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A Note on Occasionally Weakly Compatible Maps

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Abstract

We communicate some important remarks about the concept of occasionally weakly compatible (owc) maps which was introduced by Al-Thagafi and Shahzad [1]. This concept is recently becoming a topic of considerable research interest; see, e.g., [2, 3, 4, 7].

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Let f and g be selfmaps of a metric space $X := (X, d)$ and $C(f, g) := \{x \in X : f(x) = g(x)\}$. The pair (f, g) is called:

(i) weakly commuting [6] if

$d(f(g(x)), g(f(x))) \leq d(f(x), g(x))$ for all $x \in X$;

(ii) weakly compatible [5] if

$f(g(x)) = g(f(x))$ for all $x \in C(f, g)$; and

(iii) occasionally weakly compatible (owc) [1] if

$f(g(x)) = g(f(x))$ for some $x \in C(f, g)$.

The following example shows that the weakly compatible maps form a proper subclass of the owc maps.

Example. Let $X := [0, \infty)$ with the usual metric. Define $f, g : X \to X$ by $f(x) = 2x$ and $g(x) = x^2$ for all $x \in X$. Then $C(f, g) = \{0, 2\}$, $f(g(0)) = g(f(0))$, and $f(g(2)) \neq g(f(2))$. Thus (f, g) is a owc pair but not weakly compatible; see [1].

We note that the concept of owc maps was first introduced by Al-Thagafi and Shahzad [1]. This was explicitly acknowledged by Jungck and Rhoades in [4]. Consequently, the following statement of Abbas and Rhoades [2, p.1, l_{11-12}] is misleading:

"In the recent paper of Jungck and Rhoades [4], the concept of occasionally weakly compatible maps (owc) was introduced".

We also note the following useful remarks about the owc maps.

Remarks. Let $f, g : X \to X$ where X is a metric space.

(1) If $C(f, g) = \emptyset$, then (f, g) is trivially an owc pair. So, to avoid triviality, it is better to assume that $C(f, g) \neq \emptyset$.

(2) Suppose that X is a discrete metric space and $C(f, g) \neq \emptyset$. Then (f, g) is a weakly compatible pair iff it is a weakly commuting pair. Now, suppose that (f, g) is a weakly compatible pair. Then

$$d(f(x), g(x)) = d(f(g(x)), g(f(x))) = 0$$

for all $x \in C(f, g)$ and

$$d(f(x), g(x)) = 1 \geq d(f(g(x)), g(f(x)))$$

for all $x \notin C(f, g)$. So

$$d(f(g(x)), g(f(x))) \leq d(f(x), g(x))$$

for all $x \in X$. Conversely, suppose that (f, g) is a weakly commuting pair. Then

$$d(f(g(x)), g(f(x))) \leq d(f(x), g(x)) = 0$$

for all $x \in C(f,g)$ and hence (f,g) is a weakly compatible pair. Thus, whenever the concept of weak compatibility is used, it is necessary to assume that X is non-discrete and that $C(f,g) \neq \emptyset$.

(3) Remark (2) does not hold for owc maps. To see this, let $X := [0,\infty)$ with the discrete metric. Define f and g by $f(x) = 2x$ and $g(x) = x^2$ for all $x \in X$. Then $C(f,g) = \{0,2\}$, $f(g(0)) = g(f(0))$, and $f(g(2)) \neq g(f(2))$. So (f,g) is an owc pair but $d(f(2),g(2)) = 0 < 1 = d(f(g(2)),g(f(2)))$. Note that every weakly commuting pair of maps is owc.

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