
- Repeating this procedure 1000 times, the average of these estimates are taken to be the MLE of the above three parameters for each combination.
- Both the AB and the MSE are computed.
- Also, both the LCB and the UCB for each parameter are obtained.

Tables 1 and 2, represent the estimates, AB, MSE, the lower bound LCB and the upper bound UCB for the parameters β, α and θ for three different combinations of the initial values of the parameters. Table 1 represents the case when the shape parameter α is greater than 1. While Table 2 represents the case when α is less than 1.

Table 1: The MLE, AB, MSE, LCB and UCB ($\beta = 1.2, \alpha = 3, \theta = 2$)

n	$Parameter$	$Estimate$	AB	MSE	LCB	UCB
60	β	1.276489	.08334017	.14245760	.5520659	2.000913
	α	3.059458	.05945826	.32171710	1.953869	4.165047
	θ	2.010858	.02828932	.04857165	1.579419	2.442298
100	β	1.283340	.07648933	.13803430	.5736993	1.992981
	α	3.049877	.05359554	.24272360	2.089205	4.010549
	θ	2.028289	.01085830	.04195912	1.630652	2.425927
200	β	1.233404	.03340375	.07903329	.6862953	1.780512
	α	3.053596	.04987693	.18009570	2.228477	3.878714
	θ	2.008741	.00874090	.02535853	1.697094	2.320388
500	β	1.213702	.01370192	.02722804	.8914008	1.536003
	α	3.021178	.02117777	.06987313	2.504746	3.537610
	θ	2.002461	.00246119	.00940735	1.812419	2.192503
700	β	1.211511	.01151049	.01926063	.9404337	1.482587
	α	3.008441	.01270223	.04560853	2.590187	3.426694
	θ	2.002292	.00229239	.00673842	1.841463	2.163122
1000	β	1.204650	.00464952	.01382696	.9743571	1.434942
	α	3.012702	.00844073	.03398598	2.652229	3.373175
	θ	2.000365	.00036526	.00473129	1.865550	2.135181

Table 2: The MLE, AB, MSE, LCB and UCB ($\beta = 1.3, \alpha = 0.5, \theta = 1$)

n	*Parameter*	*Estimate*	*AB*	*MSE*	*LCB*	*UCB*
60	β	1.053991	.24600860	.32064680	.0543402	2.053642
	α	.5724056	.07240564	.02042343	.3309129	.8138984
	θ	.7969797	.20302030	.19443860	.0297671	1.564192
100	β	1.231330	.06867039	.25679090	.2472704	2.215389
	α	.5319416	.03194159	.00721677	.3776546	.6862286
	θ	.9308613	.06913871	.15337370	.1753238	1.686399
200	β	1.313025	.05113733	.19877630	.4395450	2.186505
	α	.5150225	.01502246	.00355625	.4019087	.6281363
	θ	1.000653	.03650486	.12653460	.3034490	1.697857
500	β	1.351137	.04761136	.10869190	.7127762	1.989498
	α	.5024747	.00247473	.00127562	.4326399	.5723096
	θ	1.036505	.02791238	.06923318	.5257732	1.547237
700	β	1.347611	.03022623	.08217767	.7935486	1.901674
	α	.5005719	.00084460	.00083438	.4439673	.5571765
	θ	1.027912	.02090430	.05154650	.5862926	1.469532
1000	β	1.330226	.01302505	.04792240	.9052686	1.755184
	α	.5008446	.00057191	.00060281	.4527510	.5489382
	θ	1.020904	.00065327	.03070837	.6798903	1.361918

As shown from the above tables one can observe that the numerical results support the theoretical findings. That is, the MLEs have good statistical properties. As the sample size increases, it is observed that the ML estimates approach the true values of the parameters. Also, when the sample size increases the estimated AB and MSE of the MLEs decrease and the confidence bounds of the parameters are to be much narrow.

The optimum value of the stress change point

To get the optimum value of the stress change point π_1^* that minimized the GAV of the MLE of the parameters β, α, and θ, the following procedure is applied:

- Using the MLE of the parameters obtained in Tables 1 and 2 as initial points, samples of different sizes are generated from Weibull distribution.
- For each sample, equation (4.2) is computed and put equal to zero.
- By solving this equation, the optimum value π_1^* is obtained.
- Then, the value of n_1 is determined.
- It is assumed that $\pi_c = 0.35$ for each case.

Tables 3 and 4 introduce the optimum values of π_1 for different combinations of the parameter values. Besides, the corresponding values of n_1, n_2, n_c and GAV are presented.

Table 3: The optimum value of π_1 and the corresponding values of n_1, n_2, n_c and GAV ($\beta = 1.2, \alpha = 3, \theta = 2$)

n	π_1^*	n_1	n_2	n_c	GAV
60	.079984	5	34	21	.0002702565
100	.138975	14	51	35	.0000656845
200	.104611	21	109	70	.0000071263
500	.106751	53	272	175	.0000056691
700	.106348	74	381	245	.0000011075
1000	.106327	106	544	350	.0000000927

Table 4: The optimum value of π_1 and the corresponding values of n_1, n_2, n_c and GAV ($\beta = 1.3, \alpha = 0.5, \theta = 1$)

n	π_1^*	n_1	n_2	n_c	GAV
60	.400053	24	15	21	.0001293299
100	.411987	41	24	35	.0000505192
200	.404969	81	49	70	.0000088734
500	.395672	198	127	175	.0000003981
700	.392288	275	180	245	.0000001487
1000	.393616	394	256	350	.0000000471

The design problem is to determine the optimal numbers (the optimal proportion) of test units that should be failed at each stress level according to a certain optimality criterion. Since the value of π_1 was obtained to minimize the GAV of the MLEs of the model parameters, the developed test plans are statistically optimum.

It is seen from Tables 3 and 4 that π_1 is approximately takes a value from 0.1 to 0.5. This means that nearly less than half of the observations will fail under the normal use conditions. The others will fail at high conditions. For example, in Table 4, when $n = 500$, 198 observations will fail under the stress x_1, 127 will fail under x_2 and 175 will be censored. As shown via the optimal value of π_1, the partial accelerating is important and needed. That is, testing will be run not only at normal condition but also at accelerated condition.

Concerning the optimal GAV of the MLEs of the model parameters in the above Tables, it is observed from the numerical results that it decreases as the sample size n increases.

6 Conclusion

In practice, the failure-step stress test can require constant monitoring and might be inconvenient. But it may be more appropriate than the time-step stress test. It enables the experimenter to collect sufficient amount of failure information needed to make a good statistical inference about the population parameters. That is, to make a reliability prediction with high level of accuracy.

This paper presented the case of two stress levels under the failure-step stress partially accelerated life testing assuming type-II censoring. A two-parameter Weibull distribution was used. The MLEs were studied together with some further properties.

Moreover, optimum plans for the FSPALT were obtained numerically using the D-optimality via a simulation study. It is noted via the optimal value of π_1 that the PALT model is more appropriate model. That is, testing at both normal and accelerated conditions.

Further work is needed to extend the results to other distributions and other censored schemes such as progressive censoring.

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