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Some Remarks about Abstract Uryson Operators

Nariman Abasov

MATI – Russian State Tecnological University
str. Orshanski 3, Moscow, 121552 Russia

Marat Pliev[1]

Southern Mathematical Institute of the Russian Academy of Sciences
str. Markusa 22, Vladikavkaz, 362027 Russia

Abstract

We continue to investigation the space of abstract Uryson operators $\mathcal{U}(E, F)$, acting between vector lattices E and F. We introduce a new class of orthogonally additive, disjointness preserving operators which called Uryson lattice homomorphisms. We consider some examples of this operators and prove the Meyer type theorem.

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1. Introduction

The theory of linear disjointness preserving operators in vector lattices is the old and well-studied area of Functional Analysis (see, for instance [6] and references therein). In [12] were established that every regular order continuous linear operator T from order complete lattice E to order continuous Banach

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lattice F has a unique representation $T = T_N + T_D$, where T_D belongs to the band, generated by the disjointness preserving operators and T_N is a narrow operator. The theory of narrow operators is represented in the monograph [20] (see also [2, 7]). On the other hand orthogonally additive operators in vector lattices and lattice-normed spaces, which had been introduced in [13], now are the field of intensive investigations ([1, 5, 9, 10, 16, 17, 18, 19, 21]). The aim of this note is to introduce the new class of disjointness preserving, orthogonally additive operators in vector lattices, which called Uryson lattice homomorphisms. We discuss some examples of this operators and prove the Meyer type theorem.

2. Preliminary information

The goal of this section is to introduce some basic definitions and facts. For standard information on vector lattices we refer to [3, 11]. All vector lattices below are assumed to be Archimedean.

Definition 2.1. Let E be a vector lattice, and let F be a real linear space. An operator $T : E \to F$ is called *orthogonally additive* if $T(x + y) = T(x) + T(y)$ whenever $x, y \in E$ are disjoint.

It follows from the definition that $T(0) = 0$. It is immediate that the set of all orthogonally additive operators is a real vector space with respect to the natural linear operations.

Definition 2.2. Let E and F be vector lattices. An orthogonally additive operator $T : E \to F$ is called:

- *positive* if $Tx \geq 0$ holds in F for all $x \in E$;
- *order bounded* it T maps order bounded sets in E to order bounded sets in F.

An orthogonally additive order bounded operator $T : E \to F$ is called an *abstract Uryson* operator.

The set of all abstract Uryson operators from E to F we denote by $\mathcal{U}(E, F)$. We shall consider some examples. The most famous one is the nonlinear integral Uryson operator.

Example 1. Let (A, Σ, μ) and (B, Ξ, ν) be σ-finite complete measure spaces, and let $(A \times B, \mu \times \nu)$ denote the completion of their product measure space. Let $K : A \times B \times \mathbb{R} \to \mathbb{R}$ be a function satisfying the following conditions[2]:

(C_0) $K(s, t, 0) = 0$ for $\mu \times \nu$-almost all $(s, t) \in A \times B$;
(C_1) $K(\cdot, \cdot, r)$ is $\mu \times \nu$-measurable for all $r \in \mathbb{R}$;
(C_2) $K(s, t, \cdot)$ is continuous on $\mathbb{R}$ for $\mu \times \nu$-almost all $(s, t) \in A \times B$.

Given $f \in L_0(B, \Xi, \nu)$, the function $|K(s, \cdot, f(\cdot))|$ is ν-measurable for μ-almost all $s \in A$ and $h_f(s) := \int_B |K(s, t, f(t))| \, d\nu(t)$ is a well defined and μ-measurable

[2] (C_1) and (C_2) are called the Carathéodory conditions

function. Since the function h_f can be infinite on a set of positive measure, we define

$$\mathrm{Dom}_B(K) := \{f \in L_0(\nu) : \; h_f \in L_0(\mu)\}.$$

Then we define an operator $T : \mathrm{Dom}_B(K) \to L_0(\mu)$ by setting

$$(Tf)(s) := \int_B K(s,t,f(t))\, d\nu(t) \quad \mu-\text{a.e.} \quad (\star)$$

Let E and F be order ideals in $L_0(\nu)$ and $L_0(\mu)$ respectively, K a function satisfying (C_0)-(C_2). Then $(\star)$ defines an *orthogonally additive order bounded integral operator* acting from E to F if $E \subseteq \mathrm{Dom}_B(K)$ and $T(E) \subseteq F$.

Example 2. We consider the vector space $\mathbb{R}^m$, $m \in \mathbb{N}$ as a vector lattice with the coordinate-wise order: for any $x, y \in \mathbb{R}^m$ we set $x \le y$ provided $e_i^*(x) \le e_i^*(y)$ for all $i = 1, \dots, m$, where $(e_i^*)_{i=1}^m$ is the coordinate functionals on $\mathbb{R}^m$. Let $T : \mathbb{R}^n \to \mathbb{R}^m$. Then $T \in \mathcal{U}(\mathbb{R}^n, \mathbb{R}^m)$ if and only if there are real functions $T_{i,j} : \mathbb{R} \to \mathbb{R}$, $1 \le i \le m$, $1 \le j \le n$ satisfying $T_{i,j}(0) = 0$ such that

$$e_i^*\big(T(x_1, \dots, x_n)\big) = \sum_{j=1}^n T_{i,j}(x_j),$$

In this case we write $T = (T_{i,j})$.

Let E be a vector lattice and $x \in E$. Recall that an element $z \in E$ is called a *component* or a *fragment* of x if $z \perp (x - z)$. The set of all fragments of an element x is denoted by $\mathcal{F}_x$. The notations $z \sqsubseteq x$ means that z is a fragment of x. The equality $x = \bigsqcup\limits_{i=1}^n x_i$ means that $x = \sum\limits_{i=1}^n x_i$ and $x_i \perp x_j$ if $i \ne j$. Consider the following order in $\mathcal{U}(E, F)$: $S \le T$ whenever $T - S$ is a positive operator. Then $\mathcal{U}(E, F)$ becomes an ordered vector space. If vector lattice F is Dedekind complete we have the following theorem.

Theorem 2.3. *([13], Theorem 3.2). Let E and F be a vector lattices, F Dedekind complete. Then $\mathcal{U}(E, F)$ is a Dedekind complete vector lattice. Moreover for $S, T \in \mathcal{U}(E, F)$ and for $f \in E$ following hold*

(1) $(T \vee S)(f) := \sup\{Tg + Sh : \; f = g \sqcup h\}$.
(2) $(T \wedge S)(f) := \inf\{Tg + Sh : \; f = g \sqcup h\}$.
(3) $(T)^+(f) := \sup\{Tg : \; g \sqsubseteq f\}$.
(4) $(T)^-(f) := -\inf\{Tg : \; g; \; g \sqsubseteq f\}$.
(5) $|Tf| \le |T|(f)$.

3. Examples and some properties of disjointness preserving operators

In this section we consider some examples of disjointness preserving abstract Uryson operators in vector lattices and describe some of their properties.

An abstract Uryson operator $T : E \to F$ is called *disjointness preserving* if $T(x) \perp T(y)$ for all $x, y \in E$ with $x \perp y$. The set of all disjointness preserving abstract Uryson operators is solid ([19], Theorem 7.3). In particular, an abstract Uryson operator $T : E \to F$ is disjointness preserving if and only if $|T|$ is. The most important example of a nonlinear disjointness preserving operator is a following one.

Example 3. Let (Ω, Σ, μ) be a σ-finite and complete measure space. Let E be a vector sublattice of the space $L_0(\mu)$ of all μ-measurable and μ-almost everywhere finite functions on A, where μ-a.e. equal functions are identified. Consider a function $N : A \times \mathbb{R} \to \mathbb{R}$ satisfying:

(1) $N(t, 0) = 0$ for μ-almost all $t \in A$;
(2) $N(\cdot, x(\cdot))$ is μ-measurable for every $x \in E$.

Then the operator $T : E \to L_0(\mu)$ defined by $(Tx)(t) := N(t, x(t))$ is projection commuting. Indeed, if the operator $\sigma : L_0(\mu) \to L_0(\mu)$ is an order projection, then there is a μ-measurable subset $V \subset A$, so that $\sigma x = x\mathbf{1}_V$ for every $x \in L_0(\mu)$. Recall that $\mathbf{1}_V$ is a characteristic function of the set V. Then we may write

$$(T\sigma x)(t) = T(x\mathbf{1}_V)(t) = N(t, x(t)\mathbf{1}_V(t)) =$$
$$N(t, x(t))\mathbf{1}_V(t) = (Tx)(t)\mathbf{1}_V(t) = (\sigma Tx)(t).$$

Thus T is a disjointness preserving orthogonally additive operator.

These operators are known in a literature as a nonlinear superposition operators or Nemytskii operators (see [4]).

In the linear case the well known example of disjointness preserving operator is a lattice homomorphism. Now, we introduce an similar class of operators in the case of an abstract Uryson operators.

Definition 3.1. Let E be a vector lattice and X a vector space. An orthogonally additive map $T : E \to X$ is called even if $T(x) = T(-x)$ for every $x \in E$. If E, F are vector lattices, the set of all even abstract Uryson operators from E to F we denote by $\mathcal{U}^{ev}(E, F)$.

If E, F are vector lattices with F Dedekind complete, the space $\mathcal{U}^{ev}(E, F)$ is not trivial (see [13], Prop. 3.4).

Lemma 3.2. *(*[19]*, Lemma 3.2) Let E, F be vector lattices with F Dedekind complete. Then $\mathcal{U}^{ev}(E, F)$ is a Dedekind complete sublattice of $\mathcal{U}(E, F)$.*

Let us see how the theory of linear regular operators is related with the theory of abstract Uryson operators. Let F be Dedekind complete. Define $\varphi : L_r(E, F) \to \mathcal{U}(E, F)$ by $\varphi(T)(f) := |T|f$. It is clear that $\varphi(T)$ is an even positive Uryson operator, for every $T \in L_+(E, F)$. Take a lattice homomorphism $T \in L_+(E, F)$. Then $\varphi(T) \in \mathcal{U}^{ev}_+(E, F)$ and

1) $\varphi(T)(x \vee y) = \varphi(T)(x) \vee \varphi(T)(y)$ for every $x, y \in E_+$.
2) $\varphi(T)(x \wedge y) = \varphi(T)(x) \wedge \varphi(T)(y)$ for every $x, y \in E_+$.

This example give us a motivation for the following definition.

Definition 3.3. Let E and F be a vector lattices and F is order complete. Operator $T \in \mathcal{U}_+^{ev}(E,F)$ is called *Uryson lattice homomorphism*, if the following conditions hold

1) $T(x \vee y) = T(x) \vee T(y)$ for every $x, y \in E_+$.
2) $T(x \wedge y) = T(x) \wedge T(y)$ for every $x, y \in E_+$.

The set of all Uryson lattice homomorphisms from E to F is denoted by $\mathcal{UH}(E,F)$. It is clear that every $T \in \mathcal{UH}(E,F)$ is an increasing operator in E_+.

Example 4. If $E = F = \mathbb{R}$, then $\mathcal{UH}(\mathbb{R})$ coincides with the set all even, nondecreasing functions $f : \mathbb{R} \to \mathbb{R}_+$, such that $f(0) = 0$ and $f([a,b])$ is a bounded set, for every $a, b \in \mathbb{R}$.

Example 5. Let E, F be vector lattices with F Dedekind complete, $\varphi : L_r(E,F) \to \mathcal{U}(E,F)$ be an embedding of the space of all linear regular operators to the space of abstract Uryson operators from E to F, and $T \in L_+(E,F)$ be a linear lattice homomorphism. Then $\varphi(T)$ is a Uryson lattice homomorphism.

Lemma 3.4. *Let (A, Σ, μ) be a σ-finite complete measure space and E be an order dense ideal in $L_0(\mu)$. Let $N : A \times \mathbb{R} \to \mathbb{R}_+$ be a function satisfying the following conditions:*

(1) *$N(t, 0) = 0$ for μ-almost all $t \in A$;*
(2) *$N(\cdot, f(\cdot))$ is μ-measurable for all $f \in E$;*
(3) *$N(t, \cdot)$ is increasing on $\mathbb{R}$ and $N(t, r) = N(t, -r)$ for μ-almost all $t \in A$ and all $r \in \mathbb{R}$.*

Then the operator $T : E \to L_0(\mu)$, defined by the formula $T(f)(t) := N(t, f(t))$ is an Uryson lattice homomorphism.

Proof. It is clear that $T \in \mathcal{U}_+^{ev}(E,F)$. Take $f, g \in E_+$ and let $B := \{t \in A : f(t) \geq g(t)\}$, $G := \{t \in A : g(t) > f(t)\}$. Remark that A and B are disjoint measurable sets and T is the disjointness preserving operator. Since N is the increasing function for for μ-almost all $t \in A$ we have that $T(f) \vee T(g) \leq T(f \vee g)$. We must to prove the converse inequality. Now, we may write

$$f \vee g = f\mathbf{1}_A + g\mathbf{1}_G;\, T(f \vee g) = T(f\mathbf{1}_A + g\mathbf{1}_G) = T(f\mathbf{1}_A) + T(g\mathbf{1}_G);$$
$$T(f) = T(f\mathbf{1}_A) + T(f\mathbf{1}_G); T(g) = T(g\mathbf{1}_A) + T(g\mathbf{1}_G);$$
$$T(f) \vee T(g) = \Big(T(f\mathbf{1}_A) + T(f\mathbf{1}_G)\Big) \bigvee \Big(T(g\mathbf{1}_A) + T(g\mathbf{1}_G)\Big) =$$
$$\Big(T(f\mathbf{1}_A) \vee T(f\mathbf{1}_G)\Big) \bigvee \Big(T(g\mathbf{1}_A) \vee T(g\mathbf{1}_G)\Big) \geq$$
$$T(f\mathbf{1}_A) \vee T(g\mathbf{1}_G) = T(f\mathbf{1}_A) + T(g\mathbf{1}_G) = T(f \vee g)$$

and therefore $T(f \vee g) = T(f) \vee T(g)$. The second equality from 3.3 can be proved analogically. □

Lemma 3.5. *Let E, F be vector lattices and F be Dedekind complete. Let $T \in \mathcal{U}(E,F)$ be Uryson lattice homomorphism. Then T is a disjointness preserving operator.*

Proof. Take $x, y \in E$, such that $|x| \perp |y|$. Then we have

$$|Tx| = Tx = T(x_+ - x_-) = T(x_+) + T(-x_-) = T(x_+) + T(x_-) = T(x_+ + x_-) = T|x|$$

and therefore

$$|Tx| \wedge |Tx| = T|x| \wedge T|y| = T(|x| \wedge |y|) = 0.$$

□

It is well known that in the case, when the vector lattice F is Dedekind complete, the space of all linear order bounded operators $L_b(E,F)$ coincides with the set of all linear regular operators $L_r(E,F)$ (i.e., operates majorized by a positive one). For abstract Uryson operators the similar result hold.

Lemma 3.6. *([13], Prop. 3.4). Let E, F be vector lattices and F be Dedekind complete, $T : E \to F$ be orthogonally additive operator. Then $T \in \mathcal{U}(E,F)$ if and only if there exists operator $\tilde{T} \in \mathcal{U}_+^{ev}(E,F)$, satisfying the following conditions:*

1. *$\tilde{T}$ is increasing in E_+;*
2. *$|Te| \leq \tilde{T}(e)$, for every $e \in E$.*

Operator $\tilde{T} : E \to F$ is defined by the formula $\tilde{T}e = \sup\{|T|g : |g| \leq |e|\}$. The following theorem is nonlinear version of the Meyer theorem [14].

Theorem 3.7. *Let E, F be vector lattices and F be Dedekind complete, $T : E \to F$ be an abstract Uryson operator. Then T is a disjointness preserving operator if and only if $\widetilde{T} \in \mathcal{UH}(E,F)$.*

Proof. Let $\widetilde{T} \in \mathcal{UH}(E,F)$ and take $x, y \in E$, such that $x \perp y$. By Lemma 3.5, $\widetilde{T}$ is a disjointness preserving operator and $|T| \leq \widetilde{T}$. Then we have

$$|Tx| \wedge |Ty| \leq |T|x \wedge |T|y \leq \widetilde{T}x \wedge \widetilde{T}y = 0.$$

Now, let T be a disjointness preserving operator. Then $|T|$ be the same. We must prove that $\widetilde{T} \in \mathcal{UH}(E,F)$. Take $x, y \in E_+$, it is clear that $\widetilde{T}(x \vee y) \geq \widetilde{T}(x) \vee \widetilde{T}(y)$. Let $\overline{E}$ be a Dedekind completion of the vector lattice E, $\mathcal{B}(\overline{E})$ be a Boolean algebra of all bands of $\overline{E}$ and let Q be a Stone compact of $\mathcal{B}(\overline{E})$. Consider the E as a vector sublattice of $C_\infty(Q)$, where $C_\infty(Q)$ is a Dedekind complete vector lattice of extended continuous real valued functions on Q. Let

$$A := \{t \in Q : x(t) \geq y(t)\}; \; B := \{t \in Q : x(t) < y(t)\}.$$

Then $x \vee y = f + h$, where $f = x\mathbf{1}_A$ and $h = y\mathbf{1}_B$, $f \perp h$. Now we may write $g = g_1 + g_2$, $g_1 \perp g_2$, $g_1 \leq x\mathbf{1}_A$, $g_2 \leq y\mathbf{1}_B$.

$$\widetilde{T}(x \vee y) = \sup\{|T|g : |g| \leq x \vee y = f + h\}.$$

By Riesz decomposition property there exist $g_1, g_2 \in E$, so that $g_1 \perp g_2$, $g_1 + g_2 = g$, $|g_1| \leq f$, $|g_2| \leq h$. Consequently

$$|T|g = |T|(g_1 + g_2) = |T|g_1 + |T|g_2,$$

where $|T|g_1 \perp |T|g_2$. Then we have

$$\begin{aligned} |T|g = |T|g_1 + |T|g_2 = |T|g_1 \vee |T|g_2 \\ \leq \widetilde{T}(x) \vee \widetilde{T}(y). \end{aligned}$$

Finally we deduce that $\widetilde{T}(x \vee y) \leq \widetilde{T}(x) \vee \widetilde{T}(y)$. Let us see that $\widetilde{T}(x \wedge y) = \widetilde{T}(x) \wedge \widetilde{T}(y)$. Remark that under our assumptions $\widetilde{T}$ is also a disjointness preserving operator. Now we may write

$$\begin{aligned} x \wedge y = x\mathbf{1}_B + y\mathbf{1}_A; x = x\mathbf{1}_B + x\mathbf{1}_A; \\ y = y\mathbf{1}_B + y\mathbf{1}_A; \widetilde{T}(x) = \widetilde{T}(x\mathbf{1}_B) + \widetilde{T}(x\mathbf{1}_A) = \\ \widetilde{T}(x\mathbf{1}_B) \vee \widetilde{T}(x\mathbf{1}_A); \widetilde{T}(y) = \widetilde{T}(y\mathbf{1}_B) + \widetilde{T}(y\mathbf{1}_A) = \\ \widetilde{T}(y\mathbf{1}_B) \vee \widetilde{T}(y\mathbf{1}_A). \end{aligned}$$

Then we have

$$\begin{aligned} \widetilde{T}(y) \wedge \widetilde{T}(x) = \\ \Big(\widetilde{T}(y\mathbf{1}_B) \vee \widetilde{T}(y\mathbf{1}_A)\Big) \wedge \Big(\widetilde{T}(x\mathbf{1}_B) \vee \widetilde{T}(x\mathbf{1}_A)\Big) = \\ \Big(\widetilde{T}(y\mathbf{1}_B) \wedge \widetilde{T}(x\mathbf{1}_B)\Big) \vee \Big(\widetilde{T}(y\mathbf{1}_B) \wedge \widetilde{T}(x\mathbf{1}_A)\Big) \vee \\ \Big(\widetilde{T}(y\mathbf{1}_A) \wedge \widetilde{T}(x\mathbf{1}_B)\Big) \vee \Big(\widetilde{T}(y\mathbf{1}_A) \wedge \widetilde{T}(x\mathbf{1}_A)\Big). \end{aligned}$$

Observe that $\widetilde{T}$ is an increasing operator in E_+. Hence

$$\begin{aligned} \widetilde{T}(y\mathbf{1}_B) \wedge \widetilde{T}(x\mathbf{1}_B) = \widetilde{T}(x\mathbf{1}_B); \widetilde{T}(y\mathbf{1}_B) \wedge \widetilde{T}(x\mathbf{1}_A) = 0; \\ \widetilde{T}(y\mathbf{1}_A) \wedge \widetilde{T}(x\mathbf{1}_A) = \widetilde{T}(y\mathbf{1}_A); \widetilde{T}(y\mathbf{1}_A) \wedge \widetilde{T}(x\mathbf{1}_B) = 0. \end{aligned}$$

And finally we have

$$\begin{aligned} \widetilde{T}(x) \wedge \widetilde{T}(y) = \widetilde{T}(x\mathbf{1}_B) \vee \widetilde{T}(y\mathbf{1}_A) = \\ \widetilde{T}(x\mathbf{1}_B) + \widetilde{T}(y\mathbf{1}_A) = \widetilde{T}(x\mathbf{1}_B + y\mathbf{1}_A) = \widetilde{T}(x \wedge y). \end{aligned}$$

□

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