Some Properties of Intuitionistic Fuzzy H-Ideals

Extension in Γ-Hemiring

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Abstract

In this paper the concept of the extension of intuitionistic fuzzy h-ideals in one sided Γ-hemiring is introduced and some of the basic properties are investigate. Finally we have explained the extension of intuitionistic fuzzy h-bi- ideal, intuitionistic fuzzy h-interior ideal and h-quasi-ideals and discussed the some of its properties.

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1 Introduction

Ideals of hemi rings play a central role in the structure theory and are very useful
for many purposes. However, they do not in general coincide with the usual ring ideals. Many results in rings apparently have no analogues in hemirings using only ideals. In applications, hemirings are useful in automata and formal languages. It is well known that the set of regular languages forms the so-called “star, semirings”. Following the introduction of fuzzy sets by L. A. Zadeh [15] the fuzzy set theory has been used for many applications in the domain of mathematics and elsewhere. The idea of “Intuitionistic Fuzzy Set” was first published by K. T. Atanassov [1] as a generalization of the notion of fuzzy set. Jun and Lee [8] applied the concept of fuzzy sets to the theory of $\Gamma$-rings. The notion of $\Gamma$-semiring was introduced by Rao [11] as a generalization of $\Gamma$-ring as well as of semiring. $\Gamma$-semirings also includes ternary semirings and provide algebraic home to non-positives cones of totally ordered rings. Henriksen [5], Iizuka [6] and La Torre [9] investigated $h$-ideals and $k$-ideals in hemirings to amend the gap between ring ideals and semiring ideals. These concepts are extended to $\Gamma$-semiring by Rao [11], Dutta and Sardar [2]. Jun et al [7] and studied these ideals in hemirings in terms of fuzzy subsets. A characterization of an $h$-hemiregular in terms of a fuzzy $h$-ideal is is discussed by Zhan et al. [17].

The notion of fuzzy $h$-ideals in $\Gamma$-hemirings are introduced and studied some of its properties by Sujit Kumar et al. [13]. In this paper the concept of the extension of intuitionistic fuzzy $h$-ideals in one sided $\Gamma$-hemiring is introduced and some of the basic properties are investigate. Finally we have explained the extension of intuitionistic fuzzy $h$-bi-ideal, intuitionistic fuzzy $h$-interior ideal and $h$-quasi-ideals and discussed the some of its properties.

2 Preliminaries

**Definition 2.1** A hemiring (respectively semiring) is a nonempty set $S$ on which operations addition and multiplication have been defined such that $(S, +)$ is a commutative monoid with identity $0$, $(S,.)$ is a semigroup (respectively monoid with identity $1_S$ ) Multiplication distributes over addition from either side, $1_S \neq 0$ and $0_s = 0 = S_0$ for all $s \in S$.

**Definition 2.2** Let $S$ and $\Gamma$ be two additive commutative semigroups with zero. Then $S$ is called a $\Gamma$-hemiring if there exists a mapping $S \times \Gamma \times S \rightarrow S((a,\alpha,b) \rightarrow a\alpha b)$ satisfying the following conditions:

(i) $(a + b)\alpha c = a\alpha c + b\alpha c$ ,

(ii) $a\alpha (b + c) = a\alpha b + a\alpha c$ ,

(iii) $a(\alpha + \beta)b = a\alpha b + a\beta b$ ,

(iv) $a\alpha (b\beta c) = (a\alpha b)\beta c$ ,
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(v) $0_s a = 0_s = a 0_s a$,
(vi) $a_\Gamma b = 0_s = b 0_\Gamma a$,

for all $a, b, c \in S$ and for all $\alpha, \beta \in \Gamma$. For simplification we write 0 instead of $0_s$ and $0_\Gamma$.

**Example 2.3** Let $S$ be the set of all $m \times n$ matrices over $\mathbb{Z}_0^-$ (the set of all non-positive integers) and $\Gamma$ be the set of all $n \times m$ matrices over $\mathbb{Z}_0^-$ then $S$ forms a $\Gamma$-hemiring with usual addition and multiplication of matrices.

**Definition 2.4** A left ideal $A$ of a $\Gamma$-hemiring $S$ is called a left h-ideal if for any $x, z \in S$ and $a, b \in A$, $x + a + z = b + z \Rightarrow x \in A$. A right h-ideal is defined analogously.

**Definition 2.5** Let $S$ be a $\Gamma$-hemiring. A proper h-ideal $I$ of $S$ is said to be prime if for any two h-ideals $H$ and $K$ of $S$, $H \Gamma K \subseteq I$ implies that either $H \subseteq I$ or $K \subseteq I$.

**Definition 2.6** [11] If $I$ is an h-ideal of a $\Gamma$-hemiring $S$ then the following condition are equivalent:

(i) $I$ is a prime h-ideal of $S$.
(ii) If $a \Gamma S b \subseteq I$ then either $a \in I$ or $b \in I$ where $a, b \in S$.

**Definition 2.7** [12] Let $\mu$ and $\theta$ be two fuzzy sets of a $\Gamma$-hemiring $S$ define a generalized h-product of $\mu$ and $\theta$ by $\mu \sigma \theta(x) = \sup \{ \min \{ \mu(a_i), \mu(c_i), \theta(b_i), \theta(d_i) \} \}$

$$x + \sum_{i=1}^{n} a_i \sigma b_i + z = \sum_{i=1}^{n} c_i \delta d_i + z$$

$= 0$, if $x$ cannot be expressed as above.

Where $x, z, a_i, b_i, c_i, d_i \in S$ and $\sigma, \delta \in \Gamma$, for $i=1,2,3,\ldots,n$.

**Definition 2.8** Let $\mu$ be a non empty fuzzy subset of a $\Gamma$-hemiring $S$ (i.e. $\mu(x) \neq 0$ for some $x \in S$). Then $\mu$ is called a fuzzy left ideal (fuzzy right ideal) of $S$ if:

(i) $\mu(x + y) \geq \min[\mu(x), \mu(y)]$ and
(ii) $\mu(xy \gamma) \geq \mu(y \gamma)$ (respectively $\mu(xy \gamma) \geq \mu(x \gamma)$) for all $x, y \in S$, $\gamma \in \Gamma$.

A fuzzy ideal of a $\Gamma$-hemiring $S$ is a non empty fuzzy subset of $S$ which is a fuzzy left ideal as well as fuzzy right ideal of $S$. 
Note that if $\mu$ is a fuzzy left or right ideal of a $\Gamma$-hemiring $S$, then $\mu(0) \geq \mu(x)$ for all $x \in S$.

**Definition 2.9** A fuzzy h-ideal $A$ of a $\Gamma$-hemiring $S$ is said to be prime (semiprime) if $A$ is not a constant function and for any two fuzzy h-ideals $\sigma$ and $\theta$ of $S$, $\sigma \Gamma_h \theta \subseteq \mu$ implies that either $\sigma \subseteq \mu$ or $\theta \subseteq \mu$ (resp. $\sigma \Gamma_h \theta \subseteq \mu$ implies $\theta \subseteq \mu$).

**Definition 2.10** Let $A$ be an intuitionistic fuzzy h-ideal of $S$ then $A$ is a prime intuitionistic fuzzy h-ideal of $S$ if and only if the following conditions hold

(i) $\mu_A(0) = 1 \& \gamma_A(1) = 0$

(ii) $\text{Im} \mu_A(t) = \{1, t\}, t \in [0, 1]$ \& $\text{Re} \gamma_A = \{0, t\}, t \in \{0, 1\}$

(iii) $A_0 = \{x \in S : \mu_A(x) = \mu_A(0) \& \gamma_A(x) = \gamma_A(1)\}$

### 3 Intuitionistic Fuzzy h-Ideal Extension in $\Gamma$-hemirings

**Definition 3.1** Let $A = \langle \mu_A, \gamma_A \rangle$ be an intuitionistic fuzzy subset of $S$ and $x \in S$, then the intuitionistic fuzzy subset $\langle x, \mu_A, \gamma_A \rangle$ of $S$ defined by

$$\langle x, \mu_A \rangle(y) = \inf_{\gamma \in \Gamma} \mu_A(xy)$$

and

$$\langle x, \gamma_A \rangle(y) = \sup_{\gamma \in \Gamma} \gamma_A(xy)$$

for all $y \in S$, is called the extension of $A$ by $x$.

**Theorem 3.2**

Let $A = \langle \mu_A, \gamma_A \rangle$ is an intuitionistic fuzzy right h-ideal of $S$ and $x \in S$. Then the extension $\langle x, \mu_A, \gamma_A \rangle$ is an intuitionistic fuzzy right h-ideal of $S$.

**Proof:** Let $p, q, a, b, z \in S$ and $\beta \in \Gamma$ then

$$\langle x, \mu_A \rangle(p + q) = \inf_{\gamma \in \Gamma} \mu_A(xy(p + q))$$

$$\geq \inf_{\gamma \in \Gamma} \min \{ \mu_A(xy(p)), \mu_A(xy(q)) \}$$

$$= \min \left\{ \inf_{\gamma \in \Gamma} \mu_A(xy(p)), \inf_{\gamma \in \Gamma} \mu_A(xy(q)) \right\}$$

$$= \min \langle x, \mu_A \rangle(p), \langle x, \mu_A \rangle(q)$$

and

$$\langle x, \gamma_A \rangle(p + q) = \sup_{\gamma \in \Gamma} \gamma_A(xy(p + q))$$

$$= \sup_{\gamma \in \Gamma} \gamma_A(xy(p))$$

$$= \sup_{\gamma \in \Gamma} \gamma_A(xy(q))$$

$$= \sup_{\gamma \in \Gamma} \langle x, \gamma_A \rangle(p), \langle x, \gamma_A \rangle(q)$$
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\[ \leq \sup_{y \in \Gamma} \max \{ \gamma_A(xy p), \gamma_A(xy q) \} \]
\[ = \max \{ \sup_{y \in \Gamma} \gamma_A(xy p), \sup_{y \in \Gamma} \gamma_A(xy q) \} \]
\[ = \max \{ \min \{ \inf \mu_A \gamma_A(xy), \inf \mu_A \gamma_A(xy) \} \} \]

Also, \( \langle x, \mu_A \rangle (p \beta q) = \inf_{y \in \Gamma} \mu_A (xy p \beta q) \geq \inf_{y \in \Gamma} \mu_A (xy p) = \langle x, \mu_A \rangle (p) \)

and \( \langle x, \gamma_A \rangle (p \beta q) = \sup_{y \in \Gamma} \gamma_A (xy p \beta q) \leq \sup_{y \in \Gamma} \gamma_A (xy p) = \langle x, \gamma_A \rangle (p) \).

Now, let \( p + a + z = b + z \) so \( xy p + xy a + xy z = xy b + xy z \). Then
\[ \langle x, \mu_A \rangle (p) = \inf_{y \in \Gamma} \mu_A (xy p) \geq \inf \min \{ \inf \mu_A xy a, \inf \mu_A xy b \} \]
\[ = \min \{ \inf \mu_A xy a, \inf \mu_A xy b \} \]
and
\[ \langle x, \gamma_A \rangle (p) = \sup_{y \in \Gamma} \gamma_A (xy p) \leq \sup \min \{ \sup \mu_A xy a, \sup \mu_A xy b \} \]
\[ = \max \{ \sup \mu_A xy a, \sup \mu_A xy b \} \]

Hence \( \langle x, \mu_A, \gamma_A \rangle \) is an intuitionistic fuzzy right ideal of S.

**Note:** If \( A = \langle \mu_A, \gamma_A \rangle \) is an intuitionistic fuzzy h-ideal of a commutative \( \Gamma \)-hemiring S and \( x \in S \), then the extension \( \langle x, \mu_A, \gamma_A \rangle \) is an intuitionistic fuzzy h-ideal of S.

**Proposition 3.3:** If \( A_i = \langle \mu_{A_i}, \gamma_{A_i} \rangle, i = 1, 2, \ldots \) be an arbitrary collection of intuitionistic fuzzy h-ideal of S, then \( \langle x \cap A_i \rangle \) is also an intuitionistic fuzzy h-ideal of S.

**Definition 3.4 [9]** Let f be a function from a set X to a set Y. A is an intuitionistic fuzzy subset of X and C be an intuitionistic fuzzy subset of Y. Then image of A under f, denoted by \( f(A) \) is an intuitionistic fuzzy subset of Y defined by

\[ f \left( \mu_A (y) \right) = \begin{cases} 
\sup_{x \in f^{-1}(y)} \mu_A & \text{if } f^{-1}(y) \neq \emptyset \\
0 & \text{otherwise}
\end{cases} \]
\[ f(\gamma_A(y)) = \begin{cases} \inf_{x \in f^{-1}(y)} \gamma_A(x) & \text{if } f^{-1}(y) \neq \emptyset \\ 0 & \text{otherwise} \end{cases} \]

The pre-image of \( C \) under \( f \), symbolized by \( f^{-1}(c) \) is an intuitionistic fuzzy subset of \( X \) defined by \( f^{-1}(c(x)) = c(f(x)) \), \( \forall x \in X \).

**Proposition 3.5:** Let \( A = (\mu_A, \gamma_A) \) be an intuitionistic fuzzy h-ideal of \( S \) and \( x \in S \) Then the following conditions hold.

(i) \( A \subseteq \langle x, \mu_A, \gamma_A \rangle \)

(ii) \( \langle (xy)^{-1}x, \mu_A, \gamma_A \rangle \subseteq \langle (xy)^n x, \mu_A, \gamma_A \rangle \) where, \( \gamma \in \Gamma \)

(iii) If \( A > 0 \) then \( \text{supp} \langle x, \mu_A, \gamma_A \rangle = S \)

Where, \( \text{supp} A \) is defined by

\[
\text{supp} A = \{ s \in S; \mu_A(s) > 0 \text{ and } \gamma_A(s) < 0 \}
\]

**Proof:**

(i) Let \( y \in S \). Now \( \langle x, \mu_A \rangle (y) = \inf_{\gamma \in \Gamma} \mu_A(xy) \geq \mu_A(y) \)

\[
\langle x, \gamma_A \rangle (y) = \sup_{\gamma \in \Gamma} \gamma_A(xy) \leq \gamma(y)
\]

Thus \( A \subseteq \langle x, \mu_A, \gamma_A \rangle \)

(ii) Let \( n \) be a positive integer and \( y \in S \), then

\[
\langle (xy)^n x, \mu_A \rangle (y) = \inf_{\gamma \in \Gamma} \mu_A \left( (xy)^n x, \gamma y \right) 
\]

\[
\geq \inf_{\gamma \in \Gamma} \mu_A \left( (xy)(xy)^{n-1} x, \gamma y \right) 
\]

\[
\geq \inf_{\gamma \in \Gamma} \mu_A \left( (xy)^{n-1} x, \gamma y \right) 
\]

\[
= \left( (xy)^{n-1} x, \mu_A \right) (y) 
\]

\[
\langle (xy)^n x, \gamma_A \rangle (y) = \sup_{\gamma \in \Gamma} \gamma_A \left( (xy)^n x, \gamma y \right) \leq \sup_{\gamma \in \Gamma} \gamma_A \left( (xy)(xy)^{n-1} x, \gamma y \right) 
\]

\[
\leq \sup_{\gamma \in \Gamma} \gamma_A \left( (xy)^{n-1} x, \gamma y \right) 
\]

\[
= \left( (xy)^{n-1} x, \gamma_A \right) (y). 
\]

So

\[
\langle (xy)^n x, \mu_A, \gamma_A \rangle \subseteq \left( (xy)^n x, \mu_A, \gamma_A \right)
\]
(iii) Let $A > 0$ and $y \in S$ then,

$$\langle x, \mu_A \rangle(y) = \inf_{y \in \Upsilon} \mu_A(xy) \geq \mu_A(x)$$

and

$$\langle x, \gamma_A \rangle(y) = \sup_{y \in \Upsilon} \gamma_A(xy) \leq \gamma_A(x)$$

Thus $y \in \text{supp} \langle x, \mu_A, \gamma_A \rangle$ and consequently, $S \subseteq \text{supp} \langle x, \mu_A, \gamma_A \rangle$.

Hence $S = \text{supp} \langle x, \mu_A, \gamma_A \rangle$.

**Proposition 3.6:** If $A = \langle \mu_A, \gamma_A \rangle$ is an intuitionistic fuzzy h-bi-ideal of $S$, then its extension by $x \in S$ is $\langle x, \mu_A, \gamma_A \rangle$ is also an intuitionistic fuzzy h-bi-ideal of $S$, provided $S$ is commutative.

**Proof:** Let $A$ be an intuitionistic fuzzy h-bi-ideal of $S$ its extension by $x \in S$ is $\langle x, \mu_A, \gamma_A \rangle$, since $A$ be an intuitionistic fuzzy h-bi-ideal it is sufficient to prove

$$\langle x, \mu_A \rangle(paqbr) \geq \min \{\langle x, \mu_A \rangle(p) \langle x, \mu_A \rangle(r)\}$$

and

$$\langle x, \gamma_A \rangle(paqbr) \leq \max \{\langle x, \gamma_A \rangle(p) \langle x, \gamma_A \rangle(r)\}$$

for all $p, q, r \in S$ and $\alpha, \beta \in \Gamma$.

Suppose $p, q, r \in S$ and $\alpha, \beta \in S$. Now

$$\langle x, \mu_A \rangle(paqbr) = \inf_{y \in \Upsilon} \mu_A(xy pqaqbr) \geq \inf_{y \in \Upsilon} \mu_A(xy p) = \langle x, \mu_A \rangle(p)$$

$$\langle x, \gamma_A \rangle(paqbr) = \sup_{y \in \Upsilon} \gamma_A(xy pqaqbr) \leq \sup_{y \in \Upsilon} \gamma_A(xy p) = \langle x, \gamma_A \rangle(p)$$

(since $S$ is commutative).

Therefore, $\langle x, \mu_A \rangle(paqbr) \geq \min \{\langle x, \mu_A \rangle(r)\}$

$$\langle x, \gamma_A \rangle(paqbr) \leq \max \{\langle x, \gamma_A \rangle(r)\}$$

So $\langle x, \mu_A, \gamma_A \rangle$ is an intuitionistic fuzzy h-bi-ideal of $S$.

**Proposition 3.7:** If $A = \langle \mu_A, \gamma_A \rangle$ is an intuitionistic fuzzy h-interior-ideal of $S$ then its extension by $x \in S$. Hence $\langle x, \mu_A, \gamma_A \rangle$ is also an intuitionistic fuzzy
h-interior ideal of \( S \), provided \( S \) is commutative and \( S \) is also \( \Gamma \)-hemiring.

**Proof:** Let \( A \) be an intuitionistic fuzzy h-interior ideal of \( S \) and its extension by \( x \in S \) is \( \langle x, \mu_A, \gamma_A \rangle \). Then it is sufficient to prove \( \langle x, \mu_A \rangle (p\alpha q\beta r) \geq \langle x, \mu_A \rangle (q) \) and \( \langle x, \gamma_A \rangle (p\alpha q\beta r) \leq \langle x, \gamma_A \rangle (q) \) for all \( p, q, r \in S \) and \( \alpha, \beta \in \Gamma \)

\[
\langle x, \mu_A \rangle (p\alpha q\beta r) = \inf_{\gamma \in \Gamma} \mu_A (x\gamma p\alpha q\beta r) \\
\langle x, \gamma_A \rangle (p\alpha q\beta r) = \sup_{\gamma \in \Gamma} \gamma_A (x\gamma p\alpha q\beta r)
\]

Hence \( \langle x, \mu_A, \gamma_A \rangle \) is also an intuitionistic fuzzy h-interior ideal of \( S \).

**Proposition 3.8.** If \( A = \langle \mu_A, \gamma_A \rangle \) is an intuitionistic fuzzy h-quasi ideal of \( S \), then its extension by \( x \in S \langle x, \mu_A, \gamma_A \rangle \) is also an intuitionistic fuzzy h-quasi ideal of \( S \).

**Proof.** Let \( A \) be an intuitionistic fuzzy h-quasi ideal of \( S \) then its extension by \( x \in S \langle x, \mu_A, \gamma_A \rangle \). Let \( p, a, b, z \in S \). Then

\[
\langle x, (\mu_A 0_h \gamma) \cap (x 0 \mu) \rangle (p) = \inf_{\gamma \in \Gamma} ((\mu_A 0_h \gamma) \cap (x 0 \mu)) (x \gamma p) \\
= \inf_{\gamma \in \Gamma} \min \{ (\mu_A 0_h \gamma), (x 0 \mu) \} (x \gamma p) \\
\leq \inf_{\gamma \in \Gamma} \mu_A (x \gamma p) = \langle x, \mu_A \rangle (p) \quad \text{and}
\]

\[
\langle x, (\mu_A 0_h \gamma) \cup (x 0 \gamma) \rangle (p) = \sup_{\gamma \in \Gamma} ((\gamma_A 0_h \gamma) \cup (x 0 \gamma)) (x \gamma p) \\
= \sup_{\gamma \in \Gamma} \max \{ (\gamma_A 0_h \gamma), (x 0 \gamma) \} (x \gamma p) \\
\geq \sup_{\gamma \in \Gamma} \gamma_A (x \gamma p) = \langle x, \gamma_A \rangle (p)
\]

Also by Theorem 3.2 we have,

\[
\langle x, \mu_A \rangle (p+ q) \geq \min \{ \mu_A (x), \mu_A (y) \} \\
\langle x, \gamma_A \rangle (p+ q) \leq \max \{ \gamma_A (x), \gamma_A (y) \} \quad \text{and}
\]
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$$p + a + z = b + z \Rightarrow \{ x, \mu_A \}(p) \geq \min \{ \{ x, \mu_A \}(a), \{ x, \mu_A \}(b) \},$$

$$\{ x, \gamma_A \}(p) \leq \max \{ \{ x, \gamma_A \}(a), \{ x, \gamma_A \}(b) \}$$

Hence $\{ x, \mu_A, \gamma_A \}$ is an intuitionistic fuzzy $h$-quasi ideal of $S$.

**Remark 3.9** We know that if $A$ is an intuitionistic fuzzy $h$-quasi-ideal of a $\Gamma$-hemiring $S$, it is also an intuitionistic fuzzy $h$-bi-ideal, in previous proposition 3.10, we show that its extension by any element $x \in S$, $\{ x, \mu_A, \gamma_A \}$ is an intuitionistic fuzzy $h$-quasi-ideal also. Now it is a routine verification to show that $\{ x, \mu_A, \gamma_A \}$ is also an intuitionistic fuzzy $h$-bi-ideal of $S$ provided $S$ is commutative.

**Proposition 3.10**: If $A$ and $B$ be any two intuitionistic fuzzy $h$-ideal of $S$, then for any $x \in S$, $\{ x, A \times B \}$ is also an intuitionistic fuzzy $h$-ideal of $S$, where

$$(A \times B)(a, b) = \min \{ \mu_A(a), \mu_A(b) \}, \quad (A \times B)(a, b) = \max \{ \gamma_A(a), \gamma_A(b) \} \quad \text{Where} \quad a, b \in S.$$

**Proof**: Since $A$ and $B$ be any two intuitionistic fuzzy $h$-ideal of $S$, by Theorem 5.3 of [4], we have $(A \times B)$ is also an intuitionistic fuzzy $h$-ideal and hence by using Theorem 3.2 we deduce that $\{ x, A \times B \}$ is also an intuitionistic fuzzy $h$-ideal of $S$.

**Theorem 3.11**: Let $A, B$ be any two intuitionistic fuzzy $h$-ideal of $S$ and $x, y \in S$. Then $\{ x, A \} \times \{ y, B \}$ is also an intuitionistic fuzzy $h$-ideal of $S$.

**Proof**: Suppose $A$ and $B$ be any two intuitionistic fuzzy $h$-ideal by Theorem 3.2 we have $\{ x, \mu_A \}$ and $\{ y, \gamma_A \}$ are intuitionistic fuzzy $h$-ideal of $S$. Hence by using Theorem 5.3 of [4] we deduce that $\{ x, A \} \times \{ y, B \}$ is also an intuitionistic fuzzy $h$-ideal of $S$.

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