\( \alpha^m \)-Closed Sets in Topological Spaces

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Abstract

In this paper, the authors introduce and study the concept of new class of closed sets called \( \alpha^m \)-closed sets. Also we investigate some of their properties.

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1 Introduction

In 1970, N. Levine introduced and investigated the concept of generalized closed sets in a topological space. A subset \( A \) in a topological space \( (X, \tau) \) is called generalized closed (briefly g-closed) if \( \text{cl}(A) \subseteq U \) whenever \( A \subseteq U \) and \( U \) is open in \( (X, \tau) \). A subset \( B \) is called g-open if the complement of \( B \) (ie \( X/B \)) is g closed. He studied their most fundamental properties and also introduced a separation axiom \( T_{1/2} \). After Levine’s works many authors defined and investigated various notions analogous to Levine’s g-closed sets and related
topics. In this paper, we investigate the behaviour of $\alpha^m$-closed sets and its various characterization are studied.

## 2 Preliminaries

Before entering into our work, we recall the following definitions which are due to Levine.

**Definition 2.1.** [6]: A subset $A$ of a topological space $(X, \tau)$ is called a pre-open set if $A \subseteq \text{int}(\text{cl}(A))$ and pre-closed set if $\text{cl}(\text{int}(A)) \subseteq A$.

**Definition 2.2.** [2] A subset $A$ of a topological space $(X, \tau)$ is called a semi-open set if $A \subseteq \text{cl}(\text{int}(A))$ and semi closed set if $\text{int}(\text{cl}(A)) \subseteq A$.

**Definition 2.3.** [7] A subset $A$ of a topological space $(X, \tau)$ is called an $\alpha$-open set if $A \subseteq \text{int}(\text{cl}(\text{int}(A)))$ and an $\alpha$-closed set if $\text{cl}(\text{int}(\text{cl}(A))) \subseteq A$.

**Definition 2.4.** [1] A subset $A$ of a topological space $(X, \tau)$ is called a semi-pre-open set ($\beta$-open set) if $A \subseteq \text{cl}(\text{int}(\text{cl}(A)))$ and semi-preclosed set ($\beta$ closed set) if $\text{int}(\text{cl}(\text{int}(A))) \subseteq A$.

**Definition 2.5.** [3] A subset $A$ of a topological space $(X, \tau)$ is called a g-closed set if $\text{cl}(A) \subseteq U$, whenever $A \subseteq U$ and $U$ is open in $(X, \tau)$.

**Definition 2.6.** [4] A subset $A$ of a topological space $(X, \tau)$ is called a generalized $\alpha$-closed (briefly $g\alpha$-closed) set if $\text{acl}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is $\alpha$-open in $(X, \tau)$.

**Definition 2.7.** [5] A subset $A$ of a topological space $(X, \tau)$ is called weakly generalized $\alpha$-closed set (briefly $wga$-closed) if $\tau^{\alpha} - \text{cl}(\text{int}(A)) \subseteq U$ whenever $A \subseteq U$ and $U$ is $\alpha$-open in $(X, \tau)$.

## 3 $\alpha^m$-closed sets

In this section we introduce the concept of $\alpha^m$-closed sets.

**Definition 3.1.** A subset $A$ of a topological space $(X, \tau)$ is called $\alpha^m$-closed set if $\text{int}(\text{cl}(A)) \subseteq U$ whenever $A \subseteq U$ and $U$ is $\alpha$-open.

**Theorem 3.1.** Every $\alpha^m$-closed set is $\alpha$-closed set.

**Proof.** Assume that $A$ is a $\alpha^m$-closed set in $X$ and let $U$ be an open set such that $A \subseteq U$. Since every open set is $\alpha$-open set and $A$ is $\alpha^m$-closed set, $\text{int}(\text{cl}(A)) \subseteq (\text{int}(\text{cl}(A))) \cup (\text{cl}(\text{int}(A))) \subseteq U$. Therefore $A$ is $\alpha$-closed set in $X$. $\square$
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Remark 3.1. The converse of the above theorem need not be true as seen from the following example.

Example 3.1. Consider X=\{a,b,c\} with τ = \{X, \phi, \{a, b\}, \{b, c\}, \{b\}\}. In topological space the subset A=\{a\} is αclosed but not αm-closed set.

Theorem 3.2. A set A is αm-closed set iff int(cl(A))-A contains no nonempty αm-closed sets.

Proof. Necessity: Suppose that F is a non empty αm-closed subset of int(cl(A)) such that F ⊂ int(cl(A)) - A. Then F ⊂ int(cl(A)) - A. Since F ⊂ int(cl(A)) ∩ Ac. Therefore F ⊂ int(cl(A)) and F ⊆ Ac. Since Fc is αm-open set and A is αm-closed set int(cl(A)) ⊆ Fc. Thus F ⊂ (int(cl(A)))c. Therefore F ⊆ [int(cl(A))] ∩ [int(cl(A))c] = φ. Therefore F = φ ⇒ int(cl(A)) - A contains no non empty αm-closed sets.

Sufficiency: Let A ⊆ U is αm-open set. Suppose that int(cl(A)) is not contained in U. Then (int(cl(A)))c is a non empty αm-closed set and contained in int(cl(A)) - A which is a contradiction. Therefore int(cl(A)) ⊆ U and hence A is αm-closed set.

□

Theorem 3.3. Let B ⊆ Y ⊆ X, if B is αm closed set relative to Y and Y is open then B is αm closed set in (X, τ)

Proof. Let U be a αm closed set in (X, τ) such that B ⊆ U. Given that B ⊆ Y ⊆ X. Therefore B ⊆ Y and B ⊆ U. This implies B ⊆ Y ∩ U. Since B is αm-closed set relative to Y, then int(cl(B)) ⊆ U. Y ∩ int(cl(B)) ⊆ Y ∩ U implies that Y ∩ (int(cl(B))) ⊆ U. Thus [Y ∩ int(cl(B))] ∪ [int(cl(B))]c ⊆ U ∪ [int(cl(B))]c. This implies that (Y ∪ (int(cl(B)))c) ∩ (int(cl(B)))c ⊆ U ∪ (int(cl(B)))c. Therefore (Y ⊂ (int(cl(B)))c) ⊆ U ∪ (int(cl(B)))c. Since Y is αm-closed set in X. int(cl(Y)) ⊆ U ∪ (int(cl(B)))c. Also B ⊆ Y implies that int(cl(B)) ⊆ int(cl(Y)). Thus int(cl(B)) ⊆ int(cl(Y)) ⊆ U ∪ (int(cl(B)))c. Therefore int(cl(B)) ⊆ U. Since int(cl(B)) is not contained in [int(cl(B))]c, B is αm-closed set relative to X.

□

Theorem 3.4. If A is a αm closed set and A ⊆ B ⊆ int(cl(A)) then B is a αm closed set.

Proof. Let A be a αm closed set such that A ⊆ B ⊆ int(cl(A)). Let U be a αm open set of X such that B ⊆ U. Since A is αm closed set, we have int(cl(A)) ⊆ U whenever A ⊆ U. Since A ⊆ B and B ⊆ int(cl(A)) then int(cl(B)) ⊆ int(cl(int(cl(A)))) ⊆ int(cl(A)) ⊆ U. Therefore int(cl(B)) ⊆ U. Thus B is αm closed set in X.

□

Theorem 3.5. The intersection of a αm closed set and a closed set is a αm closed set.
Proof. Let $A$ be a $\alpha^m$ closed set and $F$ be a closed set. Since $A$ is $\alpha^m$ closed set, $\text{int}(\text{cl}(A)) \subseteq U$ whenever $A \subseteq U$ where $U$ is $\alpha^m$ open set. To show that $A \cap F$ is $\alpha^m$ closed set. It is enough to show that $\text{int}(\text{cl}(A \cap F)) \subseteq U$ whenever $A \cap F \subseteq U$, where $U$ is $\alpha^m$ open set. Let $G = X - F$ then $A \subseteq U \cup G$. Since $G$ is open set, $U \cup G$ is $\alpha^m$ open set and $A$ is $\alpha^m$ open set. Now $\text{int}(\text{cl}(A \cap F)) \subseteq \text{int}(\text{cl}(A)) \cap \text{int}(\text{cl}(F)) \subseteq \text{int}(\text{cl}(A)) \cap F \subseteq (U \cup G) \cap F \subseteq (U \cap F) \cup (G \cap F) \subseteq (U \cap F) \cup \emptyset \subseteq U$. This implies that $A \cap F$ is $\alpha^m$ closed set.

Theorem 3.6. If $A$ and $B$ are two $\alpha^m$ closed set defined for a non empty set $X$, then their intersection $A \cap B$ is $\alpha^m$ closed set in $X$.

Proof. Let $A$ and $B$ are two $\alpha^m$ closed sets. Consider $U$ be $\alpha^m$ open set in $X$ such that $A \cap B \subseteq U$. Now $\text{int}(\text{cl}(A \cap B)) \subseteq (\text{int}(\text{cl}(A)) \cap \text{int}(\text{cl}(B))) \subseteq U$. Hence $A \cap B$ is $\alpha^m$ closed set.

Remark 3.2. The union of two $\alpha^m$ closed sets need not be $\alpha^m$ closed set.

Example 3.2. Let $X = \{a,b,c\}$ with topology $A = \{\emptyset, X, \{a\}, \{b,c\}\}$ and $B = \{\emptyset, \{b\}, \{a,c\}, X\}$. Then $A \cup B$ is not a $\alpha^m$ closed set. Since $\{c\}$ does not belong to $A \cup B$.

Theorem 3.7. Every closed sets is $\alpha^m$ closed set.

Remark 3.3. The converse of the above theorem need not be true.

Theorem 3.8. Every $\alpha^m$ closed set is w$\alpha$ closed set.

Remark 3.4. The converse of the above theorem need not be true from the following example.

Example 3.3. Consider $X = \{a,b,c\}$ with topology $A = \{\emptyset, X, \{a,b\}, \{b,c\}, \{b\}\}$. In topological space the subset $A = \{a\}$ is w$\alpha$ -closed but not $\alpha^m$-closed set.

Theorem 3.9. Every $\alpha^m$ closed set is g$\alpha$ closed set.

Remark 3.5. The converse of the above theorem need not be true from the following example.

Example 3.4. Consider $X = \{a,b,c\}$ with topology $A = \{\emptyset, X, \{a,b\}, \{b,c\}, \{b\}\}$. In topological space the subset $A = \{a\}$ is g$\alpha$ -closed but not $\alpha^m$-closed set.

Remark 3.6. The following are the implications of $\alpha^m$ -closed set and the reverse is not true.
References


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