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Regular Generalized Star b-Continuous Functions in a Bigeneralized Topological Space

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Abstract

This paper aims to introduce the concepts of $\mu^{(m,n)}$-rg^*b continuous, absolute $\mu^{(m,n)}$-rg^*b continuous, regular strongly $\mu^{(m,n)}$-rg^*b continuous and almost $\mu^{(m,n)}$-rg^*b continuous functions in bigeneralized topological spaces. Basic properties, characterizations and relationships among these functions are also considered.

Keywords: $\mu^{(m,n)}$-rg^*b continuous, absolute $\mu^{(m,n)}$-rg^*b continuous, regular strongly $\mu^{(m,n)}$-rg^*b continuous, almost $\mu^{(m,n)}$-rg^*b continuous

1 Introduction

The notion of functions is one of the most fundamental concepts in modern mathematics. Over the years, different forms of functions have been presented

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and various interesting problems arise out of it. Thus, the study on this concept has depeened and evolved constantly up to the present times.

Another important development of general topology in the recent years is the theory of bigeneralized topological space (briefly BGTS). This was initiated by C. Boonpok [2] as he studied closed and open sets in this topological space. Later, other researchers used this notion in the study of generalized sets and other related concepts.

Recently, the notion of $\mu_{(m,n)}$-regular generalized star b-closed set in BGTS was introduced and studied in [6]. In this paper, we introduce the concepts of $\mu^{(m,n)}$-rg^*b continuous, absolute $\mu^{(m,n)}$-rg^*b continuous, regular strongly $\mu^{(m,n)}$-rg^*b continuous and almost $\mu^{(m,n)}$-rg^*b continuous functions in BGTS and study their relationships, basic properties and characterizations.

2 Preliminaries

We recall some basic definitions and notations. Let X be a nonempty set and denote $P(X)$ the power set of X. A subset μ of $P(X)$ is said to be a *generalized topology* (briefly GT) on X if $\varnothing \in \mu$ and an arbitrary union of elements of μ belongs to μ [4]. If μ is a GT on X, then (X, μ) is called a *generalized topological space* (briefly *GTS*). The elements of μ are called *μ-open* sets and the complements of μ-open sets are called *μ-closed* sets. If $A \subseteq X$, then the *interior* of A denoted by $i_\mu(A)$, is the union of all μ-open sets contained in A and the *closure* of A, denoted by $c_\mu(A)$, is the intersection of all μ-closed sets containing A.

Let (X, μ) be a GTS and $A \subseteq X$. Then A is said to be *μ-regular open* if $A = i_\mu(c_\mu(A))$; *μ-regular generalized closed* (briefly μ-rg closed) if $c_\mu(A) \subseteq U$ whenever $A \subseteq U$, where U is μ-regular open [5]; and *μ-b open* if $A \subseteq c_\mu(i_\mu(A)) \cup i_\mu(c_\mu(A))$ [4]. The complement of μ-b open set is called *μ-b closed set*. The *μ-b closure* of A, denoted by $bc_\mu(A)$, is the intersection of all μ-b closed sets containing A [1].

Definition 2.1 Let (X, μ) be a GTS and $A \subseteq X$. Then

i. A is said to be *μ-regular generalized star b-closed* (briefly μ-rg^*b closed) if $bc_\mu(A) \subseteq U$ whenever $A \subseteq U$ and U is a μ-rg open set in X. A is *μ-rg^*b open* if its complement is μ-rg^*b closed.

ii. The *μ-rg^*b closure* of A, denoted by $rg^*bc_\mu(A)$, is the intersection of all μ-rg^*b closed sets containing A. Thus, $A \subseteq rg^*bc_\mu(A)$.

iii. The *μ-rg^*b interior* of A, denoted by $rg^*bi_\mu(A)$, is the union of all μ-rg^*b open sets contained in A. Hence, $rg^*bi_\mu(A) \subseteq A$.

Example 2.2 Let $X = \{a, b, c\}$ with the generalized topologies μ=$\{\varnothing$, $\{a\}$, $\{a, b\}\}$. Then $\{b\}$ is both a μ-rg^*b closed and μ-rg^*b open set of X.

Using Definition 2.1, the next remark follows.

Remark 2.3 *Let (X, μ) be a GTS and A, B and F be subsets of X.*

i. *If A is μ-open, then A is μ-rg^*b open;*

ii. *$x \in rg^*bi_\mu(A)$ if and only if there exists a μ-rg^*b open set U with $x \in U \subseteq A$;*

iii. *If A is μ-rg^*b open, then $A = rg^*bi_\mu(A)$;*

iv. *$y \in rg^*bc_\mu(A)$ if and only if for every μ-rg^*b open set U with $y \in U$, $U \cap A \neq \varnothing$;*

v. *$A \subseteq B \Rightarrow rg^*bi_\mu(A) \subseteq rg^*bi_\mu(B)$;*

vi. *$rg^*bi_\mu(A)$=$X \setminus rg^*bc_\mu(X \setminus A)$.*

Remark 2.4 *The converses of Remark* 2.3 (i) *and* (iii) *are not true since in Example* 2.2, *$\{b\}$ is a μ-rg^*b open set of X but not μ-open and for the set $X = \{a, b, c, d\}$ with the GT $\mu = \{\varnothing, \{a, b\}, \{b, c\}, \{a, b, c\}\}$, $rg^*bi_\mu(\{a, d\})$=$\{a, d\}$ but $\{a, d\}$ is not μ-rg^*b open set in X.*

Definition 2.5 [2] Let X be a nonempty set and let μ_1, μ_2 be generalized topologies on X. The triple (X, μ_1, μ_2) is said to be a *bigeneralized topological space* (briefly BGTS).

Throughout this paper, m and n are elements of the set $\{1, 2\}$ where $m \neq n$.

Definition 2.6 [6] A subset A of a BGTS (X, μ_1, μ_2) is said to be $\mu_{(m,n)}$-*regular generalized star b-closed* (briefly $\mu_{(m,n)}$-rg^*b closed) if $bc_{\mu_n}(A) \subseteq U$ whenever $A \subseteq U$ and U is a μ_m-regular generalized open set in X . The complement of $\mu_{(m,n)}$-rg^*b closed set is said to be $\mu_{(m,n)}$-*regular generalized star b-open* (briefly $\mu_{(m,n)}$-rg^*b open) set.

Definition 2.7 [3] Let $f : (X, \mu_X^1, \mu_X^2) \to (Y, \mu_Y^1, \mu_Y^2)$ be a function. Then

i. f is said to be $\mu^{(m,n)}$-*continuous* at a point $x \in X$ if for each μ_Y^m-open set V containing $f(x)$, there exists a μ_X^n-open set U containing x such that $f(U) \subseteq V$.

ii. f is said to be $\mu^{(m,n)}$-*continuous* if f is $\mu^{(m,n)}$-continuous at every point $x \in X$.

3 $\mu^{(m,n)}$-Regular Generalized Star b-Continuous Functions

In this section, we introduce $\mu^{(m,n)}$-rg^*b continuous functions in a BGTS and study some of their properties. All throughout this section (X, μ_X^1, μ_X^2) and (Y, μ_Y^1, μ_Y^2) are BGTS.

Definition 3.1 Let $f : (X, \mu_X^1, \mu_X^2) \to (Y, \mu_Y^1, \mu_Y^2)$ be a function. Then

i. f is said to be $\mu^{(m,n)}$-rg^*b *continuous at a point* $x \in X$ if for each μ_Y^m-open set V containing $f(x)$, there exists a μ_X^n-rg^*b open set U containing x such that $f(U) \subseteq V$.

ii. f is said to be $\mu^{(m,n)}$-rg^*b *continuous* if f is $\mu^{(m,n)}$-rg^*b continuous at every point $x \in X$.

iii. f is said to be *pairwise* μ-rg^*b *continuous* if f is $\mu^{(1,2)}$-rg^*b continuous and $\mu^{(2,1)}$-rg^*b continuous.

Example 3.2 Let $X = \{a, b, c\}$ and $Y = \{u, v\}$. Consider the generalized topologies μ_X^1=$\{\varnothing,\ \{a\},\ \{a, b\}\}$, μ_X^2=$\{\varnothing,\ \{c\},\ \{b, c\}\}$, μ_Y^1=$\{\varnothing,\ \{u\}\}$, and μ_Y^2=$\{\varnothing,\ \{v\},\ Y\}$. The μ_X^1-rg^*b open sets of X are $\varnothing$, X, $\{a\}$, $\{b\}$, $\{c\}$, $\{a, b\}$ and $\{a, c\}$ and the μ_X^2-rg^*b open sets of X are $\varnothing$, X, $\{a\}$, $\{b\}$, $\{c\}$, $\{a, c\}$ and $\{b, c\}$. Also, all subsets of Y are μ_Y^1-rg^*b open. Let $f : (X, \mu_X^1, \mu_X^2) \to (Y, \mu_Y^1, \mu_Y^2)$ be a function defined by $f(a)$=$f(b)$=u and $f(c)$=v. Then f is $\mu^{(1,2)}$-rg^*b continuous and $\mu^{(2,1)}$-rg^*b continuous. Therefore f is pairwise μ-rg^*b continuous.

Theorem 3.3 *Every* $\mu^{(m,n)}$*-continuous function is* $\mu^{(m,n)}$-rg^*b *continuous.*

Proof: Follows from Remark 2.3 (i).

Remark 3.4 *The converse of Theorem* 3.3 i*s not true since in Example 3.2* f *is* $\mu^{(1,2)}$-rg^*b *continuous but not* $\mu^{(1,2)}$*-continuous.*

Theorem 3.5 *For a function* $f : (X, \mu_X^1, \mu_X^2) \to (Y, \mu_Y^1, \mu_Y^2)$*, the following properties are equivalent:*

i. f *is* $\mu^{(m,n)}$-rg^*b *continuous at* $x \in X$*;*

ii. $x \in rg^*bi_{\mu_X^n}(f^{-1}(V))$ *for every* $V \in \mu_Y^m$ *containing* $f(x)$*;*

iii. $x \in rg^*bi_{\mu_X^n}(f^{-1}(B))$ *for every subset* B *of* Y *with* $x \in f^{-1}(i_{\mu_Y^m}(B))$*;*

iv. $x \in f^{-1}(F)$ *for every* μ_Y^m*-closed subset* F *of* Y *such that* $x \in rg^*bc_{\mu_X^n}(f^{-1}(F))$*.*

Proof: Let $f : X \to Y$ be a function and let $x \in X$.

(i)$\Leftrightarrow$(ii): Let $V \in \mu_Y^m$ containing $f(x)$. Since f is $\mu^{(m,n)}$-rg^*b continuous at x, there exists a μ_X^n-rg^*b open set U containing x such that $f(U) \subseteq V$. Hence, $x \in U \subseteq f^{-1}(V)$. This implies that $x \in rg^*bi_{\mu_X^n}(f^{-1}(V))$.

Conversely, let $V \in \mu_Y^m$ with $f(x) \in V$. By (ii), $x \in rg^*bi_{\mu_X^n}(f^{-1}(V))$. Hence, there exists a μ_X^n-rg^*b open set U with $x \in U \subseteq f^{-1}(V)$. Thus, $f(U) \subseteq V$. Therefore, f is $\mu^{(m,n)}$-rg^*b continuous at $x \in X$.

(ii)$\Rightarrow$(iii): Let $B \subseteq Y$ with $x \in f^{-1}(i_{\mu_Y^m}(B))$. Then $f(x) \in i_{\mu_Y^m}(B)$. Since $i_{\mu_Y^m}(B) \in \mu_Y^m$, by (ii) we have, $x \in rg^*bi_{\mu_X^n}(f^{-1}(i_{\mu_Y^m}B)) \subseteq rg^*bi_{\mu_X^n}(f^{-1}(B)$. Thus, $x \in rg^*bi_{\mu_X^n}(f^{-1}(B))$.

(iii)$\Rightarrow$(iv): Let F be a μ_Y^m-closed subset of Y such that $x \notin f^{-1}(F)$. Then $x \in X \setminus f^{-1}(F) = f^{-1}(Y \setminus F) = f^{-1}(i_{\mu_Y^m}(Y \setminus F))$ since $Y \setminus F$ is μ_Y^m open. By (iii), $x \in rg^*bi_{\mu_X^n}(f^{-1}(Y \setminus F)) = rg^*bi_{\mu_X^n}(X \setminus f^{-1}(F)) = X \setminus rg^*bc_{\mu_X^n}(f^{-1}(F))$. Hence, $x \notin rg^*bc_{\mu_X^n}(f^{-1}(F)$.

(iv)$\Rightarrow$(ii): Let $V \in \mu_Y^m$ with $f(x) \in V$. Suppose that $x \notin rg^*bi_{\mu_X^n}(f^{-1}(V))$. The $x \in X \setminus rg^*bi_{\mu_X^n}(f^{-1}(V) = rg^*bc_{\mu_X^n}(X \setminus f^{-1}(V)) = rg^*bc_{\mu_X^n}(f^{-1}(Y \setminus V))$. By (iv), $x \in f^{-1}(Y \setminus V) = X \setminus f^{-1}(V)$. This implies that $x \notin f^{-1}(V)$ which is a contradiction since $f(x) \in V$. Therefore, $x \in rg^*bi_{\mu_X^n}(f^{-1}(V))$. □

Theorem 3.6 *For a function $f : (X, \mu_X^1, \mu_X^2) \to (Y, \mu_Y^1, \mu_Y^2)$, the following properties are equivalent:*

i. *f is $\mu^{(m,n)}$-rg^*b continuous;*

ii. *$f^{-1}(V) = rg^*bi_{\mu_X^n}(f^{-1}(V))$ for every $V \in \mu_Y^m$;*

iii. *$f^{-1}(i_{\mu_Y^m}(B)) \subseteq rg^*bi_{\mu_X^n}(f^{-1}(B))$ for every $B \subseteq Y$;*

iv. *$rg^*bc_{\mu_X^n}(f^{-1}(F)) = f^{-1}(F)$ for every μ_Y^m-closed subset F of Y.*

Proof: Let $f : X \to Y$ be a function and let $x \in X$.

(i)$\Rightarrow$(ii): Let $V \in \mu_Y^m$ and $x \in f^{-1}(V)$. Then $f(x) \in V$. By Theorem 3.5 (ii), $x \in rg^*bi_{\mu_X^n}(f^{-1}(V))$. Since $rg^*bi_{\mu_X^n}(f^{-1}(V)) \subseteq f^{-1}(V)$, we have $f^{-1}(V)$ $= rg^*bi_{\mu_X^n}(f^{-1}(V))$.

(ii)$\Rightarrow$(iii): Let $B \subseteq Y$. Since $i_{\mu_Y^m}(B) \in \mu_Y^m$, by (ii) we have $f^{-1}(i_{\mu_Y^m}(B)) =$ $rg^*bi_{\mu_X^n}(f^{-1}(i_{\mu_Y^m}(B)) \subseteq rg^*bi_{\mu_X^n}(f^{-1}(B))$. Thus, $f^{-1}(i_{\mu_Y^m}(B)) \subseteq rg^*bi_{\mu_X^n}(f^{-1}(B))$.

(iii)$\Rightarrow$(iv): Let F be a μ_Y^m-closed subset of Y. Then by (iii), $f^{-1}(Y \setminus F)$ $= f^{-1}(i_{\mu_Y^m}\ (Y \setminus F)) \subseteq rg^*bi_{\mu_X^n}\ (f^{-1}(Y \setminus F)) = rg^*bi_{\mu_X^n}(X \setminus f^{-1}(F)) = X \setminus$ $rg^*bc_{\mu_X^n}(f^{-1}(F))$. Thus, $rg^*bc_{\mu_X^n}(f^{-1}(F)) \subseteq f^{-1}(F)$. Hence, $rg^*bc_{\mu_X^n}(f^{-1}(F))$ $= f^{-1}(F)$.

(iv)$\Rightarrow$(i): Let $x \in X$ and F be a μ_Y^m-closed subset of Y with $x \in rg^*bc_{\mu_X^n}(f^{-1}(F)$. By (iv), $x \in f^{-1}(F)$. Thus by Theorem 3.5(iv), f is $\mu^{(m,n)}$-rg^*b continuous at x. Since x is arbitrary, f is $\mu^{(m,n)}$-rg^*b continuous. □

Theorem 3.7 *Let $f : (X, \mu_X^1, \mu_X^2) \to (Y, \mu_Y^1, \mu_Y^2)$ be a function. If for each μ_Y^m-open set U of Y, $f^{-1}(U)$ is μ_X^n-rg^*b open in X, then f is $\mu^{(m,n)}$-rg^*b continuous.*

Proof: Let $x \in X$ and V be any μ_Y^m-open set in Y such that $f(x) \in V$. By assumption, $f^{-1}(V)$ is μ_X^n-rg^*b open in X with $x \in f^{-1}(V)$. Take $O= f^{-1}(V)$. Then $x \in O$ and $f(O) \subseteq V$. Therefore, f is $\mu^{(m,n)}$-rg^*b continuous. □

Remark 3.8 *The converse of Theorem* 3.7 *is not true.*

To see this, consider Example 3.2. Then f is $\mu^{(1,2)}$-rg^*b continuous but $f^{-1}(\{u\}) = \{a, b\}$ is not μ_X^2-rg^*b open in X.

Definition 3.9 A function $f : (X, \mu_X^1, \mu_X^2) \to (Y, \mu_Y^1, \mu_Y^2)$ is said to be *absolute $\mu^{(m,n)}$-rg^*b continuous* if for each μ_Y^m-rg^*b open set U in Y, $f^{-1}(U)$ is μ_X^n-rg^*b open in X.

Example 3.10 Let $Y = \{u, v, w\}$ and $X = \{a, b\}$. Consider the generalized topologies $\mu_Y^1=\{\varnothing, \{u\}, \{u, v\}, Y\}$, $\mu_Y^2=\{\varnothing, \{v\}, \{v, w\}, Y\}$, $\mu_X^1=\{\varnothing, \{a\}, X\}$, and $\mu_X^2=\{\varnothing, \{b\}, X\}$. The μ_Y^1-rg^*b open sets of Y are $\varnothing$, Y, $\{u, w\}$, $\{u, v\}$ and $\{u\}$ and μ_Y^2-rg^*b open sets of Y are $\varnothing$, Y, $\{v, w\}$, $\{u, v\}$ and $\{v\}$. Also, the μ_X^1-rg^*b open sets of X are $\varnothing$, X, and $\{a\}$ and the μ_X^2-rg^*b open sets of X are $\varnothing$, X, and $\{b\}$. Let $f : (Y, \mu_Y^1, \mu_Y^2) \to (X, \mu_X^1, \mu_X^2)$ be a function defined by $f(u)=b$ and $f(v)=a=f(w)$. Then f is absolute-$\mu^{(1,2)}$-rg^*b continuous.

Theorem 3.11 *If $f : (X, \mu_X^1, \mu_X^2) \to (Y, \mu_Y^1, \mu_Y^2)$ is an absolute $\mu^{(m,n)}$-rg^*b continuous function, then f is $\mu^{(m,n)}$-rg^*b continuous.*

Proof: Let V be any μ_Y^m-open set in Y. Then V is μ_Y^m-rg^*b open in Y. Since f is absolute $\mu^{(m,n)}$-rg^*b continuous, $f^{-1}(V)$ is μ_X^n-rg^*b open in X. By Theorem 3.7, f is $\mu^{(m,n)}$-rg^*b continuous. □

Remark 3.12 *The converse of Theorem* 3.11 *is not true.*

To see this, consider Example 3.2. Then f is $\mu^{(1,2)}$-rg^*b continuous but not absolute $\mu^{(m,n)}$-rg^*b continuous since for μ_Y^1-rg^*b open set $\{u\}$, $f^{-1}(\{u\}) = \{a, b\}$ is not μ_X^2-rg^*b open in X.

Theorem 3.13 *A function $f : (X, \mu_X^1, \mu_X^2) \to (Y, \mu_Y^1, \mu_Y^2)$ is absolute $\mu^{(m,n)}$-rg^*b continuous if and only if $f^{-1}(U)$ is μ_X^n-rg^*b closed in X for every μ_Y^m-rg^*b closed set U in Y.*

Proof: Suppose that f is absolute $\mu_{(m,n)}$-rg^*b continuous. Let U be a μ_Y^m-rg^*b closed in Y. Then $Y \setminus U$ is μ_Y^m-rg^*b open in Y. Hence, $f^{-1}(Y \setminus U) = X \setminus f^{-1}(U)$ is μ_X^n-rg^*b open in X. Thus, $f^{-1}(U)$ is μ_X^n-rg^*b closed in X.

Conversely, let O be a μ_Y^m-rg^*b open in Y. Then $Y \setminus O$ is μ_Y^m-rg^*b closed in Y. By assumption, $f^{-1}(Y \setminus O)= X \setminus f^{-1}(O)$ is μ_X^n-rg^*b closed in X. Therefore, $f^{-1}(O)$ is μ_X^n-rg^*b open in X. □

Theorem 3.14 *Let $f : (X, \mu_X^1, \mu_X^2) \to (Y, \mu_Y^1, \mu_Y^2)$ be an absolute $\mu^{(m,n)}$-rg^*b continuous function. Then the following hold:*

i. *For each $x \in X$ and for every μ_Y^m-rg^*b open set V in Y containing $f(x)$, there exists a μ_X^n-rg^*b open set U containing x such that $f(U) \subseteq V$;*

ii. *$f(rg^*bc_{\mu_X^n}(A)) \subseteq rg^*bc_{\mu_Y^m}(f(A))$ for every $A \subseteq X$;*

iii. *$rg^*bc_{\mu_X^n}(f^{-1}(B)) \subseteq f^{-1}(rg^*bc_{\mu_Y^m}(B))$ for every $B \subseteq Y$.*

Proof: Let $x \in X$ and let V be a μ_Y^m-rg^*b open set in Y with $f(x) \in V$. Then $Y \setminus V$ is μ_Y^m-rg^*b closed in Y. By Theorem 3.13, $f^{-1}(Y \setminus V)= X \setminus f^{-1}(V)$ is μ_X^n-rg^*b closed in X. Hence, $f^{-1}(V)$ is μ_X^n-rg^*b open. Thus, $f^{-1}(V) = rg^*bi_{\mu_X^n}(f^{-1}(V))$. It follows that $x \in rg^*bi_{\mu_X^n}(f^{-1}(V))$. Thus, (i) holds.

Let $A \subseteq X$ and let $x \in rg^*bc_{\mu_X^n}(A)$. Then $f(x) \in f(rg^*bc_{\mu_X^n}(A))$. Let V be a μ_Y^m-rg^*b open in Y with $f(x) \in V$. By (i), there exists a μ_X^n-rg^*b open set U with $x \in U$ and $f(U) \subseteq V$. Since $x \in rg^*bc_{\mu_X^n}(A)$, $A \cap U \neq \varnothing$. It follows that $\varnothing \neq f(A \cap U) \subseteq f(A) \cap f(U) \subseteq f(A) \cap V$. Hence, $f(A) \cap V \neq \varnothing$. Hence, (ii) holds.

For (iii), let $B \subseteq Y$. Take A=$f^{-1}(B)$ in (ii). Then $f(rg^*bc_{\mu_X^n}(f^{-1}(B))) \subseteq rg^*bc_{\mu_Y^m}(f(f^{-1}(B))) \subseteq rg^*bc_{\mu_Y^m}(B)$. Therefore, the conclusion holds. □

Remark 3.15 *The converse of Theorem* 3.14(i) *is not true. To see this, consider the next example.*

Example 3.16 Let $X = \{a, b, c\}$ and $Y = \{u, v\}$. Consider the generalized topologies μ_X^1=$\{\varnothing, \{a\}, \{a, b\}\}$, μ_X^2=$\{\varnothing, \{c\}, \{b, c\}\}$, μ_Y^1=$\{\varnothing, \{u\}\}$, and μ_Y^2=$\{\varnothing, \{v\}, Y\}$. The μ_X^2-rg^*b closed sets of X are $\varnothing$, X, $\{b, c\}$, $\{a, c\}$, $\{a, b\}$, $\{b\}$ and $\{a\}$ and the μ_X^2-rg^*b open sets of X are $\varnothing$, X, $\{a\}$, $\{b\}$, $\{c\}$, $\{a, c\}$ and $\{b, c\}$. Also all subsets of Y are μ_Y^1-rg^*b open and μ_Y^1-rg^*b closed sets. Let $f : (X, \mu_X^1, \mu_X^2) \to (Y, \mu_Y^1, \mu_Y^2)$ be a function defined by $f(a)$=$f(b)$=u and $f(c)$=v. Note that for each $x \in X$ and for every μ_Y^1-rg^*b open set V in Y containing $f(x)$, there exists a μ_X^2-rg^*b open set U containing x such that $f(U) \subseteq V$ however, f is not absolute $\mu^{(1,2)}$-rg^*b continuous since $f^{-1}(\{u\})$= $\{a, b\}$ is not μ_X^2-rg^*b open.

Definition 3.17 Let $f : (X, \mu_X^1, \mu_X^2) \to (Y, \mu_Y^1, \mu_Y^2)$ be a function. Then

i. f is said to be *regular strongly-*$\mu^{(m,n)}$*-*rg^*b (briefly rs-$\mu^{(m,n)}$-rg^*b) *continuous* if for each μ_Y^m-rg^*b open subset U of Y, $f^{-1}(U)$ is μ_X^n-open in X.

ii. f is said to be *pairwise rs-*μ*-*rg^*b *continuous* if f is rs-$\mu^{(1,2)}$-rg^*b continuous and rs-$\mu^{(2,1)}$-rg^*b continuous.

Example 3.18 Consider the function $f:(Y,\mu_Y^1,\mu_Y^2)\rightarrow(X,\mu_X^1,\mu_X^2)$ in Example 3.10. Then f is rs-$\mu^{(1,2)}$-rg^*b continuous and rs-$\mu^{(2,1)}$-rg^*b continuous. Therefore, f is pairwise rs-μ-rg^*b continuous.

Theorem 3.19 *If* $f:(X,\mu_X^1,\mu_X^2)\rightarrow(Y,\mu_Y^1,\mu_Y^2)$ *is an* rs-$\mu^{(m,n)}$-rg^*b *continuous function, then* f *is absolute* $\mu^{(m,n)}$-rg^*b *continuous.*

Proof: Let U be a μ_Y^m-rg^*b open set in Y. Since f is rs-$\mu^{(m,n)}$-rg^*b continuous, $f^{-1}(U)$ is μ_X^n-open in X. Thus $f^{-1}(U)$ is μ_X^n-rg^*b open in X. Therefore, f is absolute $\mu^{(m,n)}$-rg^*b continuous.

Corollary 3.20 *If* $f:(X,\mu_X^1,\mu_X^2)\rightarrow(Y,\mu_Y^1,\mu_Y^2)$ *is an* rs-$\mu^{(m,n)}$-rg^*b *continuous function, then* f *is* $\mu^{(m,n)}$-rg^*b *continuous.*

Proof: Follows from Theorem 3.19 and 3.11.

Remark 3.21 *The converses of Theorem* 3.19 *and Corollary* 3.20 *are not true. To see these, consider the following:*

Example 3.22 For Theorem 3.19, consider $X=\{a,b,c\}$ and $Y=\{u,v\}$ with the generalized topologies μ_X^1=$\{\varnothing,\ \{a\},\ \{a,b\},X\}$, μ_X^2=$\{\varnothing,\ \{c\},\ \{b,c\}\}$, μ_Y^1=$\{\varnothing,\ \{u\},\ Y\}$, and μ_Y^2=$\{\varnothing,\ \{v\},\ Y\}$. The μ_X^2-rg^*b open sets of X are $\varnothing$, X, $\{b,c\}$, $\{a,c\}$, $\{c\}$, $\{b\}$ and $\{a\}$ and the μ_Y^1-rg^*b open sets of Y are $\varnothing$, Y and $\{u\}$. Let $f:(X,\mu_X^1,\mu_X^2)\rightarrow(Y,\mu_Y^1,\mu_Y^2)$ be a function defined by $f(a)$ $=f(c)=v$ and $f(b)=u$. Then f is absolute $\mu^{(1,2)}$-rg^*b continuous but not rs-$\mu^{(1,2)}$-rg^*b continuous. For Corollary 3.20, consider Example 3.2. Then f is $\mu^{(1,2)}$-rg^*b continuous but not rs-$\mu^{(1,2)}$-rg^*b continuous.

Theorem 3.23 *A function* $f:(X,\mu_X^1,\mu_X^2)\rightarrow(Y,\mu_Y^1,\mu_Y^2)$ *is* rs-$\mu^{(m,n)}$-rg^*b *continuous if and only* $f^{-1}(A)$ *is* μ_X^n*-closed for each* μ_Y^m-rg^*b *closed set* A *in* Y.

Proof: Let A be a μ_Y^m-rg^*b closed set in Y. Then $Y\setminus A$ is μ_Y^m-rg^*b open. Since f is an rs-$\mu^{(m,n)}$-rg^*b continuous, $f^{-1}(Y\setminus A)$=$X\setminus f^{-1}(A)$ is μ_X^n-open in X. Hence, $f^{-1}(A)$ is μ_X^n-closed in X.

Conversely, suppose U is μ_Y^m-rg^*b open in Y. Then $Y\setminus U$ is a μ_Y^m-rg^*b closed in Y. Thus, $f^{-1}(Y\setminus U)$=$X\setminus f^{-1}(U)$ is μ_X^n-closed. Therefore, $f^{-1}(U)$ is μ_X^n- open. Consequently, f is rs-$\mu^{(m,n)}$-rg^*b continuous. □

Theorem 3.24 *A function $f : (X, \mu_X^1, \mu_X^2) \to (Y, \mu_Y^1, \mu_Y^2)$ is rs-$\mu^{(m,n)}$-rg^*b continuous if an only if for all $x \in X$ and for every μ_Y^m-rg^*b open set V containing $f(x)$, there exists a μ_X^n-open set U with $x \in U$ and $f(U) \subseteq V$.*

Proof: Suppose that f is an rs-$\mu^{(m,n)}$-rg^*b continuous function. Let $x \in X$ and let V be a μ_Y^m-rg^*b open set with $f(x) \in V$. Then $f^{-1}(V)$ is μ_X^n-open. Take $U = f^{-1}(V)$. Then $x \in U$ and $f(U) \subseteq V$.

Conversely, let V be a μ_Y^m-rg^*b open set and $x \in f^{-1}(V)$. Then $f(x) \in V$. By assumption, there exists a μ_X^n-open set U with $x \in U \subseteq f^{-1}(V)$. Hence, $x \in i_{\mu_X^n}(f^{-1}(V))$. It follows that, $f^{-1}(V) \subseteq i_{\mu_X^n}(f^{-1}(V))$. Therefore, $f^{-1}(V)$=$i_{\mu_X^n}(f^{-1}(V))$ implying that $f^{-1}(V)$ is μ_X^n-open. Thus, f is rs-$\mu^{(m,n)}$-rg^*b continuous. □

Theorem 3.25 *Let $f : (X, \mu_X^1, \mu_X^2) \to (Y, \mu_Y^1, \mu_Y^2)$ be an rs $\mu^{(m,n)}$-rg^*b continuous function. Then the following hold:*

i. *$f(rg^*bc_{\mu_X^n}(A)) \subseteq rg^*bc_{\mu_Y^m}(f(A))$ for every $A \subseteq X$;*

ii. *$rg^*bc_{\mu_X^n}(f^{-1}(B)) \subseteq f^{-1}(rg^*bc_{\mu_Y^m}(B))$ for every $B \subseteq Y$.*

Proof: Let f be an rs $\mu^{(m,n)}$-rg^*b continuous function, $A \subseteq X$ and $x \in rg^*bc_{\mu_X^n}(A)$. Suppose that O is a μ_Y^m-rg^*b open set in Y with $f(x) \in O$. Then $f^{-1}(O)$ is μ_X^n-open in X with $x \in f^{-1}(O)$. Since $f^{-1}(O)$ is also μ_X^n-rg^*b open, $f^{-1}(O) \cap A \neq \varnothing$. This implies that $f(f^{-1}(O) \cap A) \subseteq f(f^{-1}(O)) \cap f(A)$ $\subseteq O \cap f(A) \neq \varnothing$. Hence, $f(x) \in rg^*bc_{\mu_Y^m}(f(A))$. Therefore, (i) follows.

For (ii), let $B \subseteq Y$. By (i), $f(rg^*bc_{\mu_X^n}(f^{-1}(B))) \subseteq rg^*bc_{\mu_Y^m}(f(f^{-1}(B))) \subseteq rg^*bc_{\mu_Y^m}(B)$. Thus, $rg^*bc_{\mu_X^n}(f^{-1}(B)) \subseteq f^{-1}(rg^*bc_{\mu_Y^m}(B))$. □

Definition 3.26 Let $f : (X, \mu_X^1, \mu_X^2) \to (Y, \mu_Y^1, \mu_Y^2)$ be a function. Then

i. f is said to be *almost $\mu^{(m,n)}$-rg^*b continuous at a point $x \in X$* if for each μ_Y^m open set V containing $f(x)$, there exists a μ_X^n-rg^*b open set U containing x such that $f(U) \subseteq i_{\mu_Y^m}(c_{\mu_Y^n}(V))$.

ii. f is said to be *almost $\mu^{(m,n)}$-rg^*b continuous* if f is almost $\mu^{(m,n)}$-rg^*b continuous at every point $x \in X$.

iii. f is said to be *pairwise almost μ-rg^*b continuous* if f is almost $\mu^{(1,2)}$-rg^*b continuous and almost $\mu^{(2,1)}$-rg^*b continuous.

Example 3.27 Consider the function $f : (X, \mu_X^1, \mu_X^2) \to (Y, \mu_Y^1, \mu_Y^2)$ in Example 3.2. Then f is almost $\mu^{(1,2)}$-rg^*b continuous and almost $\mu^{(2,1)}$-rg^*b continuous. Therefore f is pairwise almost μ-rg^*b continuous.

Theorem 3.28 *If f is $\mu^{(m,n)}$-rg^*b continuous, then f is almost $\mu^{(m,n)}$-rg^*b continuous.*

Proof: Let $x \in X$ and let V be a μ_Y^m-open set with $f(x) \in V$. Then there exists a μ_X^n-rg^*b open set U with $x \in U \subseteq f^{-1}(V)$. Thus $f(U) \subseteq V \subseteq c_{\mu_Y^n}(V)$. It follows that $f(U) \subseteq i_{\mu_Y^m}(c_{\mu_Y^n}(V))$. □

Remark 3.29 *The converse of Theorem 3.28 is not true as shown by the following example:*

Example 3.30 Let $X = \{a, b, c\}$ and $Y = \{u, v, w\}$. Consider the generalized topologies μ_X^1=$\{\varnothing, \{a\}, \{b,c\}, X\}$, μ_X^2=$\{\varnothing, \{a,b\}, \{b,c\}, X\}$, μ_Y^1=$\{\varnothing, \{u\}, \{v\}, \{u,v\}\}$, and μ_Y^2=$\{\varnothing, \{w\}, \{u,w\}\}$. Then the μ_X^2-rg^*b open sets of X are $\varnothing$, X, $\{b\}$, $\{a,b\}$, $\{a,c\}$ and $\{b,c\}$. Let $f : (X, \mu_X^1, \mu_X^2) \to (Y, \mu_Y^1, \mu_Y^2)$ be a function defined by $f(a)$=u, $f(b)$=v and $f(c)$=w. Then f is almost $\mu^{(1,2)}$-rg^*b continuous but not $\mu^{(1,2)}$-rg^*b continuous.

Definition 3.31 [2] Let A be a subset of a BGTS (X, μ_X^1, μ_X^2). Then A is $\mu^{(m,n)}$-*regular* open if $A = i_{\mu_X^m}(c_{\mu_X^n}(A))$.

Theorem 3.32 *For a function $f : (X, \mu_X^1, \mu_X^2) \to (Y, \mu_Y^1, \mu_Y^2)$, the following properties are equivalent:*

i. *f is almost $\mu^{(m,n)}$-rg^*b continuous at $x \in X$;*

ii. *$x \in rg^*bi_{\mu_X^n}(f^{-1}(i_{\mu_Y^m}(c_{\mu_Y^n}(V))))$ for every $V \in \mu_Y^m$ containing $f(x)$;*

iii. *$x \in rg^*bi_{\mu_X^n}(f^{-1}(V))$ for every $\mu^{(m,n)}$-regular open subset V containing $f(x)$;*

iv. *For every $\mu^{(m,n)}$-regular open subset V containing $f(x)$, there exists μ_X^n-rg^*b open set U containing x such that $f(U) \subseteq V$.*

Proof: Let $f : X \to Y$ be a function and let $x \in X$.

(i)⇒(ii): Let $V \in \mu_Y^m$ containing $f(x)$. Then $x \in f^{-1}(V)$. Since f is almost $\mu^{(m,n)}$-rg^*b continuous at x, there exists a μ_X^n-rg^*b open set U containing x such that $f(U) \subseteq i_{\mu_Y^m}(c_{\mu_Y^n}(V))$. Hence, $x \in U \subseteq f^{-1}(i_{\mu_Y^m}(c_{\mu_Y^n}(V)))$. This implies that $x \in rg^*bi_{\mu_X^n}(f^{-1}(i_{\mu_Y^m}(c_{\mu_Y^n}(V))))$.

(ii)⇒(iii): Let V be any $\mu^{(m,n)}$-regular open subset of Y containing $f(x)$. Then $f(x) \in V$=$i_{\mu_Y^m}(c_{\mu_Y^n}(V))$. By (ii), we have $x \in rg^*bi_{\mu_X^n}(f^{-1}(i_{\mu_Y^m}(c_{\mu_Y^n}(V))))$ $= rg^*bi_{\mu_X^n}(f^{-1}(V))$.

(iii)⇒(iv): Let V be any $\mu^{(m,n)}$-regular open subset of Y containing $f(x)$. Then by (iii), $x \in rg^*bi_{\mu_X^n}(f^{-1}(V))$. Thus, there exists a μ_X^n-rg^*b open set U with $x \in U \subseteq f^{-1}(V)$. Hence, $f(U) \subseteq V$.

(iv)⇒(i): Let $V \in \mu_Y^m$ with $f(x) \in V$. Then $f(x) \in V \subseteq i_{\mu_Y^m}(c_{\mu_Y^n}(V))$. Since $i_{\mu_Y^m}(c_{\mu_Y^n}(V))$ is $\mu^{(m,n)}$-regular open, by (iv) there exists a μ_X^n-rg^*b open set containing x such that $f(U) \subseteq i_{\mu_Y^m}(c_{\mu_Y^n}(V))$. Therefore, f is almost $\mu^{(m,n)}$-rg^*b continuous at $x \in X$. □

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