Fuzzy Inventory Model under Immediate Return for Deficient Items

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Abstract

In this paper the inventory control problem with immediate return for deficient items in fuzzy senses. First, the crisp case of the proposed model is constructed in terms of annual profit. The perfective rate, demand rate and purchasing cost are fuzzified. The Function principle is used for fuzzy operation. Due to the fact that triangular fuzzy numbers are used extensively, we also provide an expression of the optimal order quantity for the case that all of the three parameters are triangular fuzzy numbers. A numerical example is provided to illustrate the proposed model and assess the effects of fuzziness of the parameters on the optimal solution.

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Keywords: Deficient items, Fuzzy inventory, economic order quantity (EOQ), Function Principle

1 Introduction

El Kassar et al. (2012) examined the case where raw materials of imperfect quality items are used in the production process. At the beginning of the inventory cycle, a supplier provides the raw material needed for the manufacture of the final product, and a proportion of this material is thought to be of imperfect quality. Those items are distinguished by a 100% screening process, yet all the raw material items are utilized in production. This will result in two types of finished products, one with perfect and another with imperfect quality. The two finished products are assumed to be demanded continuously. El Kassar et al. developed two mathematical models to demonstrate the two cases possible. They obtained an optimal production quantity and provided numerical examples to better demonstrate their models. El Kassar, Salemeh and Bitar (2012) expanded an economic production model that not only integrates the effects of imperfect quality items of raw materials, but also the time value of money. They developed a mathematical model and attained the optimal production quantity by maximizing the profit function. The solution determined is illustrated through numerical examples. Hence, several researchers pay their attention to the inventory models with deficient items. Recently, Salameh and Jaber [10] assumed that the deficient items can be sold in a single batch by the end of the 100% screening process. The result indicates that the economic order quantity tends to increase as the average percentage of imperfect quality items increases. Based on the model of Salameh and Jaber [10], many new models for deficient items were extended. Wee et al. [12] extended a to the case with shortage backordering. One of the popular ways is that the deficient items may be returned to suppliers directly. Hsu and Yu [6] have ever discussed the EOQ model with immediate return for imperfective items.

Throughout manufacturing/purchasing, two processes will be taking place: production and screening. After that, every final product is categorized as of perfect quality or imperfect quality. Let us consider $p$ as the percentage of imperfect items generated. Then $q = 1 - p$ will refer to the percentage of perfect items produced. So, we notice that the production of items with perfect and imperfect quality takes place at a stable rate. The purpose of this paper is to explore the inventory control problem with immediate return for deficient items in fuzzy senses, including fuzzy perfective rate, fuzzy demand rate and fuzzy purchasing cost. In the relevant literature, most of studies focused on fuzzifying the first two parameters (perfective rate and fuzzy demand rate). However, due to the imbalance of supply and demand, exchange-rate or price negotiations, the purchasing price of items may also be fuzzy.
In this paper, the crisp case of the proposed model is first constructed in terms of annual profit. Then, the crisp model is extended in fuzzy senses. For solving the proposed model, Function principle is utilized to rank the fuzzy annual profit and determine the optimal order quantity. Due to the fact that triangular fuzzy numbers are used extensively, we also provide an expression of the optimal order quantity for the case that all of the three parameters (perfective rate, demand rate and purchasing cost) are triangular fuzzy numbers. Finally, a numerical example is given to demonstrate the applications of the proposed model and assess the effects of fuzziness of the perfective rate, the demand rate, and the purchasing cost on the optimal solution.

2 Inventory Model With Immediate Return For Deficient Items.

In this paper, EOQ model with immediate return for imperfective items for crisp value is reviewed. Figure 1 describes the inventory with model.

The notation used in this paper is as follows:

D—demand rate (unit/ per year)
s—selling price per unit \((s > c)\)
c—purchasing cost per unit
b—holding cost rate per unit/ per unit time
h—holding cost per unit/ per unit time, \(h = bc\)
a—ordering cost per order
q—perfective rate for each order
p—deficient rate for each order \((p = 1 - q)\)
e—screening rate (unit/ per year)
w—screening cost per unit
Q—order size
T—cycle length
C—expected total cost per cycle
R—expected total revenue per cycle
P_0—expected total profit per cycle
P—expected total profit per year

2.1 Assumption made in this model

(i) A lot size of \(Q\) is replenished instantaneously at the beginning of each inventory cycle,

(ii) A 100% screening process of the lot is started at time \(t_1\) and finished at time \(t_e\). The screening process and demand proceed simultaneously, and the screening rate is greater than the demand rate \((i.e., e > D)\),
Any of deficient items found during the 100% screening process will be returned to the supplier immediately (or at least the retailer is not responsible for the safekeeping of the deficient any longer).

To avoid shortages, we have one more assumption that the number of good items is at least equal to the demand during the screening process. This leads to \( e \geq D/q \).

The other justifications and assumptions are available in the traditional EOQ model.

The inventory level behavior of the model can be illustrated as Figure 1, where \([t_1, t_2]\) denotes the time period of a cycle. Hence, the demand rate during \( t_1 \sim t_e \) can be regarded as \( D' = D + e(1 - q) \). After the screening process is finished, that is at \( t_e \), the demand rate will return to \( D \) and the remaining non-deficient items will be depleted at time \( t_2 \).

The optimal \( Q \) is determined by maximizing the following expected total profit per year:

\[
P(Q) = P_0(Q) * N;
\]

where \( P_0(Q) \) is the expected profit of a cycle and \( N \) is the number of orders per year. For ease of exposition, \( P_0(Q) \) can be expressed as follows:

\[
P_0(Q) = R(Q) - C(Q)...........................(a)
\]

where \( R(Q) \) and \( C(Q) \) denote the revenue and the total cost in each cycle, respectively. It is easily seen that

\[
R(Q) = SR + RR...........................(b)
\]

where \( SR \) and \( RR \) denote the revenues for sale and for return in each cycle, respectively.

It is clear that \( SR = sQq \) and \( RR = cQ(1 - q) \).

Further, \( C(Q) \) comprises the following four parts:

\[
C(Q) = PC + AC + WC + HC;
\]

where \( PC, AC, WC, \) and \( HC \) denote the procurement cost, ordering cost, screening cost, and holding cost in each cycle, respectively.

Obviously, \( PC = cQ, AC = a, \) and \( WC = wQ \).

To compute \( HC \), we need to calculate the total quantity of inventory in a cycle, which is equal to the sum of the areas of \( \triangle ABC \) and \( \square BCDE \) in Figure 1. By direct algebraic manipulations, we have:

\[
V = \frac{Q^2}{2} \left( \frac{q^2}{D} + \frac{(1 - q)}{e} \right).
\]
Hence,
\[ HC = h \times V = cb \times \frac{Q^2}{2} \left( \frac{q^2}{D} + \frac{(1 - q)}{e} \right). \]

The total annual profit \( P(Q) \), thus can be derived as:
\[
P(Q) = P_0(Q) \times N = [(SR + RR) - (PC + AC + WC + HC)] \times N
= \left( [sQq + cQ(1 - q)] - [cQ + a + wQ + cb \times \frac{Q^2}{2} \left( \frac{q^2}{D} + \frac{(1 - q)}{e} \right)] \right) \frac{D}{Qq}
\]
\[
P(Q) = sD - cD + \frac{bQ}{2e} \cdot cD - \frac{bQ}{2e} \cdot cq - \left( \frac{a}{Q} + w \right) \cdot \frac{D}{q} - \left( \frac{bQ}{2e} \right) \cdot \frac{cD}{q} \tag{1}
\]

Taking the first derivative of \( P(Q) \) with respect to \( Q \) and setting the result to zero, we have:
\[
P'(Q) = \frac{bcD}{2e} - \frac{bcq}{2} + \frac{aD}{Q^2q} - \frac{bcD}{2eq} = 0.
\]

By solving the above equation, the optimal solution, termed as \( Q^*c \), can be found as:
\[
Q^*c = \sqrt{\frac{2aDe}{cb[q^2 + (1 - q)D]}}.
\]

It is easy to find the second derivative of \( P(Q) \) with respect to \( Q \) is:
\[
P''(Q) = -\frac{2aD}{Q^3} < 0 \text{ for } Q > 0
\]

Thus, the \( Q^*_c \) is the global maximum solution of \( P(Q) \). We can obtain the corresponding total profit \( P^*_c(= P(Q^*_c)) \). It is easily seen that if the deficient rate is zero (i.e., \( q = 1 \)), then \( Q^* = \sqrt{\frac{2aD}{cb}} \), the traditional EOQ formula.

### 3 Fuzzy Inventory Models under generalized mean preference

In this paper, we assume that perfective rate, demand rate and purchasing cost are fuzzy numbers with parameters: \( \tilde{q}, \tilde{D}, \tilde{c} \).

Suppose \( \tilde{q} = (q_1, q_2, q_3, q_4), \tilde{D} = (d_1, d_2, d_3, d_4), \tilde{c} = (c_1, c_2, c_3, c_4) \).

Therefore the corresponding total profit (denoted by \( \tilde{P}(Q) \)) is fuzzy as follows:
\[
\tilde{P}(Q) = s\tilde{D} - (\tilde{c} \otimes \tilde{D}) + \frac{bQ}{2e} \cdot (\tilde{c} \otimes \tilde{D}) - \frac{bQ}{2} \cdot (\tilde{c} \otimes q) - \left( \frac{a}{Q} + w \right) \cdot (\tilde{D} \otimes \tilde{q}) - \frac{bQ}{2e} \cdot (\tilde{c} \otimes (\tilde{D} \otimes \tilde{q})).
\]

Note that the expression for \( \tilde{P}(Q) \) consists of five fuzzy numbers: \( \tilde{D}, (\tilde{c} \otimes \tilde{D}), (\tilde{c} \otimes q), (\tilde{D} \otimes \tilde{q}), \) and \( (\tilde{c} \otimes (\tilde{D} \otimes \tilde{q})), \)
In this study, trapezoidal is used for membership function. The trapezoidal membership function for variable:

$$
\mu_\tilde{A}(x) = \begin{cases} 
\frac{w(x-a_1)}{a_2-a_1} & \text{for } a_1 \leq x \leq a_2 \\
w & \text{for } a_2 \leq x \leq a_3 \\
\frac{w(x-a_3)}{a_4-a_3} & \text{for } a_3 \leq x \leq a_4 \\
0 & \text{otherwise}
\end{cases}
$$

Where $w < 0 \leq 1$

Function principle is used for fuzzy operation. The trapezoidal membership function $\tilde{A}$ is denoted as $(a_1, a_2, a_3, a_4; w)$ in which $a_1 \leq a_2 \leq a_3 \leq a_4$. Under the function principle, the fuzzy arithmetic operations for the two trapezoidal membership function $\tilde{A}$ and $\tilde{B}$ are described as follows:

(i) The addition of $\tilde{A}$ and $\tilde{B}$ is

$$\tilde{A} \oplus \tilde{B} = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4),$$

where $a_1, a_2, a_3, a_4, b_1, b_2, b_3$, and $b_4$ are all real numbers.

(ii) The multiplication of $\tilde{A}$ and $\tilde{B}$ is

$$\tilde{A} \otimes \tilde{B} = (t_1, t_2, t_3, t_4),$$

where $T = \{a_1 b_1, a_1 b_4, a_4 b_1, a_4 b_4\}$, $T_1 = \{a_2 b_2, a_2 b_3, a_3 b_2, a_3 b_3\}$, $t_1 = \min T$, $t_2 = \min T_1$, $t_3 = \max T_1$, $t_4 = \max T$.

If $a_1, a_2, a_3, a_4, b_1, b_2, b_3$, and $b_4$ are all nonzero positive real numbers, then $\tilde{A} \otimes \tilde{B} = (a_1 b_1, a_2 b_2, a_3 b_3, a_4 b_4)$. 
Fuzzy Inventory Model

(iii) \( \tilde{b} = (-b_4, -b_3, -b_2, -b_1) \), then the subtraction of \( \tilde{a} \) and \( \tilde{b} \) is

\[ \tilde{a} \ominus \tilde{b} = (a_1 - b_4, a_2 - b_3, a_3 - b_2, a_4 - b_1) \]

where \( a_1, a_2, a_3, a_4, b_1, b_2, b_3, \) and \( b_4 \) are all real numbers.

(iv) \( \frac{1}{\tilde{b}} = \frac{1}{b_4}, \frac{1}{b_3}, \frac{1}{b_2}, \frac{1}{b_1} \), where \( b_1, b_2, b_3, \) and \( b_4 \) are all positive real numbers. If \( a_1, a_2, a_3, a_4, b_1, b_2, b_3, \) and \( b_4 \) are all nonzero positive real numbers, then the division of \( \tilde{a} \) and \( \tilde{b} \) is

\[ \tilde{A} \oslash \tilde{B} = (a_1/b_4, a_2/b_3, a_3/b_2, a_4/b_1) \]

Where \( w \) is the minimum value of \((w_1, w_2)\). In order to defuzzify a fuzzy variable \( \tilde{A} \), median rule proposed by Park [14] is used. The median of \( a_m \) for \( \tilde{A} \) is derived from:

\[ a_m = \frac{(a_1 + a_2 + a_3 + a_4)}{4} \]

In this study, only the case in which \( a_1 < a_2 < a_3 < a_4 \) is considered. Based on the median rule, the total relevance cost can be formulated as:

\[ \tilde{P}(Q) = \frac{1}{4} \sum_{i=1}^{4} \left( s d_i - c_i d_i + \frac{b Q}{2 e} c_i d_i - \frac{b Q}{2} c_i q_i - \left( \frac{a}{Q} + w \right) \frac{d_i}{q_i} - \frac{b Q c_i d_i}{2 e q_i} \right) \]

Take into account the partial derivative of \( \tilde{P}(Q) \),

\[ \frac{\partial (\tilde{P})}{\partial Q} = \frac{1}{4} \sum_{i=1}^{4} \left( \frac{b}{2 e} c_i d_i - \frac{b}{2} c_i q_i + \frac{a}{Q^2} d_i - \frac{b}{2 e} c_i d_i \right) \]

\[ = \frac{1}{4} \sum_{i=1}^{4} \left( \frac{b}{2 e} \left( c_i d_i - c_i q_i e - c_i d_i \right) + \frac{a}{Q^2} d_i \right) \]

Optimal solution is obtained using the following conditions:

\[ \frac{\partial (\tilde{P})}{\partial Q} = 0 \]

The results is:

\[ \frac{1}{4} \sum_{i=1}^{4} \frac{a d_i}{Q^2 q_i} = \frac{1}{4} \sum_{i=1}^{4} \left( \frac{b}{2 e q_i} \left( c_i d_i + c_i q_i^2 e - c_i d_i q_i \right) \right) \]
\[
\frac{1}{Q^2} \sum_{i=1}^{4} \frac{d_i}{q_i} = \frac{b}{2ae} \sum_{i=1}^{4} \left( \frac{c_i}{q_i} \left( q_i^2 e + (1 - q_i) d_i \right) \right)
\]

\[
Q^2 = \frac{2ae \sum_{i=1}^{4} \frac{d_i}{q_i}}{b \sum_{i=1}^{4} \left( \frac{c_i}{q_i} \left( q_i^2 e + (1 - q_i) d_i \right) \right)}
\]

\[
Q^* = \sqrt{\frac{\sum_{i=1}^{4} 2aecd_i}{\sum_{i=1}^{4} (bc_i \left( q_i^2 e + (1 - q_i) d_i \right))}}
\]

When the fuzzy membership functions consist of a single value, such that \(\tilde{C} = (c_1 = c_2 = c_3 = c_4 = C)\), the values of \(\tilde{Q}\), \(\tilde{P}(Q)\) are:

\[
\tilde{Q}^* = \sqrt{\frac{2aD}{cb[eq^2 + (1 - q)D]}}.
\]

\[
\tilde{P}(Q) = sD - cD + \frac{cDbQ}{2e} - \frac{cqbQ}{2} - \left( \frac{a}{Q} + w \right) \frac{D}{q} - \frac{bQDc}{2eq}.
\]

Equation (3) is the same as Equation (1); it means that when all the fuzzy membership consists of a single value, we can consider it as crisp value, that is \(\tilde{P}(Q)\) is the same as \(P(Q)\).

4 Numerical Example:

A company needs to estimate the EOQ. However, the company have a policy with immediate return for deficient items. The company estimated that the annual demand \(\tilde{D}\) is likely to be 50000 units per year. The purchasing cost \(\tilde{c}\) is approximately 25 per order. The perfective rate \(\tilde{q}\) is 0.98 for each order. The ordering cost \(\tilde{a}\) is projected to be around 100 per order. holding cost \(\tilde{b}\) is estimated to be around 0.2 per unit. The selling price \(\tilde{s}\) is 175200. The screening cost \(\tilde{w}\) is 0.5.

To solve this problem, regular trapezoidal membership functions are used to represent the uncertainty of data. The highest possible value is approximately in the mean of the range value. The highest membership value is 1.

The membership functions for each possible cost are given as:
\[
\tilde{D} = (50000, 50000, 50000, 50000)
\]
\[
\tilde{q} = (0.98, 0.98, 0.98, 0.98)
\]
\[
\tilde{c} = (25, 25, 25, 25)
\]

By Equations (2), we have the optimal solution of the crisp model as:

\[
Q^* = 1438.806
\]
5 Conclusion

The purpose of the paper is to explore the inventory control problem with immediate return for deficient items. Practically, this case is very popular for procurement operations of retailers. Further, for enhancing the practical applications of the model, this paper also extends the model in fuzzy senses, including the fuzziness of deficient rate, demand rate and purchasing cost. In the previous studies, due to computational difficulty and complexity, there are few articles exploring an inventory model with three fuzzy parameters. The results improve the practical applications of fuzzy theory significantly. Hence these models are executable and useful in the real world.

References


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