Optimal Order Policy for Deteriorating Items with Time-Dependent Demand in Response to Temporary Price Discount Linked to Order Quantity

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Abstract

This paper presents the possible effects of a temporary price discount offered by a supplier on a retailer’s replenishment policy for deteriorating items with linear time dependent demand rate. The optimal ordered quantity of a special order policy for a selected case is obtained by maximizing the total cost saving between special and regular orders for the duration of the depletion time. An algorithm is given to find the optical solution. Numerical examples are used to illustrate the theoretical results. Truncated Taylor’s series is used for finding closed form optimal solution.

Keywords: Inventory, deteriorating items, temporary price discount, linear time dependent demand
1. Introduction

The main objective of inventory management deals with maximization of the total inventory profit for which it is required to determine the optimal stock and optimal time of replenishment of inventory to meet the future demand. The one of the important concerns of the management is to decide when and how much to order so that the total cost associated with the inventory system should be minimum. Deterioration plays an important role in the study of inventory system. Deterioration is defined as decay, spoilage loss of utility of the product. Maximum items deteriorate over time. The rate of deterioration is very small in some items like hardware, glassware, toys and steel. Some items such as fish, medicine, vegetables, blood, alcohol and food grains have finite shelf-life and deteriorate rapidly over time. As a result, while determining optimal replenishment order, the loss due to deterioration cannot be ignored. Ghare and Schrader (1963) first established an EOQ model for exponentially-decaying items, for which there is constant demand. Later, Covert and Philip (1973) extended Ghare and Schrader’s (1963) model and obtained an EOQ model for a variable deterioration rate, by assuming a two-parameter Weibull distribution. Shah (1977) extended Philip’s (1974) model and considered the circumstances in which shortage is allowed. A complete note on inventory literature for deteriorating inventory models was given by Goyal and Giri (2001) and Raafat (1991).

In addition, when the supplier offers a temporary price discount to stimulate demand, increase market share and cash flow, it is important for the retailer to determine whether or not it is advantageous to place a special forward buying order. Several researchers have studied temporary price discounts and proposed various inventory models to gain deeper insight into the relationship between price discounts and order policy. Naddor (1966), Barker (1976), and Tersine (1982) were the earliest researchers to propose inventory models in response to a temporary price discount where the special sale coincides with the replenishment time. Following this, Barker and Vilcassim (1983), Goyal (1990), Tersine (1994), and Silver et al. (1979) studied situations where the special sale occurs during the retailer’s sales period. Ardalan (1988, 1995) developed optimal order policies with possible combinations of replenishment time and sales periods. More research studying the response to a temporary sale price include Aull-Hyde (1996), Abad (1997), Bhaba and Mahmood (2006), Lev et al. (1981), Tersine and Gengler (1982), Wee and Yu (1997), Chang and Dye (2000), Bhavin (2005) and so on. In the above inventory literature, all researchers assumed that the price discount rate is independent of the special order quantity. However, in practical business situations, the supplier usually proposes a quantity discount to encourage larger orders. As a result, the retailer may trade off purchase price savings against higher total carrying cost.

As to the research considering quantity discounts, Lal and Staelin (1984) presented a model of buyer reaction to seller pricing schemes which assumed special
forms of discount price structure with multiple buyers and constant demands. Other recent studies related to the quantity discount recently included Shiue (1990), Burwell et al. (1997), Wee (1999), Papachristos and Skouri (2003), etc.

Many researchers have given considerable attention towards the situation where the demand rate is dependent on the level of the on-hand inventory. The assumption of constant demand rate is not always applicable to many inventory items such as fashionable clothes, electronic equipments, tasty foods etc. as they are fluctuated in the demand rate. Demand of a product may vary with time or price or ever with the instantaneous level of inventory displayed in a retail shop. With the progress of time, researchers developed inventory models with deteriorating items and time-dependent demand rates. In this area, the work done by various authors like Ritchie (1984), Deb and Chaudhari (1986), Goel and Aggarwal (1981). Inventory models for deteriorating items with linearly trended demand and no shortage were considered by Dave and Patel (1989), Bahari-Kashani (1989), Chung and Ting (1993), Ouyang et al. (2009) etc.

Hung (2011) developed an inventory model with generalized type demand, deterioration and backorder rates. Khanra et al. (2011) presented an EOQ model for a deteriorating item with time–dependent quadratic demand under permissible delay in payment. In this paper, an effort has been made to analyze an EOQ model for deteriorating item considering quadratic time dependent demand rate and permissible delay in payment. Sana (2010) formulated optimal selling price and lot size with time varying deterioration and partial backlogging. In this work, an EOQ model over an infinite time horizon for perishable item where demand is price dependent and partial backorder permitted is discussed. Teng et al. (2012) established an EOQ model with trade credit financing for non-decreasing demand and optimal solution and relevant managerial phenomena was also calculated.

The aim of this paper is to investigate the possible effects of a temporary price discount offered by a supplier on a retailer’s replenishment policy for deteriorating items with linear time dependent demand rate. The purpose of this study is to develop a decision process to assist retailers in deciding whether to adopt a regular or special order policy. When the special order policy is selected at the retailer’s replenishment time, the retailer’s optimal special order quantity is determined by maximizing the total cost saving between special and regular orders during a special order period. The theoretical analysis is conducted and the results categorizing the optimal solutions. Furthermore, an algorithm is established and numerical examples are used to illustrate the solution procedure.

The rest of the paper is organized as follows. In the next section notation and assumption are given followed by mathematical formulation in section 3. Theoretical results are given in section 4. In section 5, algorithm is mentioned. Numerical examples have been given in section 6. Finally, conclusion and future research directives are given in the last section 7.
# Notations and Assumptions

The mathematical model is based on the following notations and assumptions.

## 2.1 Notations

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>Ordering cost per regular or special order</td>
</tr>
<tr>
<td>$C$</td>
<td>Unit purchasing price</td>
</tr>
<tr>
<td>$D(t) = a + bt$</td>
<td>Time-dependent demand rate, where $t \geq 0, a &gt; 0, 0 &lt; b &lt; 1$</td>
</tr>
<tr>
<td>$h$</td>
<td>Holding cost per unit per unit time</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Constant deterioration rate, $0 \leq \theta &lt; 1$</td>
</tr>
<tr>
<td>$Q$</td>
<td>Order quantity under regular order policy</td>
</tr>
<tr>
<td>$Q^*$</td>
<td>Optimal order quantity under regular order policy</td>
</tr>
<tr>
<td>$T$</td>
<td>Length of replenishment cycle time under regular order policy</td>
</tr>
<tr>
<td>$T^*$</td>
<td>Optimal length of replenishment cycle time under regular order policy</td>
</tr>
<tr>
<td>$Q_s$</td>
<td>Special order quantity at discount price, decision variable.</td>
</tr>
<tr>
<td>$T_s$</td>
<td>Length of depletion time for the special order quantity $Q_s$</td>
</tr>
<tr>
<td>$I(t)$</td>
<td>Inventory level at time $t$ for $0 \leq t \leq T$</td>
</tr>
<tr>
<td>$I_d(t)$</td>
<td>Inventory level at time $t$, for $0 \leq t \leq T$,</td>
</tr>
<tr>
<td>$K(T)$</td>
<td>The total cost per unit time</td>
</tr>
</tbody>
</table>

## 2.2 Assumptions

(a) The demand rate is linearly time dependent
(b) The replenishment rate is infinite.
(c) The lead time is zero or negligible.
(d) Shortages are not allowed.
(e) There is no replacement or repair for deteriorated units during the period under consideration.
(f) The supplier offers the retailer a temporary price discount as the order quantity is larger than the regular order quantity $Q^*$. Moreover, the discount rate depends on the quantity ordered and the discount schedule, as follows

<table>
<thead>
<tr>
<th>Classification $(i)$</th>
<th>Special order quantity $(Q_s)$</th>
<th>Discount rate $(\lambda_i)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$x_1 \leq Q_s &lt; x_2$</td>
<td>$\lambda_1$</td>
</tr>
<tr>
<td>2</td>
<td>$x_2 \leq Q_s &lt; x_3$</td>
<td>$\lambda_2$</td>
</tr>
<tr>
<td>:</td>
<td>:</td>
<td>:</td>
</tr>
<tr>
<td>$m$</td>
<td>$x_m \leq Q_s &lt; x_{m+1}$</td>
<td>$\lambda_n$</td>
</tr>
</tbody>
</table>
where $x_i$ is the $i^{th}$ discount rate breaking point, $i = 1, 2, ..., m+1$ and $Q^* < x_1 < x_2 < ... < x_{m+1} = \infty$; $\lambda_i$ is the price discount fraction offered by the supplier, when the retailer’s order quantity $Q_s$ belongs to the interval $[x_i, x_{i+1})$, and $0 < \lambda_1 < \lambda_2 < ... \lambda_m$.

### 3. Mathematical Formulation

As the retailer follows the regular order policy without a temporary price discount, they adopt a simple EOQ model with deteriorating items. The depletion of inventory occurs due to combined effect of the demand and deterioration in the time interval $[0, T]$. The rate of change of inventory level is governed by the following differential equation

$$\frac{dI(t)}{dt} = -\theta I(t) - (a + bt), 0 \leq t \leq T \quad (1)$$

Given the boundary condition $I(T) = 0$, the solution of (1) may be represented by

$$I(t) = \left[ \frac{1}{\theta} a(e^{\theta(T-t)} - 1) + \frac{b}{\theta^2} (e^{\theta(T-t)}(\theta T - 1) - (\theta t - 1)) \right] \cdot 0 \leq t \leq T \quad (2)$$

Thus, the order quantity is given by

$$Q = I(0) = \left[ \frac{1}{\theta} a(e^{\theta T} - 1) + \frac{b}{\theta^2} e^{\theta T}(\theta T - 1) + 1 \right] \quad (3)$$

When the supplier does not offer the temporary price discount, the retailers follows the regular order policy with a unit purchasing cost $c$ and hence the total cost per order cycle is the sum of the ordering cost, purchasing cost and holding cost, that is

$$A + cQ + hc \int_0^T I(t) \, dt = A + \frac{c}{\theta} \left[ a(e^{\theta T} - 1) + \frac{b}{\theta} \left( e^{\theta T} - 1 \right) \right]$$

$$+ ch \left[ -\frac{a}{\theta^2} (1 - e^{\theta T} + \theta T) + \frac{b}{\theta} \left( 1 - e^{\theta T} \right) \left( \frac{1}{\theta} - 1 \right) - T \left( \frac{T \theta}{2} - 1 \right) \right]$$

(4)

Since this equation contains exponential, it is difficult to find closed form solution of $T$. Truncated Taylors series is used for finding closed form solution in exponential terms i.e. of $e^{\theta T}$, neglecting higher order terms, that is $e^{\theta T} \approx 1 + \theta T + \frac{\theta^2 T^2}{2}$ etc.
The total cost per order cycle (denoted by $K$) is given by

$$K = A + \frac{c}{\theta} \left\{ a \left( \theta T + \frac{\theta^2 T^2}{2} \right) + \frac{\theta b T^2}{2} \right\} + \frac{achT^2}{2}$$

(5)

Therefore, the total cost per unit time (denoted by $K(T)$) without temporary price discount is

$$K(T) = \frac{1}{T} \left[ A + \frac{c}{\theta} \left\{ a \left( \theta T + \frac{\theta^2 T^2}{2} \right) + \frac{\theta b T^2}{2} \right\} + \frac{achT^2}{2} \right]$$

(6)

Differentiating $K(T)$ with respect to ‘$T$’ two times we obtain

$$\frac{dK(T)}{dT} = -\frac{A}{T^2} + \frac{c}{2} (a\theta + b) + \frac{ach}{2}$$

(7)

$$\frac{d^2K(T)}{dT^2} = \frac{2A}{T^3} > 0$$

(8)

Which shows that $K(T)$ is a convex function of $T$, hence, there exists a unique value of $T$ (say $T^*$) that minimizes $K(T)$. $T^*$ can be found by solving the equation $\frac{dK(T)}{dT} = 0$

We obtain $T = T^* = \sqrt[2]{\frac{2A}{c\{a(\theta + h) + b\}}}$

(9)

When the optimal length of replenishment cycle time, $T^*$, is obtained, the optimal order quantity, $Q^*$, is obtained as follows:

$$Q^* = aT^* + \frac{T^*}{2} (a\theta + b)$$

(10)

The purpose of this study is to determine the optimal special order quantity by maximizing the total cost saving between special and regular orders during the length of depletion time for the special order quantity. When the supplier offers a temporary price discount, the retailer may order a quantity greater than $Q^*$ to take advantage of the discount price. Alternatively, the retailer may ignore this discount and adopt a regular order policy. Here, we formulate the corresponding total cost saving function when the special order time occurs at the retailer’s replenishment time.

If the retailer decides to adopt a special order policy and orders $Q_s$ units, the inventory level at time $t$ is
Optimal order policy for deteriorating items

\[ I_s(t) = \frac{1}{\theta} \left[ a \left( e^{\theta(T_s-t)} - 1 \right) + \frac{b}{\theta} \left( e^{\theta(T_s-t)}(\theta t - 1) - (\theta t - 1) \right) \right] , 0 \leq t \leq T_s \]  

(11)

and the special order quantity is

\[ Q_s = \frac{1}{\theta} \left[ a \left( e^{\theta T_s} - 1 \right) + \frac{b}{\theta} \left( e^{\theta T_s}(\theta t - 1) + 1 \right) \right] \]  

(12)

Taking Taylor's expansion

\[ Q_s = \frac{1}{\theta} \left[ a\theta T_s \left( 1 + \frac{\theta T_s}{2} \right) + b \left( \frac{\theta T_s^2}{2} \right) \right] \]  

(13)

As the price discount rate being dependent on the special order quantity, for the given price discount rate \( \lambda_s \), the total cost of the special order during the time interval \([0, T_s]\) (denoted by \( K_{sl}(T_s) \)) consists of the ordering cost \( A \), purchasing cost

\[ (1-\lambda_s)cQs = (1-\lambda_s)c \left[ a \left( e^{\theta T_s} - 1 \right) + \frac{b}{\theta} \left( e^{\theta T_s}(\theta T_s - 1) + 1 \right) \right] \]  

and holding cost

\[ (1-\lambda_s)hc \int_0^T I(t) dt \], i.e.

\[ K_{sl}(T_s) = A + (1-\lambda_s)cT_s \left[ a \left( 1 + \frac{\theta T_s}{2} \right) + b \frac{\theta T_s^2}{2} \right] + (1-\lambda_s) \frac{achT_s^2}{2} \]  

(14)

On the other hand, if the retailer adopts regular order policy instead of placing a large special order policy, then the total cost during the time interval \([0, T_s]\) can be obtained by using the average cost approach which was asserted by Tersine [12], and used by Goyal. The total cost of a regular order during the time interval \([0, T_s]\) (denoted by \( K_{RN}(T_s) \)) is

\[ K_{RN}(T_s) = \frac{T_s}{T^*} \left[ A + c \left( \frac{\theta T^*}{2} + \frac{\theta^2 T^*^2}{2} \right) + \frac{b\theta T^*^2}{2} + \frac{achT^*^2}{2} \right] \]  

(15)

Comparing equation (10) with (11), for the fixed price discount rate \( \lambda_s \), the total cost saving can be formulated as follows (denoted by \( G_{sl}(T_s) \))

\[ G_{sl}(T_s) = K_{RN}(T_s) - K_{sl}(T_s) \]

\[ = \frac{T_s}{T^*} \left[ A + c \left( \frac{\theta T^*}{2} + \frac{\theta^2 T^*^2}{2} \right) + \frac{b\theta T^*^2}{2} + \frac{achT^*^2}{2} \right] - \left[ A - (1-\lambda_s)c \left( 1 + \frac{\theta T_s}{2} \right) + \frac{b T_s}{2} \right] \]

(16)
4. Theoretical Results

In this section, the optimal value of $T_s$ that maximizes the total cost saving is determined. For the fixed price discount rate $\lambda$, taking the first and second order derivatives of $G_i(T_s)$ in (12) with respect to $T_s$, gives

$$
\frac{dG_i(T_s)}{dT_s} = \frac{1}{T^*} \left[ A + \left( a \left( \theta T^* + \frac{\theta^2 T^{*2}}{2} \right) + \frac{b T^{*2}}{2} \right) \frac{achT^{*2}}{2} \right] -(1-\lambda_i)c(a(1+\theta T_s)+bT_s)-(1-\lambda_i)achT_s
$$

and

$$
\frac{d^2G_i(T_s)}{dT_s^2} = -(1-\lambda_i)c(a\theta+b+ah) < 0 \tag{17}
$$

It can be easily shown that $G_i(T_s)$ is a concave function of $T_s$; hence, a unique value of $T_s$ (say $T_{s1}^*$) exist that maximizes $G_i(T_s)$. $T_{s1}^*$ can be found by solving the equation $\frac{dG_i(T_s)}{dT_s} = 0$. Given by

$$
T_{s1}^* = \frac{y - (1-\lambda_i)ac}{c(1-\lambda_i)(a\theta+b+ah)} \tag{18}
$$

where $y = \frac{1}{T^*} \left[ A + \left( a \left( \theta T^* + \frac{\theta^2 T^{*2}}{2} \right) + \frac{b T^{*2}}{2} \right) \frac{achT^{*2}}{2} \right] > 0$

To ensure $Q^*<Q_{s1i}$ (i.e. $T^*<T_{s1i}$, where $T^*$ can be found by solving equation (9)), we substitute (18) in to the inequality $T^*<T_{s1i}$, and it result in

if $\delta_i > 0$, then $T^*<T_{s1i}$, \tag{19}

where $\delta_i = y - (1-\lambda_i)c(a + T^*(a\theta+b+ah)]$

Again, substituting (18) into (15) the corresponding maximum total cost saving can be obtained as

$$
G_i(T_{s1i}) = (1-\lambda_i)\frac{cT^*}{2}(a\theta+b+ah) - A \tag{20}
$$

It is worth placing a special order only if $G_i(T_{s1i})>0$. Otherwise the retailer will adopt the regular order policy (i.e. the order quantity is $Q^*$). Let

$$
\delta_{2i} \equiv G_i(T_{s1i}) = (1-\lambda_i)\frac{cT^*}{2}(a\theta+b+ah) - A \tag{21}
$$

Then from the above arguments, we can obtain the optimal value of $T_{s1i}$ (denoted by $T_{s1i}^*$) as
To obtain the optimal solution, we can develop an algorithm as follows:

5. Algorithm

Step 1: Determine \( T^* \) from equation (9).

Step 2: For each \( \lambda_i, i = 1, 2, \ldots, m \) calculate \( T_{sl_i} \) in (18),

\[
\delta_i = y - (1 - \lambda_i)c \left\{ a + T^*(a\theta + b + ah) \right\}
\]

and

\[
\{ a + T^*(a\theta + b + ah) \} \delta_2 = (1 - \lambda_i) \frac{cT^2_1}{2} (a\theta + b + ah) - A.
\]

If \( \delta_i > 0 \) and \( \delta_2 > 0 \), then substitute \( T_{sl_i} \) in (12) to evaluate the corresponding lot size, \( Q_{sl_i} \) and check \( Q_{sl_i} \) under \( \delta_i \).

(i) If \( x_i \leq Q_{sl_i} < x_{i+1}, Q_{sl_i} \) is a feasible solution. Set \( Q_{sl_i}^* = Q_{sl_i} \) and substitute \( T_{sl_i}^* = T_{sl_i} \) into (20) to evaluate \( G_{li}(T_{sl_i}^*) \).

(ii) If \( Q_{sl_i} \geq x_{i+1} \), we can get a larger price discount rate which is greater than \( \lambda_i \) and thus \( Q_{sl_i}^* \) not a feasible solution.

(iii) If \( Q_{sl_i} < x_i \) set \( Q_{sl_i}^* = x_i \) and substitute \( Q_{sl_i}^* \) into (9) to find \( T_{sl_i}^* \). Then, substitute \( T_{sl_i}^* \) into (20) to evaluate \( G_{li}(T_{sl_i}^*) \). If \( G_{li}(T_{sl_i}^*) > 0 \), go to step 3; otherwise, set \( T_{sl_i} = T^*, Q_{sl_i}^* = Q^* \) and \( G_{li}(T_{sl_i}^*) = 0 \).

Otherwise, set \( T_{sl_i}^* = T^*, Q_{sl_i}^* = Q^* \) and \( G_{li}(T_{sl_i}^*) = 0 \).

Step 3: Find \( \max_{i = 1, 2, \ldots, m} \max_{G_{li}(T_{sl_i}^*)} \). If \( G_{li}(T_{sl_i}^*) = \max_{i = 1, 2, \ldots, m} \) \( G_{li}(T_{sl_i}^*) \), \( T_{sl_i}^* \) is the optimal solution and thus the optimal order quantity \( Q_{sl_i}^* \) can also be determined.

Step 4: Stop.

6. Numerical Examples

**Example 1**: Consider \( a = 1000 \) units/year, \( b = 0.2, A = 150 \) per order, \( c = 10 / \) unit, \( h = 0.3 / \) units/year, \( \theta = 0.01 / \) year.

From the above data, the optimal solutions for the regular order quantity are \( T^* = 0.31098 \) year and \( Q^* = 311.473 \) units.

The price discount rate schedule offered by the supplier is tabulated in Table 1. And using the algorithm, the solution produce and computational results are shown in Table 2.
Table 1. Price discount rate schedule

<table>
<thead>
<tr>
<th>Classification</th>
<th>Special order quantity</th>
<th>Discount rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>i</td>
<td>Q_s</td>
<td>$\lambda_i$</td>
</tr>
<tr>
<td>1</td>
<td>500 ≤ Q_s &lt; 1000</td>
<td>0.1</td>
</tr>
<tr>
<td>2</td>
<td>1000 ≤ Q_s</td>
<td>0.15</td>
</tr>
</tbody>
</table>

Table 2. Optimal Solutions of Example 1

<table>
<thead>
<tr>
<th>$\lambda_i$</th>
<th>$Q_{s1i}^*$</th>
<th>$T_{s1i}^*$</th>
<th>$Q_{SIi}^*$</th>
<th>$G_{SIi}^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>706.212</td>
<td>0.703</td>
<td>706.212</td>
<td>541.29</td>
</tr>
<tr>
<td>→ 0.15</td>
<td>939.216</td>
<td>0.93476</td>
<td>1000.00</td>
<td>1001.94</td>
</tr>
</tbody>
</table>

Note: "→" denotes the maximum total cost saving.

From Table 2, we can see that the optimal order policy is as follows:

$T_{s1i}^* = 0.93476$, $Q_{s1i}^* = 1000$ and $G_{SIi}^* = 1001.94$.

From the above tables 1 and 2, all the observations can be summed up as follows:

From Table 1, increase of special order quantity $Q_s$ results increase in discount rate $\lambda_i$.

From Table 2, it can be easily seen that increase in discount rate results increase in $Q_{s1i}^*$, $T_{s1i}^*$, $Q_{SIi}^*$ and $G_{SIi}^*$.

Example 2: Consider $a = 600$ units/year, $b = 0.2$, $A = 50$ per order, $c = 30 / \text{unit}$, $h = 1/ \text{units/year}$, $\theta = 0.20/\text{year}$.

From the above data, the optimal solutions for the regular order quantity are $T^* = 0.06804$ year and $Q^* = 41.0776$ units.

The price discount rate schedule offered by the supplier is tabulated in Table 3. And using the algorithm, the solution produce and computational results are shown in Table 4.

Table 3. Price discount rate schedule

<table>
<thead>
<tr>
<th>Classification</th>
<th>Special order quantity</th>
<th>Discount rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>i</td>
<td>$Q_s$</td>
<td>$\lambda_i$</td>
</tr>
<tr>
<td>1</td>
<td>100 ≤ Q_s &lt; 600</td>
<td>0.10</td>
</tr>
<tr>
<td>2</td>
<td>600 ≤ Q_s &lt; 1100</td>
<td>0.15</td>
</tr>
<tr>
<td>3</td>
<td>Q_s ≥ 1100</td>
<td>0.25</td>
</tr>
</tbody>
</table>
Table 4. Optimal Solutions of Example 2

<table>
<thead>
<tr>
<th>$\lambda_i$</th>
<th>$Q_{si_i}$</th>
<th>$T_{si_i}^*$</th>
<th>$Q_{si_i}^*$</th>
<th>$G_{si_i}^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.10</td>
<td>102.2</td>
<td>0.167</td>
<td>102.2</td>
<td>1579.13</td>
</tr>
<tr>
<td>0.15</td>
<td>138.93</td>
<td>0.226</td>
<td>600</td>
<td>2029.11</td>
</tr>
<tr>
<td>$\rightarrow$ 0.25</td>
<td>228.66</td>
<td>0.367</td>
<td>600</td>
<td>2928.38</td>
</tr>
</tbody>
</table>

Note: "$\rightarrow$" denotes the maximum total cost saving.

From Table 4, we can see that the optimal order policy is as follows:

$T_{si_i}^* = 0.367$, $Q_{si_i}^* = 600$ and $G_{si_i}^* = 2928.38$.

From the above tables 3 and 4, all the observations can be summed up as follows:

From Table 3, increase of special order quantity $Q_i$ results increase in discount rate $\lambda_i$.

From Table 4, it can be easily seen that increase in discount rate results increase in $Q_{si_i}^*, T_{si_i}^*, Q_{si_i}^*$ and $G_{si_i}^*$.

7. Conclusion and Future Research

This model incorporates realistic features that are likely to be related and connected with special kinds of inventory. First deterioration with respect to time is a realistic feature for at most goods. Secondly, demand rate is usually influenced by the time. This model is very useful in the retail business. In this study we discussed the possible effects of a temporary price discount offered by a supplier on a retailer’s replenishment policy for deteriorating items with linear time-dependent demand rate. By analyzing the total cost saving under special and regular order policies, results were developed to characterize the optimal solution. Finally, the result shows that as we increase the price discount, the total cost saving increases. Differential calculus is used for finding optimal solution. For finding closed form, optimal solution truncated Taylor’s series is used. From managerial point of view, this is helpful in seasonal products. The model presented in this study provides a basis for several possible extensions. For future research, this model can be extended for shortages and variable ordering cost. Another extension possible would be considering the holding cost as time dependent or stock dependent etc.

Remark

When the optimal length of depletion time for the special order quantity is $T_{si_i}^* = T^*$, it means that it is not worthwhile to place a special order; instead the retailer should adopt its regular order policy with the original unit purchasing cost $c$. 


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References


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[38] V. P. Goel and S. P. Aggarwal, Order level inventory system with power demand pattern for deteriorating items, proceedings *All India Seminar on Operational Research and Decision Making, University of Delhi* (1981), Delhi-110007.


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