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Implicit Iteration Process with Mean Errors for Common Fixed Points of a Finite Family of Strictly Pseudocontrative Maps[1]

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Abstract

The purpose of this paper is to study the weak and strong convergence of an implicit iterative precess with mean errors to a common fixed

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point for a finite family of strictly pseudocontractive maps in Hilbert spaces. The results presented in this paper extend and generalize some results in the literature.

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1. Introduction and Preliminaries

Let H be a real Hilbert space with inner product $\langle .,. \rangle$ and induced norm $\|.\|$, K be a convex subset of H. A mapping $T : K \to K$ is said to be strictly pseudocontractive in the terminology of Browder and Petryshyn [1] if there exists $0 < k < 1$ such that

$$\|Tx - Ty\|^2 \leq \|x - y\|^2$$

$$+k\|(I - T)x - (I - T)y\|^2, \forall x, y \in K, \tag{1.1}$$

where I denotes the identity operator on K.

(1.1) can be equivalently written in the form

$$\langle (I - T)x - (I - T)y, x - y \rangle$$

$$\geq \lambda\|(I - T)x - (I - T)y\|^2, \forall x, y \in K, \tag{1.2}$$

where $\lambda = (\frac{1-k}{2})$. From (1.2), we have

$$\|x - y\| \geq \lambda\|(I - T)x - (I - T)y\|$$

$$\geq \lambda(\|Tx - Ty\| - \|x - y\|), \forall x, y \in K,$$

so that

$$\|Tx - Ty\| \leq \frac{1 + \lambda}{\lambda}\|x - y\|, \forall x, y \in K. \tag{1.3}$$

Hence a strictly pseudocontrative map is also a L-Lipschitzian map, where $L = \frac{1+\lambda}{\lambda}$.

The class of strictly pseudocontractive maps has been studied by several authors (see [1-6]). It follows from (1.3) that the important class of nonexpansive maps is a subclass of the class of strictly pseudocontractive maps.

We recall that T is said to be demiclosed at a point p if whenever $\{x_n\}$ is a sequence in K such that $\{x_n\}$ converges weakly to $x \in K$ and $\{Tx_n\}$ converges strongly to p, then $Tx = p$. Furthermore, T is said to be demicompact if whenever $\{x_n\}$ is a bounded sequence in K such that $\{x_n - Tx_n\}$ converges strongly, then $\{x_n\}$ has a subsequence which converges strongly.

Definition 1.1. Let K be a nonempty convex subset of a Hilbert space H, and let $\{T_i\}_{i=1}^N$ be a family of strictly pseudocontractive self-maps of K. Let $x_0 \in K$ and $\{\alpha_n\}_{n=1}^\infty$, $\{\beta_n\}_{n=1}^\infty$, $\{\gamma_n\}_{n=1}^\infty$ be three sequences in [0,1] satisfying $\alpha_n + \beta_n + \gamma_n = 1$ for all $n \geq 1$, and $\{u_n\}_{n=1}^\infty$ be a sequence in K. Then the sequence $\{x_n\}_{n=1}^\infty \subset K$ generated by

$$\begin{cases} x_1 = \alpha_1 x_0 + \beta_1 T_1 x_1 + \gamma_1 u_1, \\ x_2 = \alpha_2 x_1 + \beta_2 T_2 x_2 + \gamma_2 u_2, \\ \cdots \\ x_N = \alpha_N x_{N-1} + \beta_N T_N x_N + \gamma_N u_N, \\ x_{N+1} = \alpha_{N+1} x_N + \beta_{N+1} T_1 x_{N+1} + \gamma_{N+1} u_{N+1}, \\ \cdots. \end{cases} \tag{1.4}$$

is called the implicit iteration process with mean errors for a finite family of strictly pseudocontractive maps $\{T_i\}_{i=1}^N$.

The scheme (1.4) can be expressed in a compact form as

$$x_n = \alpha_n x_{n-1} + \beta_n T_n x_n + \gamma_n u_n, \quad n \geq 1,$$

where $T_n = T_{n \bmod N}$.

Remark 1.1. If $\gamma_n \equiv 0$, $\beta_n = 1 - \alpha_n$ $(\forall n \geq 1)$, then the implicit iteration process with mean errors generated by (1.4) reduces to the process (5) in [7].

Recently, concerning the convergence of implicit iteration process to a common fixed point of a finite family of strictly pseudocontractive maps or asymptotically nonexpansive maps or nonexpansive maps have been considered by several authors (see [2], [7-9] and the references therein).

The purpose of this paper is to study the weak and strong convergence of the implicit iterative sequences generated by (1.4) to a common fixed point for a finite family of strictly pseudocontractive maps in Hilbert spaces. The results presented in this paper extend and generalize some results in [2] and [7].

In order to prove the main results of this paper, we need the following lemmas:

Lemma 1.1 (see [10, p.560]). Let E be an inner product space. Then, for all $x, y, z \in E$ and $\alpha, \beta, \gamma \in [0, 1]$ with $\alpha + \beta + \gamma = 1$, we have

$$\|\alpha x + \beta y + \gamma z\|^2 = \alpha\|x\|^2 + \beta\|y\|^2 + \gamma\|z\|^2$$

$$-\alpha\beta\|x - y\|^2 - \alpha\gamma\|x - z\|^2 - \beta\gamma\|y - z\|^2.$$

Remark 1.2. If $y = 0$ in Lemma 1.1, then we have for all $x, z \in E$ and $\alpha, \beta, \gamma \in [0, 1]$ with $\alpha + \beta + \gamma = 1$, we have

$$\|\alpha x + \gamma z\|^2 = \alpha\|x\|^2 + \gamma\|z\|^2 - \alpha\beta\|x\|^2$$

$$-\alpha\gamma\|x - z\|^2 - \beta\gamma\|z\|^2$$

$$\leq \alpha\|x\|^2 + \gamma\|z\|^2 - \alpha\gamma\|x - z\|^2$$

$$\leq \alpha\|x\|^2 + \gamma\|z\|^2.$$

Lemma 1.2 (see [11,12]). Let $\{a_n\}_{n=1}^{\infty}$, $\{b_n\}_{n=1}^{\infty}$, $\{\delta_n\}_{n=1}^{\infty}$ be nonnegative sequences satisfying the following inequality

$$a_{n+1} \leq (1 + \delta_n)a_n + b_n, \forall n \geq 1.$$

If $\sum_{n=1}^{\infty} \delta_n < \infty$ and $\sum_{n=1}^{\infty} b_n < \infty$, then $lim_{n\to\infty} a_n$ exists.

Lemma 1.3 (see [6, p.444]). Let E be a real q-uniformly smooth Banach space which is also uniformly convex. Let K be a nonempty closed convex subset of E and $T : K \to K$ a strictly pseudocontractive map. Then $(I - T)$ is demiclosed at zero.

Lemma 1.4(see [13]). A Hilbert space H has Opial's property: $x_n \to x$ weakly implies that lim sup$\|x_n - z\|$ <lim sup $\|x_n - z\|$ for any $z \in H$, $z \neq x$.

2.The Main Results

Theorem 2.1. Let H be a real Hilbert space and let K be a nonempty closed convex subset of H. Let $\{T_i\}_{i=1}^{N}$ be N strictly pseudocontractive self-maps of K such that $F = \bigcap_{i=1}^{N} F(T_i) \neq \emptyset$, where $F(T_i) = \{x \in K : T_i x = x\}$. Let $x_0 \in K$ and $\{u_n\}$ be a bounded sequence in K, let $\{\alpha_n\}, \{\beta_n\}, \{\gamma_n\}$ be three sequences in $[0, 1]$ satisfying the following conditions:

(i) $\{\alpha_n\} + \{\beta_n\} + \{\gamma_n\} = 1, \forall n \geq 1;$

(ii) There exist constant σ_1, σ_2 such that $0 < \sigma_1 \leq \beta_n \leq \sigma_2 < 1, \forall n \geq 1$;

(iii) $\sum_{n=1}^{\infty} \gamma_n < \infty$.

Then the implicit iterative sequence $\{x_n\}_{n=1}^{\infty}$ defined by (1.4) converges weakly to a common fixed point of the maps $\{T_i\}_{i=1}^{N}$.

Proof. Since $F = \bigcap_{i=1}^{N} F(T_i) \neq \emptyset$, for any given $p \in F$, it follows from Lemma 1.1 that

$$\|x_n - p\|^2 = \|\alpha_n x_{n-1} + \beta_n T_n x_n + \gamma_n u_n - p\|^2$$
$$= \|\alpha_n(x_{n-1} - p) + \beta_n(T_n x_n - p) + \gamma_n(u_n - p)\|^2$$
$$= \alpha_n\|x_{n-1} - p\|^2 + \beta_n\|T_n x_n - p\|^2 + \gamma_n\|u_n - p\|^2$$
$$- \alpha_n\beta_n\|x_{n-1} - T_n x_n\|^2 - \alpha_n\gamma_n\|x_{n-1} - u_n\|^2$$
$$- \beta_n\gamma_n\|T_n x_n - u_n\|^2. \qquad (2.1)$$

Since each T_i is strictly pseudocontractive, then there exists $k_i \in (0,1)$ such that for $i = 1, 2, ..., N$, we have

$$\|T_i x - T_i y\|^2 \leq \|x - y\|^2 + k_i\|x - T_i x - (y - T_i y)\|^2.$$

Let $k = max_{1\leq i\leq N}\{k_i\}$. Then $k \in (0,1)$ and

$$\|T_i x - T_i y\|^2 \leq \|x - y\|^2$$
$$+ k\|x - T_i x - (y - T_i y)\|^2. \qquad (2.2)$$

Thus we obtain from (2.1) and (2.2) that

$$\|x_n - p\|^2 \leq \alpha_n\|x_{n-1} - p\|^2 + \beta_n[\|x_n - p\|^2$$
$$+k\|x_n - T_n x_n\|^2] + \gamma_n\|u_n - p\|^2$$
$$- \alpha_n\beta_n\|x_{n-1} - T_n x_n\|^2 - \beta_n\gamma_n\|T_n x_n - u_n\|^2. \qquad (2.3)$$

It follows from Remark 1.1 that

$$\|x_n - T_n x_n\|^2$$
$$= \|(\alpha_n x_{n-1} + \beta_n T_n x_n + \gamma_n u_n) - T_n x_n\|^2$$
$$= \|\alpha_n(x_{n-1} - T_n x_n) + \gamma_n(u_n - T_n x_n)\|^2$$
$$\leq \alpha_n\|x_{n-1} - T_n x_n\|^2 + \gamma_n\|u_n - T_n x_n\|^2. \qquad (2.4)$$

We obtain from (2.3) and (2.4) that

$$\|x_n - p\|^2$$
$$\leq \alpha_n\|x_{n-1} - p\|^2 + \beta_n\|x_n - p\|^2$$
$$+k\beta_n\alpha_n\|x_{n-1} - T_n x_n\|^2 + k\beta_n\gamma_n\|u_n - T_n x_n\|^2$$
$$+ \gamma_n\|u_n - p\|^2 - \alpha_n\beta_n\|x_{n-1} - T_n x_n\|^2$$
$$- \beta_n\gamma_n\|T_n x_n - u_n\|^2.$$

Hence

$$\|x_n - p\|^2 \leq \frac{\alpha_n}{1-\beta_n}\|x_{n-1} - p\|^2 - \frac{(1-k)\alpha_n\beta_n}{1-\beta_n}\|x_{n-1} - T_n x_n\|^2 - \frac{(1-k)\beta_n\gamma_n}{1-\beta_n}\|u_n - T_n x_n\|^2 + \frac{\gamma_n}{1-\beta_n}\|u_n - p\|^2 \leq \|x_{n-1} - p\|^2 - \frac{(1-k)\alpha_n\beta_n}{1-\beta_n}\|x_{n-1} - T_n x_n\|^2 + \frac{\gamma_n}{1-\beta_n}\|u_n - p\|^2.$$

By the condition (ii), we have

$$\|x_n - p\|^2 \leq \|x_{n-1} - p\|^2 - \frac{(1-k)\alpha_n\beta_n}{1-\beta_n}\|x_{n-1} - T_n x_n\|^2 + \frac{\gamma_n}{1-\sigma_2}\|u_n - p\|^2. \quad (2.5)$$

Clearly, there exists a number $M > 0$ such that

$$\frac{1}{1-\sigma_2}\|u_n - p\|^2 \leq M. \quad (2.6)$$

By (2.5) and (2.6), we obtain

$$\|x_n - p\|^2 \leq \|x_{n-1} - p\|^2 + M\gamma_n. \quad (2.7)$$

It now follows from (2.7), the condition (iii) and Lemma 1.2 that $lim_{n\to\infty}\|x_n - p\|$ exists. For example let

$$lim_{n\to\infty}\|x_n - p\| = d. \quad (2.8)$$

By (2.5), (2.6) and the condition (ii), we also obtain

$$\frac{(1-k)\sigma_1(1-\sigma_2-\gamma_n)}{1-\sigma_1}\|x_{n-1} - T_n x_n\|^2 \leq \|x_{n-1} - p\|^2 - \|x_n - p\|^2 + \frac{\gamma_n}{1-\sigma_2}\|u_n - p\|^2 \leq \|x_{n-1} - p\|^2 - \|x_n - p\|^2 + M\gamma_n, \quad \forall n \geq 1.$$

By condition (iii), we know that for a fixed number $\varepsilon \in (0, 1-\sigma_2)$, there is a positive integer N_0 such that if $n \geq N_0$, then

$$0 \leq \gamma_n \leq \varepsilon.$$

Thus

$$\frac{(1-k)\sigma_1(1-\sigma_2-\varepsilon)}{1-\sigma_1}\|x_{n-1} - T_n x_n\|^2 \leq \|x_{n-1} - p\|^2 - \|x_n - p\|^2 + M\gamma_n, \quad \forall n \geq N_0.$$

So,

$$\frac{(1-k)\sigma_1(1-\sigma_2-\varepsilon)}{1-\sigma_1}\sum_{n=N_0}^{\infty}\|x_{n-1} - T_n x_n\|^2 \leq \|x_{N_0-1} - p\|^2 + M\sum_{n=N_0}^{\infty}\gamma_n. \quad (2.9)$$

It follows the condition (iii) and (2.9) that $\sum_{n=1}^{\infty}\|x_{n-1}-T_nx_n\|^2<\infty$. Thus

$$\lim_{n\to\infty}\|x_{n-1}-T_nx_n\|=0. \tag{2.10}$$

Since $\|x+y\|^2\le 2[max(\|x\|,\|y\|)]^2\le 2(\|x\|^2+\|y\|^2), \forall x,y\in H$, by Remark 2.1 and (2.2), we have

$$\begin{aligned}
&\|x_n-T_nx_n\|^2\\
&=\|(\alpha_nx_{n-1}+\beta_nT_nx_n+\gamma_nu_n)-T_nx_n\|^2\\
&=\|\alpha_n(x_{n-1}-T_nx_n)+\gamma_n(u_n-T_nx_n)\|^2\\
&\le\alpha_n\|x_{n-1}-T_nx_n\|^2+\gamma_n\|u_n-T_nx_n\|^2\\
&\le\alpha_n\|x_{n-1}-T_nx_n\|^2+2\gamma_n\|u_n-p\|^2\\
&\qquad+2\gamma_n\|T_nx_n-p\|^2\\
&\le\alpha_n\|x_{n-1}-T_nx_n\|^2+2\gamma_n\|u_n-p\|^2\\
&\quad+2\gamma_n(\|x_n-p\|^2+k\|x_n-T_nx_n\|^2).
\end{aligned}$$

Thus

$$\begin{aligned}(1-2\gamma_nk)\|x_n-T_nx_n\|^2&\le\alpha_n\|x_{n-1}-T_nx_n\|^2\\
&+2\gamma_n\|u_n-p\|^2+2\gamma_n\|x_n-p\|^2.\end{aligned} \tag{2.11}$$

It follows from (2.8), (2.10), (2.11) and the condition (iii) that

$$\lim_{n\to\infty}\|x_n-T_nx_n\|=0. \tag{2.12}$$

Moreover, since

$$\begin{aligned}
&\|x_n-x_{n-1}\|\\
&=\|\beta_n(T_nx_n-x_{n-1})+\gamma_n(u_n-x_{n-1})\|\\
&\le\beta_n\|T_nx_n-x_{n-1}\|+\gamma_n\|u_n-x_{n-1}\|.\\
&\le\beta_n\|T_nx_n-x_{n-1}\|+\gamma_n(\|u_n-p\|+\|x_{n-1}-p\|).
\end{aligned}$$

It also follows from (2.8), (2.10) and the condition (iii) that

$$\lim_{n\to\infty}\|x_n-x_{n-1}\|=0. \tag{2.13}$$

Thus

$$\lim_{n\to\infty}\|x_n-x_{n+i}\|=0, \forall i=1,2,...,N. \tag{2.14}$$

Since

$$\begin{aligned}
&\|x_n-T_{n+i}x_n\|\\
&\le\|x_n-x_{n+i}\|+\|x_{n+i}-T_{n+i}x_{n+i}\|
\end{aligned}$$

$$+ \|T_{n+i}x_{n+i} - T_{n+i}x_n\|. \tag{2.15}$$

And

$\|T_i x - T_i y\| \le L_i\|x-y\|, \quad \forall i = 1,2,...,N.$

If we choose $L = max_{1\le i\le N}\{L_i\}$, then

$\|T_i x - T_i y\| \le L\|x-y\|, \quad \forall i = 1,2,...,N. \quad (2.16)$

It follows from (2.15) and (2.16) that

$\|x_n - T_{n+i}x_n\|$

$$\le \|x_n - x_{n+i}\| + \|x_{n+i} - T_{n+i}x_{n+i}\| + L\|x_{n+i} - x_n\|$$

$$\le (1+L)\|x_n - x_{n+i}\| + \|x_{n+i} - T_{n+i}x_{n+i}\|.$$

It follows from (2.12) and (2.14) that for all $i = 1,2,...,N$,

$$\lim_{n\to\infty} \|x_n - T_{n+i}x_n\| = 0. \tag{2.17}$$

Since $\{x_n\}$ is bounded in K, it has a subsequence $\{x_{n_k}\}_{k=1}^{\infty}$ which converges weakly to some $q \in K$. Without loss of generality, we may assume that $n_k \equiv j$ mod N for all k and some $j \in \{1,2,...,N\}$. For any fixed $l \in \{1,2,...,N\}$, we can find an $i \in \{1,2,...,N\}$, independent of k, such that $n_k + i = l$ mod N for all k. It then follows from (2.17) that

$$\lim_{k\to\infty} \|x_{n_k} - T_l x_{n_k}\| = 0. \tag{2.18}$$

Since every Hilbert space is uniformly convex and 2-uniformly smooth (see for example [7]), it follows Lemma 1.3 that $(I - T_l)$ is demiclosed at zero, so that $q \in F(T_l)$. Since $l \in \{1,2,...,N\}$ is arbitrary, then $q \in F$. Thus we have a subsequence $\{x_{n_k}\}_{k=1}^{\infty}$ of the sequence $\{x_n\}$ which converges weakly to a common fixed point q of $\{T_i\}_{i=1}^{N}$. If $\{x_n\}$ has another subsequence $\{x_{n_j}\}_{j=1}^{\infty}$ which converges weakly to q_1 and $q_1 \ne q$. Then by the same method as given above, we can also prove that $q_1 \in F$.

Taking $p = q$ and $p = q_1$ and by using the same method as given in the proof of (2.8) we can prove that the following two limits exist and

$$\lim_{n\to\infty} \|x_n - q\| = d_1; \qquad \lim_{n\to\infty} \|x_n - q_1\| = d_2;$$

where d_1 and d_2 are two nonnegative numbers. It follows from Lemma 1.4 that the Hilbert space H satisfies the Opial condition, so we have

$$d_1 = \limsup_{n_k\to\infty} \|x_{n_k} - q\| < \limsup_{n_k\to\infty} \|x_{n_k} - q_1\| = d_2$$

$$= \limsup_{n_j \to \infty} \|x_{n_j} - q_1\| < \limsup_{n_j \to \infty} \|x_{n_j} - q\| = d_1.$$

This is a contradiction. Hence, $q = q_1$. This implies that $\{x_n\}$ converges weakly to q. This completes the proof.

Theorem 2.2. Let H be a real Hilbert space and let K be a nonempty closed convex subset of H. Let $\{T_i\}_{i=1}^N$ be N strictly pseudocontractive self-maps of K such that $F = \bigcap_{n=1}^N F(T_i) \neq \emptyset$. Let $x_0 \in K$ and $\{u_n\}$ be a bounded sequence in K, let $\{\alpha_n\}, \{\beta_n\}, \{\gamma_n\}$ be three sequences in $[0, 1]$ satisfying the following conditions:

(i) $\{\alpha_n\} + \{\beta_n\} + \{\gamma_n\} = 1, \forall n \geq 1$;

(ii) There exist constant σ_1, σ_2 such that $0 < \sigma_1 \leq \beta_n \leq \sigma_2 < 1$;

(iii) $\sum_{n=1}^{\infty} \gamma_n < \infty$;

(iv) There exists $i_0 \in \{1, 2, ..., N\}$ such that T_{i_0} is demicompact.

Then the implicit iterative sequence $\{x_n\}_{n=1}^{\infty}$ defined by (1.4) converges strongly to a common fixed point of the maps $\{T_i\}_{i=1}^N$.

Proof. From the Proof of Theorem 2.1, we know that there exists a subsequence $\{x_{n_k}\} \subset \{x_n\}$ such that $\{x_{n_k}\}$ converges weakly to some $q \in K$ and for any fixed $l \in \{1, 2, ..., N\}$, we have

$$\lim_{k \to \infty} \|x_{n_k} - T_l x_{n_k}\| = 0. \tag{2.18}$$

Since $T_{i_0} (1 \leq i_0 \leq N)$ is demicompact, there exists a subsequence of $\{x_{n_k}\}$ (we denote it still by $\{x_{n_k}\}$) such that

$$\lim_{k \to \infty} \|x_{n_k} - q\| = 0.$$

Again by (2.18) and the arbitrariness of l, we obtain that $q \in F$. From the proof of (2.8), we know that the limit $\lim_{n \to \infty} \|x_n - q\|$ exists, and then $\lim_{n \to \infty} \|x_n - q\| = 0$, i.e., $x_n \to q$. This completes the proof.

Remark 2.1. Theorem 2.1 extends and generalizes Theorem 1 in [2] and Theorem 2 in [7].

Remark 2.2. Theorem 2.2 answered the open question given by Xu and Ori [7] to some extent.

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