Lightlike Submanifolds of an
Indefinite Kaehler Manifold with
a Non-metric ($\phi$, $\theta$)-Connection

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Abstract

We define a new connection on semi-Riemannian manifolds ($\bar{M}$, $\bar{g}$), which is called a non-metric ($\phi$, $\theta$)-connection. Non-metric $\theta$-connection is an example of this connection such that $\phi = \bar{g}$. The purpose of this paper is to study two types of 1-lightlike submanifolds $M$, so called lightlike hypersurface and half lightlike submanifold, of an indefinite Kaehler manifold $\bar{M}$ with a non-metric ($\phi$, $\theta$)-connection.

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1 Introduction

An affine connection $\bar{\nabla}$ on a semi-Riemannian manifold ($\bar{M}$, $\bar{g}$) is called a non-metric ($\phi$, $\theta$)-connection if it satisfies

\[
(\bar{\nabla}_X \bar{g})(Y, Z) = -\phi(X, Y)\theta(Z) - \phi(X, Z)\theta(Y),
\]

for any vector fields $X$, $Y$ and $Z$ on $\bar{M}$, where $\phi$ is a tensor field of type $(0, 2)$, $\theta$ is a 1-form associated with a vector field $\zeta$, which is called the characteristic...
vector field, by $\theta(X) = \bar{g}(X, \zeta)$. In case $\phi = \bar{g}$, we say that $\bar{\nabla}$ is a non-metric $\theta$-connection [11, 12] on $\bar{M}$. The semi-symmetric non-metric connection [1] and the quarter-symmetric non-metric connection [2, 3, 7, 8, 13] are two important examples of this connection. These two special connections are important for the mathematical study and the applications to physics.

The theory of lightlike submanifolds is an important topic of research in differential geometry due to its application in mathematical physics, especially in the general relativity. 1-lightlike submanifold is a particular case of $r$-lightlike submanifold [5]. Much of its geometry will be immediately generalized in a formal way to arbitrary $r$-lightlike submanifolds. Moreover the theory of 1-lightlike submanifold is more simple than that of $r$-lightlike submanifold. Due to this reason, we study only 1-lightlike submanifolds in this paper.

The equations (2.32) in [5, Section 4.2] and (4.13) in [4] indicates that the induced connection $\nabla$ of any 1-lightlike submanifold $M$ of a semi-Riemannian manifold $(\bar{M}, \bar{g})$ is an important example of non-metric $(\phi, \theta)$-connection. Motivated by the existing lightlike geometry of non-metric $(\phi, \theta)$-connection, the objective of this paper is to study two types of 1-lightlike submanifolds of an indefinite Kaehler manifold with a non-metric $(\phi, \theta)$-connection, in which the tensor field $\phi$ of (1.1) is identical with the fundamental tensor field $\phi$ associated with the indefinite almost complex structure $J$ of $\bar{M}$, that is,

$$\phi(X,Y) = \bar{g}(X, JY). \quad (1.2)$$

2 Lightlike hypersurfaces

Let $\bar{M} = (\bar{M}, \bar{g}, J)$ be an indefinite Kaehler manifold with a non-metric $(\phi, \theta)$-connection, where $\bar{g}$ is a semi-Riemannian metric and $J$ is an indefinite almost complex structure satisfying, for any vector field $X$ and $Y$ of $\bar{M}$,

$$J^2 X = -X, \quad \bar{g}(JX, JY) = \bar{g}(X, Y), \quad (\bar{\nabla}_X J)Y = 0. \quad (2.1)$$

Let $(M, g)$ be a lightlike hypersurface of $\bar{M}$. In this case, the normal bundle $TM^\perp$ of $M$ is a vector subbundle of the tangent bundle $TM$ of $M$, of rank 1, and coincides with the radical distribution $\text{Rad}(TM) = TM \cap TM^\perp$. A complementary vector bundle $S(TM)$ of $\text{Rad}(TM)$ in $TM$ is non-degenerate distribution on $M$, which is called a screen distribution on $M$, such that

$$TM = \text{Rad}(TM) \oplus_{\text{orth}} S(TM),$$

where $\oplus_{\text{orth}}$ denotes the orthogonal direct sum. We denote such a lightlike hypersurface by $M = (M, g, S(TM))$. Denote by $F(M)$ the algebra of smooth functions on $M$ and by $\Gamma(E)$ the $F(M)$ module of smooth sections of any vector bundle $E$ over $M$. Also denote by (2.1), the $i$-th equation of the three
equations in (2.1). We use same notations for any others. For any null section \( \xi \) of \( \text{Rad}(TM) \) on a coordinate neighborhood \( U \subset M \), there exists a unique null section \( N \) of a unique lightlike vector bundle \( tr(TM) \) in \( S(TM) \) \cite{5} satisfying

\[
\bar{g}(\xi, N) = 1, \quad \bar{g}(N, N) = \bar{g}(N, X) = 0, \quad \forall X \in \Gamma(S(TM)).
\]

We call \( tr(TM) \) and \( N \) the transversal vector bundle and the null transversal vector field of \( M \) with respect to the screen distribution \( S(TM) \), respectively. The tangent bundle \( TM \) of \( M \) is decomposed as follow:

\[
TM = TM \oplus tr(TM) = \{\text{Rad}(TM) \oplus tr(TM)\} \oplus_{\text{orth}} S(TM).
\]

In the sequel, let \( X, Y, Z \) and \( W \) be the vector fields on \( M \), unless otherwise specified. Let \( P \) be the projection morphism of \( TM \) on \( S(TM) \). Then the local Gauss and Weingarten formulas of \( M \) and \( S(TM) \) are given respectively by

\[
\begin{align*}
\bar{\nabla}_XY &= \nabla_XY + B(X,Y)N, \quad (2.2) \\
\bar{\nabla}_XN &= -A_NX + \tau(X)N; \quad (2.3) \\
\bar{\nabla}_XPY &= \nabla_PY + C(X, PY)\xi, \quad (2.4) \\
\bar{\nabla}_X\xi &= -A^*_X - \sigma(X)\xi, \quad (2.5)
\end{align*}
\]

where \( \nabla \) and \( \nabla^* \) are the induced linear connections on \( TM \) and \( S(TM) \) respectively, \( B \) and \( C \) are the local second fundamental forms on \( TM \) and \( S(TM) \) respectively, \( A_N \) and \( A^*_\xi \) are the shape operators on \( TM \) and \( S(TM) \) respectively and \( \tau \) and \( \sigma \) are 1-forms on \( TM \).

For a lightlike hypersurfaces \( M \) of an indefinite a Hermition manifold \( \bar{M} \), it is known (\cite{5, Section 6.2}, \cite{9}) that \( J(\text{Rad}(TM)) \) and \( J(tr(TM)) \) are subbundles of \( S(TM) \), of rank 1 such that \( J(\text{Rad}(TM)) \cap J(tr(TM)) = \{0\} \). Thus there exist two non-degenerate almost complex distributions \( D_o \) and \( D \) on \( M \) with respect to \( J \), i.e., \( J(D_o) = D_o \) and \( J(D) = D \), such that

\[
S(TM) = J(\text{Rad}(TM)) \oplus J(tr(TM)) \oplus_{\text{orth}} D_o, \\
D = \{\text{Rad}(TM) \oplus_{\text{orth}} J(\text{Rad}(TM))\} \oplus_{\text{orth}} D_o.
\]

In this case, the decomposition form of \( TM \) is reduced to

\[
TM = D \oplus J(tr(TM)). \quad (2.6)
\]

Consider two null vector fields \( U \) and \( V \), and two 1-forms \( u \) and \( v \) such that

\[
U = -JN, \quad V = -J\xi, \quad u(X) = g(X, V), \quad v(X) = g(X, U). \quad (2.7)
\]

Denote by \( S \) the projection morphism of \( TM \) on \( D \). Any vector field \( X \) of \( M \) is expressed as \( X = SX + u(X)U \). Applying \( J \) to this form, we have

\[
JX = FX + u(X)N, \quad (2.8)
\]
where $F$ is a tensor field of type $(1, 1)$ globally defined on $M$ by $F = J \circ S$. Applying $J$ to (2.8) and using (2.1) and (2.7), we have
\[ F^2 X = -X + u(X)U. \] (2.9)

**Theorem 2.1.** Let $M$ be a lightlike hypersurface of an indefinite Kaehler manifold with a non-metric $(\phi, \theta)$-connection. Then the characteristic vector field $\zeta$ of $\bar{M}$ is a null vector field and it belongs to $J(\text{Rad}(TM))$.

**Proof.** Replacing $Y$ by $\xi$, $N$, $V$ and $U$ to (1.2) by turns, we obtain
\[ \phi(X, \xi) = -u(X), \quad \phi(X, N) = -v(X), \quad \phi(X, V) = 0, \quad \phi(X, U) = \eta(X). \] (2.10)

Put $a = \theta(N)$ and $b = \theta(\xi)$. The connection $\nabla$ is not metric and satisfies
\[
(\nabla_X g)(Y, Z) = B(X, Y)\eta(Z) + B(X, Z)\eta(Y)
- \phi(X, Y)\theta(Z) - \phi(X, Z)\theta(Y),
\] (2.11)
where $\eta$ is a 1-form on $TM$ such that
\[ \eta(X) = \bar{g}(X, N). \]

From the fact that $B(X, Y) = \bar{g}(\nabla_X Y, \xi)$, we know that $B$ is independent of the choice of the screen distribution $S(TM)$ and satisfies
\[ B(X, \xi) = -bu(X). \]

From this equation, (2.2) and (2.5), we obtain
\[ \nabla_X \xi = -A^*_\xi X - \sigma(X)\xi - bu(X)N. \] (2.12)

Using (1.1), (2.12) and the fact that $B(X, Y) = \bar{g}(\nabla_X Y, \xi)$, we obtain
\[ B(X, Y) = g(A^*_\xi X, Y) + b\phi(X, Y) - \{\theta(Y) - b\eta(Y)\}u(X). \] (2.13)

Applying $\nabla_X$ to $\bar{g}(N, \xi) = 1$ and using (2.3), (2.11) and (2.12), we have
\[ \sigma(X) = \tau(X) + au(X) + bv(X). \] (2.14)

Applying $\nabla_X$ to (2.7)$_2$ and using (2.1)$_3$, (2.2), (2.8) and (2.12), we have
\[ \nabla_X V = F(A^*_\xi X) - \sigma(X)V - bu(X)U, \quad B(X, V) = u(A^*_\xi X). \] (2.15)

On the other hand, taking $Y = V$ to (2.13) and using (2.10)$_3$, we have
\[ B(X, V) = u(A^*_\xi X) - \theta(V)u(X). \]
Comparing this and (2.15)_2, we get $\theta(V)u(X) = 0$. Thus we obtain

$$\theta(V) = 0. \quad (2.16)$$

Applying $\nabla_X$ to (2.8) and using (2.1)_3, (2.2), (2.3) and (2.8), we have

$$(\nabla_X F)Y = u(Y)A_N X - B(X,Y)u,$$
$$(\nabla_X u)Y = -u(Y)\tau(X) - B(X,FY).$$

On the other hand, applying $\nabla_X$ to $u(Y) = g(Y,V)$ and using (2.10)_3, (2.11), (2.13), (2.14), (2.15)_1,2 and (2.16), we have

$$(\nabla_X u)(Y) = -u(Y)\tau(X) - B(X,FY)$$
$$+ b\phi(X,FY) - \theta(FY)u(X) - \{au(X) + bv(X)\}u(Y).$$

Comparing the last two equations, we obtain

$$b\phi(X,FY) - \theta(FY)u(X) - \{au(X) + bv(X)\}u(Y) = 0. \quad (2.17)$$

Taking $Y = U$ to (2.17) and using the fact that $FU = 0$, we obtain

$$au(X) + bv(X) = 0.$$

It follows that $a = 0$ and $b = 0$. Thus the vector field $\zeta$ belongs to $S(TM)$.

As $a = b = 0$, taking $X = U$ to (2.17), we have

$$\theta(FY) = 0, \quad \forall Y \in \Gamma(TM).$$

Taking $Y = FX$ to this equation and using (2.9), we have

$$\theta(X) = \theta(U)u(X). \quad (2.18)$$

Taking $X = \zeta$ to this and using the fact that $u(\zeta) = \theta(V) = 0$, we obtain

$$\theta(\zeta) = \theta(U)u(\zeta) = 0. \quad (2.19)$$

Thus $\zeta$ is a null vector field. Taking $X \in \Gamma(D_o)$, then $u(X) = 0$. Thus we have $\theta(X) = 0$ by (2.18). From this result and (2.16), we see that the characteristic vector field $\zeta$ belongs to $J(Rad(TM))$.

From (2.19), we have the following result:

**Corollary.** There exist no lightlike hypersurfaces of an indefinite Kaehler manifold $(\overline{M}, \overline{g})$ with a non-metric $(\phi, \theta)$-connection such that $\zeta$ is non-null.
3 Half lightlike submanifolds

Let \((M, g)\) be a half lightlike submanifold of an indefinite Kaehler manifold \(\bar{M}\) with a non-metric \((\phi, \theta)\)-connection, of codimension 2. Then the radical distribution \(\text{Rad}(TM)\) of \(M\) is a vector subbundle of \(TM\), of rank 1. There exist two complementary non-degenerate distributions \(S(TM)\) and \(S(TM^\perp)\) of \(\text{Rad}(TM)\) in \(TM\) and \(TM^\perp\) respectively, which are called the screen distribution and co-screen distribution of \(M\), such that \(TM = \text{Rad}(TM) \oplus_{\text{orth}} S(TM)\), \(TM^\perp = \text{Rad}(TM) \oplus_{\text{orth}} S(TM^\perp)\).

We denote such a half lightlike submanifold by \(M = (M, g, S(TM), S(TM^\perp))\). Choose \(L \in \Gamma(S(TM^\perp))\) as a unit spacelike vector field, no loss of generality. Consider the orthogonal complementary distribution \(S(TM)^\perp\) to \(S(TM)\) in \(TM\). Certainly, \(\text{Rad}(TM)\) and \(S(TM^\perp)\) are vector subbundles of \(S(TM)^\perp\). As the co-screen distribution \(S(TM^\perp)\) is non-degenerate, we have \(S(TM)^\perp = S(TM^\perp) \oplus_{\text{orth}} S(TM^\perp)^\perp\), where \(S(TM^\perp)^\perp\) is the orthogonal complementary to \(S(TM^\perp)\) in \(S(TM)^\perp\).

For any null section \(\xi\) of \(\text{Rad}(TM)\), there exists a uniquely defined lightlike vector bundle \(ltr(TM)\) and a null vector field \(N\) of \(ltr(TM)\) [4] satisfying \(\bar{g}(\xi, N) = 1, \bar{g}(N, N) = \bar{g}(N, X) = \bar{g}(N, L) = 0, \forall X \in \Gamma(S(TM))\).

We call \(N, ltr(TM)\) and \(tr(TM) = S(TM^\perp) \oplus_{\text{orth}} ltr(TM)\) the lightlike transversal vector field, lightlike transversal vector bundle and transversal vector bundle of \(M\) with respect to \(S(TM)\) respectively [6]. Thus \(T\bar{M}\) is decomposed as

\[
T\bar{M} = TM \oplus tr(TM) = \{\text{Rad}(TM) \oplus tr(TM)\} \oplus_{\text{orth}} S(TM) = \{\text{Rad}(TM) \oplus ltr(TM)\} \oplus_{\text{orth}} S(TM) \oplus_{\text{orth}} S(TM^\perp).
\]

The local Gauss and Weingarten formulas of \(M\) and \(S(TM)\) are given by

\[
\nabla_X Y = \nabla_X Y + B(X, Y)N + D(X, Y)L, \quad (3.1)
\]

\[
\nabla_X N = -A_N X + \tau(X)N + \rho(X)L, \quad (3.2)
\]

\[
\nabla_X L = -A_L X + \mu(X)N + \nu(X)L; \quad (3.3)
\]

\[
\nabla_X PY = \nabla_X^* PY + C(X, PY)\xi, \quad (3.4)
\]

\[
\nabla_X \xi = -A_\xi X - \sigma(X)\xi, \quad (3.5)
\]

respectively, where \(\nabla\) and \(\nabla^*\) are linear connections on \(TM\) and \(S(TM)\) respectively, \(B\) and \(D\) are called the local second fundamental forms of \(M\), \(C\) is called the local second fundamental form on \(S(TM)\). \(A_N, A_L^*\) and \(A_L\) are linear operators on \(TM\) and \(\tau, \rho, \mu, \nu\) and \(\sigma\) are 1-forms on \(TM\)
For a half lightlike submanifold \( M \) of an indefinite almost Hermitian manifold \( \bar{M} \), it is known [10] that \( J(\text{Rad}(TM)), J(\text{ltr}(TM)) \) and \( J(S(TM^\perp)) \) are vector subbundles of \( S(TM) \) with trivial intersections, of rank 1. Thus there exist two non-degenerate almost complex distribution \( H_o \) and \( H \) on \( M \) with respect to the indefinite complex structure \( J \) such that

\[
S(TM) = J(\text{Rad}(TM)) \oplus J(\text{ltr}(TM)) \oplus_{\text{orth}} J(S(TM^\perp)) \oplus_{\text{orth}} H_o,
\]

\[
H = \{\text{Rad}(TM) \oplus_{\text{orth}} J(\text{Rad}(TM))\} \oplus_{\text{orth}} H_o.
\]

In this case, the decomposition form of \( TM \) is reduced to

\[
TM = H \oplus J(\text{ltr}(TM)) \oplus_{\text{orth}} J(S(TM^\perp)).
\]  

(3.6)

Consider the two null and one spacelike vector fields \( \{U, V\} \) and \( W \) such that

\[
U = -JN, \quad V = -J\xi, \quad W = -JL.
\]

(3.7)

Denote by \( S \) the projection morphism of \( TM \) on \( H \) with respect to the decomposition (3.6). Any vector field \( X \) on \( M \) is expressed as follow:

\[
X = SX + u(X)U + w(X)W,
\]

where \( u, v \) and \( w \) are 1-forms locally defined on \( M \) by

\[
u(X) = g(X, V), \quad v(X) = g(X, U), \quad w(X) = g(X, W). \]

(3.8)

Using (3.7), the action \( JX \) of \( X \) by \( J \) is expressed as follow:

\[
JX = FX + u(X)N + w(X)L,
\]

where \( F \) is a tensor field of type (1, 1) globally defined on \( M \) by \( F = J \circ S \).

Applying \( J \) to (3.9) and using (2.1) and (3.7), we have

\[
F^2X = -X + u(X)U + w(X)W. \]

(3.10)

**Theorem 3.1.** Let \( M \) be a half lightlike submanifold of an indefinite Kaehler manifold with a non-metric \((\phi, \theta)\)-connection. Then the characteristic vector field \( \zeta \) of \( M \) is a null vector field and it belongs to \( J(\text{Rad}(TM)) \).

**Proof.** Replacing \( Y \) by \( \xi, N, L, V, U \) and \( W \) to (1.2) by turns, we obtain

\[
\phi(X, \xi) = -u(X), \quad \phi(X, N) = -v(X), \quad \phi(X, L) = -w(X), \quad \phi(X, V) = 0, \quad \phi(X, U) = \eta(X), \quad \phi(X, W) = 0.
\]

(3.11)

Denote \( a, b \) and \( c \) the functions given by \( a = \theta(N) \), \( b = \theta(\xi) \) and \( c = \theta(L) \). The induced connection \( \nabla \) on \( M \) is not metric and satisfies

\[
(\nabla_X g)(Y, Z) = B(X, Y)\eta(Z) + B(X, Z)\eta(Y) - \phi(X, Y)\theta(Z) - \phi(X, Z)\theta(Y).
\]

(3.12)
From the facts that $B(X,Y) = \bar{g}(\nabla_X Y, \xi)$ and $D(X,Y) = \bar{g}(\nabla_X Y, L)$, we know that $B$ and $D$ are independent of the choice of $S(TM)$ and satisfy

$$B(X, \xi) = -bu(X), \quad D(X, \xi) = -\lambda(X),$$

where we set $\lambda(X) = bw(X) + cu(X) + \mu(X)$. From (3.1) and (3.5), we get

$$\bar{\nabla}_X \xi = -A^* X - \sigma(X) \xi - bu(X) N - \lambda(X) L. \quad (3.13)$$

Also, by using (3.3), (3.12) and (3.13), we obtain

$$B(X,Y) = g(A^*_X Y) + b\phi(X,Y) - \{\theta(Y) - b\eta(Y)\} u(X), \quad (3.14)$$
$$D(X,Y) = g(A_L X, Y) + c\phi(X,Y) - \theta(Y) w(X) - \eta(Y) \mu(X). \quad (3.15)$$

Applying $\bar{\nabla}_X$ to $\bar{g}(N, \xi) = 1$ and using (3.2), (3.12) and (3.13), we have

$$\sigma(X) = \tau(X) + au(X) + bv(X). \quad (3.16)$$

Applying $\bar{\nabla}_X$ to (3.7) and using (2.1), (3.1), (3.3), (3.7) and (3.9), we have

$$\nabla_X W = F(A_L X) + \mu(X) U + \nu(X) W, \quad (3.17)$$
$$D(X,W) = w(A_L X). \quad (3.18)$$

On the other hand, taking $Y = V$ to (3.13) and using (3.11), we have

$$B(X,V) = u(A^*_X V) - \theta(V) u(X).$$

From the last two equations, we obtain $\theta(V) u(X) = 0$. Thus we get $\theta(V) = 0$. Applying $\bar{\nabla}_X$ to (3.7) and using (2.1), (3.1), (3.3), (3.7) and (3.9), we have

$$\nabla_X W = F(A_L X) + \mu(X) U + \nu(X) W, \quad (3.17)$$
$$D(X,W) = w(A_L X). \quad (3.18)$$

On the other hand, taking $Y = W$ to (3.14) and using (3.11), we have

$$D(X,W) = w(A_L X) - \theta(W) w(X).$$

From the last two equations, we obtain $\theta(W) w(X) = 0$. Thus we have

$$\theta(V) = 0, \quad \theta(W) = 0. \quad (3.19)$$

Applying $\bar{\nabla}_X$ to (3.9) and using (3.1)~(3.3), (3.7) and (3.9), we have

$$(\nabla_X F) Y = u(Y) A_N X + w(Y) A_L X - B(X,Y) U - D(X,Y) W,$$
$$(\nabla_X u) Y = - u(Y) \tau(X) - w(Y) \mu(X) - B(X,F Y).$$
On the other hand, applying $\nabla_X$ to $u(Y) = g(Y, V)$ and using (3.11)$_4$, (3.12), (3.14) and (3.16)$\sim$(3.19), we have
\[
(\nabla_X u)(Y) = -u(Y)\tau(X) - w(Y)\mu(X) - B(X, FY) \\
+ b\phi(X, FY) - \theta(FY)u(X) - \{au(X) + bv(X)\}u(Y) \\
- \{bw(X) + cu(X)\}w(Y).
\]
From the last two equations, we obtain
\[
b\phi(X, FY) - \theta(FY)u(X) - \{au(X) + bv(X)\}u(Y) \\
- \{bw(X) + cu(X)\}w(Y) = 0. \tag{3.20}
\]
Taking $Y = U$ and $Y = W$ to (3.20) by turns, since $FU = FW = 0$, we have
\[
au(X) + bv(X) = 0, \quad bw(X) + cu(X) = 0.
\]
From these two equations, we obtain $a = b = c = 0$. Thus $\zeta$ belongs to $S(TM)$.

As $a = b = c = 0$, taking $X = U$ to (3.20), we have
\[
\theta(FY) = 0, \quad \forall Y \in \Gamma(TM).
\]
Taking $Y = FX$ to this result and using (3.10), we have
\[
\theta(X) = \theta(U)u(X) + \theta(W)w(X). \tag{3.21}
\]
Taking $X = \zeta$ to this equation and using the facts that $u(\zeta) = \theta(V) = 0$ and $w(\zeta) = \theta(W) = 0$, we obtain
\[
\theta(\zeta) = \theta(U)u(\zeta) + \theta(W)w(\zeta) = 0. \tag{3.22}
\]
Thus $\zeta$ is a null vector field. Taking $X \in \Gamma(H_o)$, then $u(X) = w(X) = 0$. Thus we have $\theta(X) = 0$ by (3.21). From this and (3.19), we see that the characteristic vector field $\zeta$ belongs to $J(Rad(TM))$.

From (3.22), we have the following result:

**Corollary.** There exist no half lightlike submanifolds $M$ of an indefinite Kaehler manifold $(\bar{M}, \bar{g})$ with a non-metric $(\phi, \theta)$-connection such that the characteristic vector field $\zeta$ of $\bar{M}$ is a non-null vector field.

**Note.** (1) Although $\zeta$ is a null vector field, by the methods of [5, Section 6.2] and [9, 10], we can obtain all results in [5, Section 6.2] and [9, 10]

(2) In case $\phi = \bar{g}$, we shown [11] that there exist no 1-lightlike submanifolds of an indefinite Kaehler manifold $(\bar{M}, \bar{g})$ with a non-metric $\theta$-connection such that the characteristic vector field $\zeta$ of $\bar{M}$ is a non-null vector field.

(3) By (2) we see that there exist no 1-lightlike submanifolds of an indefinite Kaehler manifold with a semi-symmetric non-metric connection or a quarter-symmetric non-metric connection such that $\zeta$ is a non-null vector field.
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