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Korovkin Type Theorems for Weighted Approximation

Jorge Bustamante

Facultad de Ciencias Fí́sico-Matemáticas,
Universidad Autónoma de Puebla, Puebla, Mexico
jbusta@fcfm.buap.mx

Lorena Morales de la Cruz

Facultad de Ciencias Fí́sico-Matemáticas,
Universidad Autónoma de Puebla, Puebla, Mexico

Abstract

We present some new results related with approximation of functions in weighted spaces. We show that the theory of approximation by linear operators in the spaces $C^*_\rho[0,\infty)$ can be reduced the same theory for the space $C[0,1]$ and that there are not finite Korovkin families for the space $C_\rho[0,\infty)$.

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1 Introduction

Throughout the paper $\rho : [0,\infty) \to (0,\infty)$ is a fixed continuous function. By $C_\rho[0,\infty)$ we denote the space of all continuous functions $f : [0,\infty) \to \mathbb{R}$ such that $\rho(x)f(x)$ is bounded. This is a Banach space with the norm

$$\|f\|_\rho = \sup_{x\in[0,\infty)} \mid \rho(x)f(x) \mid .$$

Moreover, $C^*_\rho[0,\infty)$ is the closed subspace of $C_\rho[0,\infty)$ formed by the functions f for which the limit

$$\rho(\infty)f(\infty) = \lim_{x\to\infty} \rho(x)f(x)$$

exists and is finite.

In last years there have been some interest in studying linear positive operators acting on $C^*_\rho[0,\infty)$ for some particular weights ρ. In particular, some new moduli of continuity were introduced in order to present quantitative theorems for the rate of convergence of some sequences of linear positive operators.

In this paper we present some new results related with approximation of functions in weighted spaces. First, the theory of approximation by linear operators in the spaces $C^*_\rho[0,\infty)$ can be reduced the same theory for the space $C[0,1]$ of continuous functions on the interval $[0,1]$ (with the sup norm $\|\cdot\|_\infty$). Second, as it was expected, moduli of smoothness of Ditzian-Totik type are a good instrument to estimate the rate of convergence of sequences of linear positive operators in the spaces $C^*_\rho[0,\infty)$. Third, there are not finite Korovkin families for the space $C_\rho[0,\infty)$.

The paper is organized as follows. In the second section we prove that there exists a linear positive isomorphism $\Phi : C^*_\rho[0,\infty) \to C[0,1]$. Moreover, Φ can be chosen is such a way that $\|\Phi(f)\|_\infty = \|f\|_\rho$, for any $f \in C^*_\rho[0,\infty)$. Then, we used this isomorphism to define different moduli of smoothness for functions in $C^*_\rho[0,\infty)$. In the third section we present some estimations for linear positive operators on $C^*_\rho[0,\infty)$ and we compare they with previous known results. In Section 4 we discuss the case of linear positive operators on $C_\rho[0,\infty)$. Finally, the last section is devoted to analyze some extensions.

2 Properties of the spaces $C^*_\rho[0,\infty)$

Let us denote

$$\psi(y) = \frac{y}{1-y}, \qquad y \in [0,1). \tag{1}$$

It is clear that $\psi : [0,1) \to [0,\infty)$ is a homeomorphism with inverse function

$$\psi^{-1}(x) = \frac{x}{1+x}, \qquad x \in [0,\infty).$$

For $f \in C^*_\rho[0,\infty)$ define

Theorem 2.1 *If the operator $\Phi : C^*_\rho[0,\infty) \to C[0,1]$ is define by*

$$\Phi(f,y) = \begin{cases} \rho(\psi(y))f(\psi(y)), & \text{if} \quad y \in [0,1), \\ \lim\limits_{x\to\infty} \rho(x)f(x), & \text{if} \quad y = 1, \end{cases} \qquad f \in C^*_\rho[0,\infty), \tag{2}$$

then Φ is a positive linear isomorphism, with positive linear inverse $\Phi^{-1} : C[0,1] \to C^*_\rho[0,\infty)$ given by

$$\Phi^{-1}(g,x) = \frac{g(\psi^{-1}(x))}{\rho(x)}, \quad g \in C[0,1], \quad x \in [0,\infty).$$

Moreover, for each $f \in C^*_\rho[0,\infty)$

$$\|f\|_\rho = \|\Phi(f)\|_\infty.$$

Proof. Fix $f \in C^*_\rho[0,\infty)$. Since ρ is continuous, the function $g(x) = \rho(x)f(x)$ is continuous on $[0,\infty)$. Then $g \circ \psi$ is continuous on $[0,1)$. Moreover

$$\lim_{y\to 1} g(\psi(y)) = \lim_{x\to\infty} \rho(x)f(x) = \Psi(f,1).$$

Thus $\Psi(f) \in C[0,1]$ and the operator Ψ is well defined.

It is easy to verify that Ψ is a linear operator. Define $N : C[0,1] \to C[0,\infty)$ by

$$N(g,x) = g(\psi^{-1}(x))/\rho(x), \qquad x \in [0,\infty), \quad g \in C[0,1].$$

If $g \in C[0,1]$,

$$\lim_{x\to\infty} \rho(x)N(g,x) = \lim_{x\to\infty} g(\psi^{-1}(x)) = \lim_{x\to\infty} g\left(\frac{x}{1+x}\right) = g(1).$$

Therefore $N(g) \in C^*_\rho[0,\infty)$. For $y \in [0,1)$

$$\Psi(N(g),y) = \rho(\psi(y))N(g,\psi(y)) = \rho(\psi(y))\frac{g(\psi^{-1}(\psi(y))}{\rho(\psi(y))} = g(y).$$

For $y = 1$,

$$\Psi(N(g),1) = \lim_{x\to\infty} \rho(x)N(g,x) = g(1).$$

This proves that $\Psi : C^*_\rho[0,\infty) \to C[0,1]$ is surjective and $\Psi(N(g)) = g$.

On the other hand, if $f \in C^*_\rho[0,\infty)$ and $x \in [0,\infty)$, then

$$N(\Psi(f),x) = \frac{\Psi(f,\psi^{-1}(x))}{\rho(x)} = \frac{1}{\rho(x))}\rho(\psi(\psi^{-1}(x))f(\psi(\psi^{-1}(x)) = f(x).$$

Thus $N = \Phi^{-1}$.

Since $\rho(x) > 0$, the operators Φ and Φ^{-1} are positives.

Finally, for $f \in C^*_\rho[0,\infty)$,

$$\|f\|_\rho = \sup_{x\geq 0} \rho(x) \mid f(x) \mid = \sup_{y\in[0,1)} \rho(\psi(y)) \mid f(\psi(y)) \mid$$

$$= \sup_{y\in[0,1)} \mid M(f,y) \mid = \sup_{y\in[0,1]} \mid \Psi(f,y) \mid = \|\Psi f\|. \blacksquare$$

3 Moduli of smoothness in $C^*_\rho[0,\infty)$

In the first part of this section we recall some the moduli of smoothness with have been used to estimate lineal operators on weighted subspaces of $C[0,\infty)$.

For $f : I \to \mathbb{R}$ and $h > 0$ the first and second (symmetric) differences are defined by

$$\Delta_h f(x)) = f(x+h) - f(x-h)$$

and

$$\Delta_h^2 f(x)) = f(x+h) - 2f(x) + f(x-h)$$

respectively, whenever $x+h, x-h, x \in I$ and $\Delta_h f(x) = 0$ otherwise.

For $f \in C_\rho[0,\infty)$ and $h > 0$, set (see [4])

$$\omega_1(f,h)_\rho = \sup_{0<t\leq h} \|\Delta_t f\|_\rho \quad \text{and} \quad \omega_2(f,h)_\rho = \sup_{0<t\leq h} \|\Delta_t^2 f\|_\rho. \tag{3}$$

This is a natural way to introduce a modulus and we shall restrict the definition to functions f such that ρf is uniformly continuous. Such a modulus has been used to estimate the rate of uniform convergence of some particular sequences of linear positive operators in polynomial and exponential weighted spaces (see [10] and [11]). However these moduli do not satisfy several of the classical properties of the moduli on a compact set. This fact motivated Lopez-Moreno [8] to define

$$\Omega(f,h) = \sup_{0<t\leq h} \sup_{x\geq 0} \rho(h+x) \mid f(x+h) - f(x) \mid,$$

for the case $\rho(x) = (1+x^m)^{-1}$, $m \in \mathbb{N}$.

Amanov presented Ditzian-Totik moduli [2]. He considered continuous, non increasing function ρ, such that $0 < \rho(x) \leq 1$ on $[0,\infty)$ and defined

$$\omega_1^\varphi(f,h)_\rho = \sup_{0<t\leq h} \sup_{x\geq 0} \rho(x+t\varphi(x)) \mid f(x+t\varphi(x)) - f(x) \mid,$$

where $\varphi(x) = \sqrt{x}$ (whit usual variation for higher order moduli).

Now, if Θ is any modulus of smoothness in $C[0,1]$, we obtain a modulus in $C^*_\rho[0,\infty)$ by seeting

$$\Theta(f,h)_\rho = \Theta(\Phi f, h), \quad h \in (0,1], \quad f \in C^*_\rho[0,\infty). \tag{4}$$

Moduli as the ones defined by equation (4) are sufficient to present estimations for the rate of approximation in $C^*_\rho[0,\infty)$, but there are some inconveniences. First, if we want to compute the modulus for a concrete function f,

we need to know the limits of the function $\rho(x)f(x)$ at infinity. Second, since the moduli are defined by composition with the function ψ it is difficult to compare it with other known moduli. In any case we can construct equivalent moduli in $C^*_\rho[0,\infty)$ which make not references to ψ. In our next result we show how this can be accomplished in two particular (but important) cases.

Theorem 3.1 *If $f \in C^*_\rho[0,\infty)$ and Φf is defined by (2), then for $h \in (0,1]$*

$$\omega_1^\varphi(\Phi f, h) = \omega^\lambda(f,h)_\rho, \tag{5}$$

where

$$\omega_1^\varphi(\Phi f, h) = \sup_{0<t\le h/2} \sup_{y\pm\varphi(y)\in[0,1]} \mid \Delta_{t\varphi(y)}\Phi(f)(y) \mid \tag{6}$$

with $\varphi(y) = 1-y$ $(y \in [0,1])$ and

$$\omega^\lambda(f,h)_\rho = \sup_{t\in(0,h/2]} \sup_{\lambda(x)t\le x} \mid \Delta_{t\lambda(x)}(\rho f)(x) \mid, \tag{7}$$

with $\lambda(x) = 1+x$, $x \in [0,\infty)$.

Proof. Fix $h \in (0,1]$, $t \in (0,h/2]$ and $y \in [0,1]$ such that $y \pm h(1-y) \in (0,1)$. Set

$$x = \frac{y+(1-y)t^2}{(1-y)(1-t^2)} \quad y \quad s = \frac{t}{(1-y)(1-t^2)},$$

Notice that $x - s = \psi(y-(1-y)t)$, $x+s = \psi(y-(1-y)t)$,

$$1+x = \frac{1}{(1-y)(1-t^2)} \quad \text{and} \quad s = (1+x)t.$$

Thus

$$\begin{aligned}
&\mid M(f, y+(1-y)t) - M(f, y-(1-y)t) \mid \\
&\qquad = \mid \rho(x+s)f(x+s) - \rho(x-s)f(x-s) \mid \\
&\qquad \le \sup_{t\in(0,h/2]} \sup_{\lambda(x)t\le x} \mid (\rho f)(x+\lambda(x)t) - (\rho f)(x-\lambda(x)t) \mid \\
&\qquad = \omega^\lambda(f,h)_\rho.
\end{aligned}$$

This proves that $\omega_1^\varphi(Mf,h) \le \omega^\lambda(f,h)_\rho$.

For the converse inequality, fix $s \in (0, h/2]$ and x such that $\varphi(x)s \leq x$ and set

$$z = \frac{x - \varphi(x)s^2}{(1+x)(1-s^2)} \quad \text{and} \quad u = \frac{s}{(1+x)(1-s^2)}.$$

One has $\psi^{-1}(x - \varphi(x)s) = z - u$, $\psi^{-1}(x + \varphi(x)s) = z + u$, and $s(1-z) = u$. Hence

$$\rho(x+\varphi(x)s)f(x+\varphi(x)s) - \rho(x-\varphi(x)s)f(x-\varphi(x)s)$$

$$= M(f, \psi(z + (1-z)s) - M(f, \psi(z - (1-z)s).$$

It is sufficient to prove that $\omega^{\lambda}(f,h)_{\rho} \leq \omega_1^{\varphi}(Mf,h)$.■

Theorem 3.2 *If $f \in C_{\rho}^{*}[0,\infty)$ and Φf is defined by (2), then for $h \in (0, 1/2]$*

$$\omega_1^{*}(f,h)_{\rho} \leq \omega_1^{\varphi}(\Phi f, h) \leq \omega_1^{*}(f, 2h)_{\rho}. \tag{8}$$

where $\omega_1^{\varphi}(\Phi f, h)$ is defined by (6) with $\varphi(y) = (1-y)^2$ and

$$\omega_1^{*}(f,h)_{\rho} = \sup_{t \in (0,h/2]} \sup_{x \geq t} \mid \Delta_t(\rho f)(x) \mid . \tag{9}$$

Proof. Fix $t \in (0, h/2]$ and $x \geq h$ and define

$$y = \frac{x + x^2 - t^2}{(1-x)^2 - t^2}$$

and

$$s = \frac{t}{(1-x)^2 - t^2} = (1-y)^2 \frac{(1+x)^2 - t^2}{(1+x)^2} t = (1-y)^2 u.$$

It follows that $\Phi(y \pm s) = x \pm h$. Therefore

$$\mid \Delta_t(\rho f)(x) = \mid \Delta_{(1-y)^2 u} \Phi(f)(y) \mid \leq \omega_1^{\varphi}(\Phi f, h).$$

It is sufficient to prove the first inequality in (8).

For the second one, fix $t \in (0, h/4]$ and y such that $y \pm (1-y)^2 h \in [0,1]$. Set

$$x = \frac{y + (1-y)^3 t^2}{(1-y)(1-(1-y)^2 t^2} \quad \text{and} \quad s = \frac{t}{1-(1-y)^2 t^2}.$$

It follows that $\Phi(y \pm (1-y)^2 h) = x \pm s$. Since $2t^2 \leq 1$, then $s \leq 2t \leq h$. Hence

$$\mid \Delta_{\varphi(y)t}(\Phi f)(y) = \mid \Delta_s(\rho f)(x) \mid \leq \omega_1^{*}(f, 2h)_{\rho}.■$$

Remark 3.3 Theorems 3.1 and 3.2 help us to compare the moduli induced by $C[0,1]$ with some of the usual ones recalled at the beginning of this section. For instance, the first moduli presented in (3) use the expressions

$$\rho(x) \mid f(x+t) - f(x) \mid,$$

while in (9) we consider the term

$$\mid \rho(x+t)f(x+t) - \rho(x-t)f(x-t) \mid .$$

Thus the difference between the corresponding moduli is given by

$$\sup_{0<t\le h} \sup_{x\ge t} \left| 1 - \frac{\rho(x-t)}{\rho(x+t)} \right| \mid \rho(x+t)f(x+t) \mid \le \|f\|_\rho \sup_{0<t\le h} \sup_{x\ge t} \left| 1 - \frac{\rho(x-t)}{\rho(x+t)} \right| .$$

If there exist a positive constant C such that

$$\sup_{0<t\le h} \sup_{x\ge t} \left| 1 - \frac{\rho(x-t)}{\rho(x+t)} \right| \le Ct,$$

then both moduli have a similar behavior. For the case of polynomial weights $\rho(x) = (1+x^m)^{-1}$ $(m \in \mathbb{N})$ and exponential weights $\rho(x) = e^{-\alpha x}$ $(\alpha > 0)$, this condition holds.

Remark 3.4 If we define a modulus of smoothness of second order in $C^*_\rho[0,\infty)$ using symmetric differences, then the arguments used in the proof of the last theorem do not provide a characterization as the one given in (5). But we can use the unusual idea of nonsymmetric differences. In fact one has

$$\sup_{\{y\,:\,y\pm(1-y)t\in[0,1]\}} \mid M(f, y+(1-y)t) - 2M(f,y) + M(f, y-(1-y)t) \mid$$

$$= \sup_{\{x\,:\,x>0,(1+x)t\le x\}} \mid (\rho f)(x+(1+x)t) - 2(\rho f)(x-(1+x)t^2) + (\rho f)(x-(1+x)t) \mid .$$

Of course, we may return to symmetric differences by considering an extra term given by

$$\sup_{0<t\le h} \sup_{(1+x)t\le x} \mid (\rho f)(x) - (\rho f)(x-(1+x)t^2) \mid .$$

$$= \sup_{0<t\le h} \sup_{(1+x)t\le x} \left| \Psi\left(f, \frac{x}{1+x}\right) - \Psi\left(f, \frac{x-(1+x)t^2}{(1+x)(1-t^2)}\right) \right|$$

$$= \sup_{0<t\le h} \sup_{y\pm(1-y)t\in[0,1]} \left| \Psi\left(f, y + \frac{(1-y)t^2}{2-t^2}\right) - \Psi\left(f, y - \frac{(1-y)t^2}{2-t^2}\right) \right|$$

$$= \omega_1^\varphi\left(\Psi(f), \frac{2h^2}{2-h^2}\right) = \omega^\lambda\left(f, \frac{2h^2}{2-h^2}\right)_\rho ,$$

for $h \le \sqrt{2/3}$, with $\varphi(y) = 1-y$ and $\lambda(x) = 1+x$.

4 Linear positive operators in $C^*_\rho[0,\infty)$

There are many papers were linear positive operators are considered in weighted spaces as $C_\rho[0,1\infty)$. In all of them some special computations are used in order to estimate the rate of convergence and the arguments can not be used for abstract positive linear operators. In this section we show that the estimations can be obtained in a simple form by using well known results for positive linear operators in the space $C[0,1]$. First we show that there exists a one-to-one correspondence between liner positive operators in $C^*_\rho[0,\infty)$ and in $C[0,1]$. Then we show two the possible way of using results on $C[0,1]$.

Theorem 4.1 *A linear operator $L : C^*_\rho[0,\infty) \to C^*_\rho[0,\infty)$ is positive if and only the operator $L_\Phi : C[0,1] \to C[0,1]$ given by*

$$L_\Phi(g) = (\Phi \circ L \circ \Phi^{-1})(g), \qquad g \in C[0,1].$$

is positive, where Φ and Φ^{-1} are defined as in Theorem 2.1.

Moreover, for $f \in C^*_\rho[0,\infty)$, one has

$$\|f - L(f)\|_\rho = \|\Phi f - L_\Phi(\Phi f)\|_\rho.$$

Proof. If L is positive, then L_Φ is a composition of positive operators (see Theorem 2.1). On the other hand $L = \Phi^{-1} \circ L_\Phi \circ \Phi$. The last assertion follows form the equalities ($\psi(y) = x$)

$$\rho(x) \mid f(x) - L(f)(x) \mid = \mid \Phi(f - L(f))(y) \mid$$

$$\mid (\Phi f)(y) - \Phi(Lf)(y) \mid = \mid (\Phi f)(y) - L_\Phi(\Phi f, y) \mid . \blacksquare$$

In what follows for every $\alpha \in [0,+\infty)$, set $e_\alpha(t) = t^\alpha$. Moreover set

$$f_0(z) = \frac{1}{\rho(z)}, \quad f_1(z) = \frac{z}{\rho(z)(1+z)}, \text{ and } f_2(z) = \frac{z^2}{\rho(z)(1+z)^2}.$$

Notice that $\psi(e_i) = f_i$, $i = 0,1,2$.

Theorem 4.2 *If $L : C^*_\rho[0,\infty) \to C^*_\rho[0,\infty)$ is a linear positive operator and $f \in C^*_\rho[0,\infty)$, then*

$$\|f - L(f)\|_\rho \le \|f\|_\rho \|f_0 - L(f_0)\|_\rho + (\|L\| + 2\sqrt{\|L\|})\omega_1(\Psi f, \sqrt{\gamma}),$$

where $\|L\|$ is the norm of the operator L and $\gamma = \sup_{x\in[0,\infty)} \rho(x)L((f_1 - xf_0)^2, x)$ and for $x \in [0,\infty)$ and $h \in (0,1/2]$,

$$\rho(x) \mid f(x) - L(f,x) \mid \le \mid (\rho f)(x) \mid\mid \rho(x)L(f_0,x) - 1 \mid$$

$$+\rho(x) \mid L(f_1 - \frac{x}{1+x} f_0, x) \mid \frac{1}{h}\omega_1(\Psi f, h)$$

$$+ \left(1 + \frac{\rho(x) L((f_1 - (x/(1+x) f_0)^2, x)}{2h^2}\right) \omega_2(\Phi f, h),$$

where ω_1 and ω_2 denote the first and second usual moduli of smoothness respectively.

Proof. It is known that if $M : C[0,1] \to C[0,1]$ is a linear positive operator and $g \in C[0,1]$, then

$$\|g - M(g)\|_\infty \leq \|g\|_\infty \|e_0 - M(e_0)\|_\infty + (\|M\| + 2\sqrt{\|M\|})\omega_1(g, \sqrt{\lambda})$$

where $\lambda = \sup_{y\in[0,1]} M((e_1 - ye_0)^2, y)$ and ω_1 is the usual first modulus of continuity ([5], p. 278-279) and for $h \in (0, 1/2]$ and $y \in [0,1]$ one has

$$\mid g(x) - M(g,x) \mid \leq \mid g(x) \mid\mid M(e_0, x) - 1 \mid + \mid M(e_1 - ye_0, y) \mid \frac{1}{h}\omega_1(g,h)$$

$$+ \left(1 + \frac{M((e_1 - ye_0)^2, y)}{2h^2}\right) \omega_2(g,h),$$

where $\omega_1(f,h)$ and $\omega_2(f,h)$ denote the first and second moduli of smoothness respectively (see [9]). The result follows from the arguments above (with $M = L_\Phi$), Theorem 4.1 and the definition of the functions f_i. In fact, if $x \in [0,\infty)$ and $\psi(y) = x$, then

$$L_\Phi(e_0, y) = \rho(\psi(y)) L(\Phi^{-1} e_0, \psi(y)) = \rho(x) L(f_0, x), \tag{10}$$

$$L_\Phi(e_1 - ye_0, y) = \rho(\psi(y)) L(\Phi^{-1}(e_1 - ye_0), \psi(y)) \tag{11}$$

$$= \rho(x) L(f_1 - \frac{x}{1+x} f_0, x),$$

and

$$L_\Phi((e_1 - ye_0)^2, y) = \rho(\psi(y)) L(\Phi^{-1}(e_1 - ye_0)^2, \psi(y)) \tag{12}$$

$$= \rho(x) L\left(f_2 - \frac{2x}{1+x} f_1 + \frac{x^2}{(1+x)^2} f_0, x\right). \blacksquare$$

Similar expressions are available in terms of Ditzian-Totik type moduli of smoothness. If we want an estimation in terms of the first moduli in (3), then we shall use the estimation in $C[0,1]$ in terms of $\omega_1^\varphi(\Phi f, h)$ (see Theorem 3.2).

5 Other compactifications of the semi ray

It is clear that $[0,1]$ is homeomorphic to a compactification of the semi ray $[0,\infty)$. This fact was a fundamental argument for obtaining the results presented in the previous sections. Thus, it is natural to look at other compactification of the semi ray.

Let X be a closed subspace of $C_\rho[0,\infty)$. A subset H of X is called a Korovkin subset with respect to positive operators (briefly, a K_+-subset) if for every equicontinuous net $(L_i)_{i\in I}$ of positive linear operators on X satisfying $\lim_{i\in I} L_i(h) = h$ for every $h \in H$, one has $\lim_{i\in I} L_i(f) = f$ for every $f \in X$ (see [1] for more information).

For our next result we need some fact related with the Stone-Cech compactifications. Recall that the Stone-Cech compactification βX of a Tikhonov space X is the largest element in the family $\mathcal{C}(X)$ of all compactifications of X. In particular, it is known that a compactification Y of X is (homeomorphic to) the Stone-Cech compatification of βX if and only if every continuous and bounded function $g : X \to \mathbb{R}$ has a continuous extension to Y (see [7], Corollary 3.6.3).

Theorem 5.1 *There is not any finite Korovkin subset for $C_\rho[0,\infty)$.*

Proof- As in the proof of Theorem 2.1, we can prove that there exists isometric positive isomorphism $T : C_b[0,1) \to C_\rho[0,\infty)$, where $C_b[0,1)$ is the family of all bounded and continuous functions $g : [0,1) \to \mathbb{R}$. Then there exists a isometric positive isomorphism $\Gamma : C_\rho[0,\infty) \to C(\beta[0,1))$. Henceforth, positive linear operators on $C_\rho[0,1)$ are in one-to-one correspondence with positive linear operators on $C(\beta[0,1))$. But, since the Stone-Cech compactification of $[0,1)$ is not metrizable, $C(\beta[0,1))$ has not finite Korovkin subset (see [1]).■

Remark 5.2 *Others compactification of $[0,\infty)$ can be used to defined more general weighted spaces where a Korovkin type theorem can be obtained.*

References

[1] Altomare, F., Campiti, M., *Korovkin-Type Approximation Theory and its Applications*, De Gruyter Series Studies in Mathematics, Vol. 17, Walter de Gruyter & Co., Berlin, New York, 1994

[2] N. T. Amanov, *On the weighted approximation by Szász-Mirakjan operators*, Anal. Math. 18 3 (1992), 167-184.

[3] V. A. Baskakov, *An example of a sequence of linear positive operators in the space of continuous functions*, Dokl. Akad. Nauk SSSR, **113** (1957), 249–251.

[4] M. Becker, *Global approximation theorems for Szász–Mirakjan and Baskakov type operators*, Indiana Univ. Math. J., 27 (1978), 127–138.

[5] R. A. DeVore, G. G. Lorentz, *Constructive Approximation*, Springer-Verlag, Berlin Heidelberg New York, (1993).

[6] Z. Ditzian, V. Totik, *Moduli of Smoothness*, Springer, New York (1987).

[7] R. Engelking, *General Topology*, Berlin, Heldermann, 1989.

[8] A. J. Lopez-Moreno, *Weigthed simultaneous approximation with Baskakov type operators*, Acta. Math. Hungar., 104 (1-2) (2004), 143–151.

[9] R. Păltănea, *Optimal estimates with moduli of continuity*, Result. Math., 32 (1997), 318–331.

[10] Z. Walczak, *On certain linear positive operators in polynomial weighted spaces*, Acta Math. Hungar., 101(3) (2003), 179-191.

[11] Z. Walczak, *On some linear positive operators in exponential weighted spaces*, Math. Commun., 8 (2003), no. 1, 77-84.

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