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α^m-Closed Sets in Topological Spaces

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Abstract

In this paper , the authors introduce and study the concept of new class of closed sets called $\alpha^{\ m}$ -closed sets. Also we investigate some of their properties.

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1 Introduction

In 1970,N.Levine introduced and investigated the concept of generalized closed sets in a topological space.A subset A in a topological space(X, τ) is called generalized closed (briefly g-closed) if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in(X, τ). A subset B is called g-open if the complement of B (ie X/B) is g closed.He studied their most fundamental properties and also introduced a separation axiom $T_{1/2}$. After Levine's works many authors defined and investigated various notions analogous to Levine's g-closed sets and related

topics. In this paper, we investigate the behaviour of α^{m} -closed sets and its various characterization are studied.

2 Preliminaries

Before entering into our work, we recall the following definitions which are due to Levine.

Definition 2.1. *[6]: A subset A of a topological space (X, τ) is called a pre-open set if $A \subseteq int(cl(A))$ and pre-closed set if $cl(int(A)) \subseteq A$.*

Definition 2.2. *[2] A subset A of a topological space (X, τ) is called a semi-open set if $A \subseteq cl(int(A))$ and semi closed set if $int(cl(A)) \subseteq A$.*

Definition 2.3. *[7] A subset A of a topological space (X, τ) is called an α-open set if $A \subseteq int(cl(int(A)))$ and an α -closed set if $cl(int(cl(A))) \subseteq A$.*

Definition 2.4. *[1] A subset A of a topological space (X, τ) is called a semi pre-open set (β-open set) if $A \subseteq cl(int(cl(A)))$ and semi-preclosed set (β closed set) if $int(cl(int(A))) \subseteq A$.*

Definition 2.5. *[3] A subset A of a topological space (X, τ) is called a g-closed if $cl(A) \subseteq U$, whenever $A \subseteq U$ and U is open in (X, τ).*

Definition 2.6. *[4] A subset A of a topological space (X, τ) is called a generalized α -closed (briefly $g\alpha$ -closed) if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is α-open in (X, τ).*

Definition 2.7. *[5] A subset A of a topological space (X, τ) is called weakly generalized α -closed set (briefly $wg\alpha$-closed) if $\tau^{\alpha} - cl(int(A)) \subset U$ whenever $A \subset U$ and U is α - open in (X, τ).*

3 α^{m}-closed sets

In this section we introduce the concept of α^{m}-closed sets .

Definition 3.1. *A subset A of a topological space (X, τ) is called α^{m}-closed set if $int(cl(A)) \subseteq U$ whenever $A \subseteq U$ and U is α-open.*

Theorem 3.1. *Every α^{m}-closed set is α-closed set.*

Proof. Assume that A is a α^{m}-closed set in X and let U be an open set suchthat $A \subseteq U$ .Since every open set is α-open set and A is α^{m}-closed set,$int(cl(A)) \subseteq (int(cl(A))) \cup (cl(int(A))) \subseteq U$.Therefore A is α -closed set in X. □

Remark 3.1. *The converse of the above theorem need not be true as seen from the following example.*

Example 3.1. *Consider X={a,b,c} with* $\tau = \{X, \phi, \{a, b\}, \{b, c\}, \{b\}\}$*.In topological space the subset A={a} is* α *-closed but not* α^m*-closed set.*

Theorem 3.2. *A set A is* α^m*-closed set iff int(cl(A))-A contains no nonempty* α^m*-closed sets.*

Proof. **Necessity:**Suppose that F is a non empty α^m-closed subset of int(cl(A)) suchthat $F \subset int(cl(A)) - A$.Then $F \subseteq int(cl(A)) - A$.Then $F \subseteq int(cl(A))$ $\cap A^c$.Therefore $F \subseteq int(cl(A))$ and$F \subseteq A^c$.Since F^c is α^m-open set and A is α^m-closed set ,$int(cl(A)) \subseteq F^c$ Thus $F \subseteq (int(cl(A)))^c$.Therefore $F \subseteq [int(cl(A))] \cap [int(cl(A))]^c = \phi$. Therefore $F = \phi \Rightarrow int(cl(A)) - A$ contains no non emptyα^m-closed sets.
Sufficiency: Let $A \subseteq U$ is α^m -open set. Suppose that int(cl(A)) is not contained in U. Then $(int(cl(A)))^c$ is a non empty α^m closed set and contained in $int(cl(A)) - A$ which is a contradiction. Therefore $int(cl(A)) \subseteq U$ and hence A is α^m closed set. □

Theorem 3.3. *Let* $B \subseteq Y \subseteq X$*,if B is* α^m *closed set relative to Y and Y is open then B is* α^m *closed set in* (X, τ)

Proof. Let U be a α^m closed set in (X, τ) suchthat $B \subseteq U$. Given that $B \subseteq Y \subseteq X$. Therefore $B \subseteq Y$ and $B \subseteq U$. This implies $B \subseteq Y \cap U$. Since B is α^m-closed set relative to Y, then $int(cl(B)) \subseteq U$. $Y \cap int(cl(B)) \subseteq Y \cap U$ implies that $Y \cap (int(cl(B))) \subseteq U$. Thus $[Y \cap int(cl(B))] \cup [int(cl(B))]^c \subseteq U \cup [int(cl(B))]^c$. This implies that $(Y \cup (int(cl(B)))^c) \cap (int(cl(B))) \cup (int(cl(B)))^c \subseteq U \cup (int(cl(B)))^c$. Therefore $(Y \subset (int(cl(B)))^c) \subseteq U \cup (int(cl(B)))^c$. Since Y is α^m closed set in X. $int(cl(Y)) \subseteq U \cup (int(cl(B)))^c$. Also $B \subseteq Y$ implies that $int(cl(B)) \subseteq int(cl(Y))$. Thus $int(cl(B)) \subseteq int(cl(Y)) \subseteq U \cup (int(cl(B)))^c$. Therefore $int(cl(B)) \subseteq U$. Since int(cl(B)) is not contained in $[int(cl(B))]^c$, B is α^m closed set relative to X. □

Theorem 3.4. *If A is a* α^m *closed set and* $A \subseteq B \subseteq int(cl(A))$ *then B is a* α^m *closed set .*

Proof. Let A be a α^m closed set suchthat $A \subseteq B \subseteq int(cl(A))$. Let U be a α^m open set of X suchthat $B \subseteq U$.Since A is α^m closed set ,we have $int(cl(A)) \subseteq U$ whenever $A \subseteq U$. Since $A \subseteq B$ and $B \subseteq int(cl(A))$ then $int(cl(B)) \subseteq int(cl(int(cl(A))) \subseteq int(cl(A)) \subseteq U$.Therefore $int(cl(B)) \subseteq U$. Thus B is α^m closed set in X. □

Theorem 3.5. *The intersection of a* α^m *closed set and a closed set is a* α^m *closed set.*

Proof. Let A be a α^m closed set and F be a closed set.Since A is α^m closed set $int(cl(A)) \subseteq U$ whenever $A \subseteq U$ where U is α^m open set .To show that $A \cap F$ is α^m closed set.It is enough to show that $int(cl(A \cap F)) \subseteq U$ whenever$A \cap F \subseteq U$, where U is α^m open set .Let G =X - F then $A \subseteq U \cup G$. Since G is open set, $U \cup G$ is α^m open set and A is α^m open set $int(cl(A)) \subseteq U \cup G$. Now $int(cl(A \cap F) \subseteq int(cl(A)) \cap int(cl(F)) \subseteq int(cl(A)) \cap F \subseteq (U \cup G) \cap F \subseteq (U \cap F) \cup (G \cap F) \subseteq (U \cap F) \cup \phi \subseteq U$. This implies that $A \cap F$ is α^m closed set. □

Theorem 3.6. *If A and B are two α^m closed set defined for a non empty set X,then their intersection $A \cap B$ is α^m closed set in X.*

Proof. Let A and B are two α^m closed sets. Consider U be α^m open set in X suchthat $A \cap B \subseteq U$.Now $int(cl(A \cap B)) \subseteq (int(cl(A)) \cap int(cl(B))) \subseteq U$. Hence $A \cap B$ is α^m closed set. □

Remark 3.2. *The union of two α^m closed sets need not be α^m closed set. .*

Example 3.2. *Let X= {a,b,c} with topology $A = \{\Phi, X, \{a\}, \{b, c\}\}\}$ and $B = \{\Phi, \{b\}, \{a, c\}, X\}$. Then $A \cup B$ is not a α^m closedset .Since $\{c\}$ does not belong to $A \cup B$.*

Theorem 3.7. *Every closed sets is α^m closedset.*

Remark 3.3. *The converse of the above theorem need not be true .*

Theorem 3.8. *Every α^m closedset is wgα closed set.*

Remark 3.4. *The converse of the above theorem need not be true from the following example.*

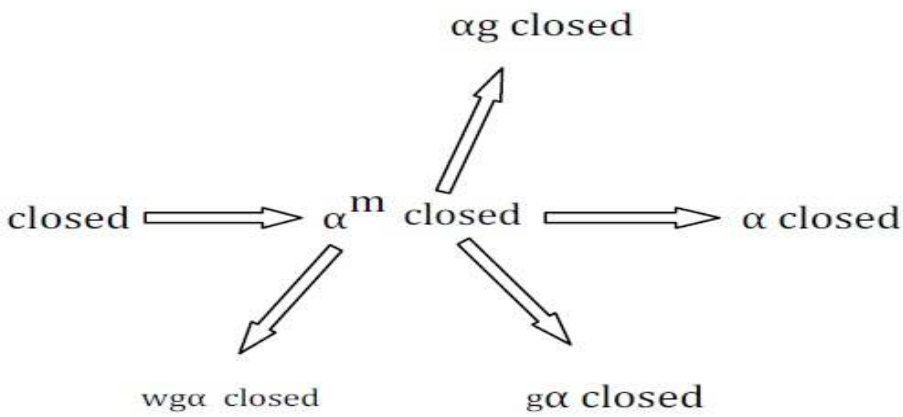
Example 3.3. *Consider X={a,b,c} with $\tau = \{X, \phi, \{a, b\}, \{b, c\}, \{b\}\}$.In topological space the subset A={a} is wgα -closed but not α^m-closed set*

Theorem 3.9. *Every α^m closedset is gα closed set.*

Remark 3.5. *The converse of the above theorem need not be true from the following example.*

Example 3.4. *Consider X={a,b,c} with $\tau = \{X, \phi, \{a, b\}, \{b, c\}, \{b\}\}$.In topological space the subset A={a} is gα -closed but not α^m-closed set*

Remark 3.6. *The following are the implications of α^m -closed set and the reverse is not true*



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