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Boundary Layer Flow and Heat Transfer over a Continuous Surface in the Presence of Hydromagnetic Field

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Abstract

Two – dimensional steady flow of an electrically conducting, viscous incompressible fluid past a continuously moving surface is considered in the presence of uniform transverse magnetic field. The non-linear partial differential equations, governing the problem under consideration have been transformed by a similarity transformation into a system of ordinary differential equations, which is solved numerically by applying Runge-Kutta Gill method along with shooting technique. The effects of the parameters such as reference temperature, exponent, magnetic, Prandtl number and Eckert number are discussed graphically on velocity and temperature distributions.

Keywords: Boundary layer, Hydro magnetic Flow, Heat transfer, Moving surface

INTRODUCTION

In recent years, the problems of free convective heat transfer flows through a porous medium under the influence of a magnetic field have been attracted the attention of a number of researchers because of their possible applications in many branches of science and technology, such as its applications in transportation cooling of re-entry vehicles and rocket boosters, cross-hatching on ablative surfaces and film vaporization in combustion chambers. Flow through a porous medium have numerous engineering and geophysical applications, for example, in chemical engineering for filtration and purification process; in agriculture engineering to study the underground water resources; in petroleum technology to study the movement of natural gas, oil and water through the oil reservoirs. In view of these applications, many researchers have studied MHD free convective heat and mass transfer flow in a porous medium.

Rahman and Sattar [22] have investigated the effect of heat generation or absorption on convective flow of a micropolar fluid past a continuously moving vertical porous plate in presence of a magnetic field. The chemical reaction effect on heat and mass transfer flow along a semi-infinite horizontal plate has been studied by Anjalidevi and Kandaswamy [1] and later it was extended for Hiemenz flow by Seddeek et al. [25] and for polar fluid by Patil and Kulkarni [21]. Salem and Abd El-Aziz [24] have reported the effect of hall currents and chemical reaction on hydromagnetic flow of a stretching vertical surface with internal heat generation or absorption. Ibrahim et al. [7] studied the effect of chemical reaction and radiation absorption on the unsteady MHD free convection flow past a semi-infinite vertical permeable moving plate with heat source and suction. A detailed numerical study has been carried out for unsteady hydromagnetic natural convection heat and mass transfer with chemical reaction over a vertical plate in rotating system with periodic suction by Parida et al. [20]. Rajeswari et al. [23] have investigated chemical reaction, heat and mass transfer on nonlinear MHD boundary layer flow through a vertical porous surface in presence of suction. Mahdy [11] has studied the effect of chemical reaction and heat generation or absorption on double diffussive convection from vertical truncated cone in a porous media with variable viscosity. Pal and Talukdar [19] have studied perturbation analysis of unsteady magnetohydrodynamic convective heat mass transfer in boundary layer slip flow past a vertical permeable plate with a thermal radiation and chemical reaction. Further the effect of thermal radiation, heat and mass transfer flow of a variable viscosity fluid past a vertical porous plate in presence of transverse magnetic field was investigated by Makinde and Ogulu [15]. The analysis of MHD mixed-convection interaction with thermal radiation and higher order chemical reaction is carried out by Makinde [16]. Aziz [2] theoretically examined a similarity solution for a laminar thermal boundary layer over a flat plate with a convective surface boundary condition. He found an interesting result that a similarity solution is possible if the convective heat transfer along with the hot fluid on the lower surface of the plate is inversely proportional to the square root of the axial distance. Recently, the combined effects of an exponentially decaying internal heat generation and a convective boundary condition on the thermal boundary layer over a flat plate are investigated by Olanrewaju et al. [18]. In their study authors have neglected the Sherwood effect. Similar analysis has been carried out by Makinde [12, 13] without heat source and with heat source [14], neglecting chemical reaction effect. There has been considerable interest in studying the effect of chemical reaction [6] and heat source effect on the boundary layer flow problem with heat and mass transfer of an electrically conducting fluid in different geometry [17,4 and 3]. The problem of flow and heat transfer over a moving surface has drawn considerable attention and a very good amount of literature has been generated on this problem (Fang [5] and Ishaket al. [8]). Kumari and Nath [10] discussed the problem of MHD boundary layer flow of a non-Newtonian fluid over a continuously moving surface with a parallel free stream. Recently, Jat and Chaudhary [9] studied the flow of incompressible viscous conducting fluid past a continuous moving surface in the presence of transverse magnetic field. Soundalgekar et al. [26] studied the flow of incompressible viscous fluid past a continuously moving semi-infinite plate by considering variable viscosity and variable temperature.

The objective of the present paper is to study the effects on both momentum and heat transfer problem with viscous dissipation and Joule heat transfer for an electrically conducting fluid past a continuously moving plate in the presence of a uniform transverse magnetic field.

MATHEMATICAL FORMULATION

Consider the two-dimensional steady flow of an electrically conducting, viscous, incompressible fluid past a continuously moving surface with uniform velocity U in the presence of uniform transverse magnetic field of strength B_0. The magnetic Reynolds number is assumed to be small so that the induced magnetic field can be neglected. The x-axis is taken along the surface and y-axis normal to it as shown in fig.1. The fluid properties are assumed to be isotropic and constant. Therefore, under the usual boundary layer approximations, the governing equations of motion are

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \quad \text{…………………………………………………………………}(1)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{1}{\rho}\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right)-\frac{\sigma_e B_0{}^2 u}{\rho} \quad \text{………………………………………………}(2)$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2}+\frac{\mu}{\rho c_p}\left(\frac{\partial u}{\partial y}\right)^2+\frac{\sigma_e B_0{}^2 u^2}{\rho c_p} \quad \text{…………………………………..}(3)$$

where ρis the density, μ be the coefficient of viscosity, σ_e the electrical conductivity, k thermal conductivity of the fluid and c_p the specific heat at constant pressure. The other symbols have their usual meanings.

The corresponding boundary conditions are:

y =0 ; u = U; v = 0; T= $T_w(x)$

y = ∞; u = 0; T = T_∞; ……………………..(4)

The continuity equation (1) is satisfied by introducing the stream function ψ(x, y), such that

$$u=\frac{\partial\psi}{\partial y}, \qquad v=-\frac{\partial\psi}{\partial x} \quad \text{……………………….…………………}(5)$$

The momentum and energy equations (2)and (3) can be transformed to the corresponding ordinary differential equations by introducing the following similarity transformations (Blasius (1908)):

$$\psi=\sqrt{Ux\upsilon_\infty}\; f(\eta) \quad \text{……………………………………………………..}(6)$$

$$\eta=y\sqrt{\frac{U}{\upsilon_\infty x}} \quad \text{………………………………………………………..}(7)$$

$$\theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty}$$ ……………….……………………………………..(8)

where υ_∞ is a reference kinematic viscosity.

It will be assumed that the temperature difference between the moving surface and the free stream varies as Ax^n.

i.e.$T_W(x)-T_\infty=Ax^n$……………….. ……………….………………….(9)

where A is a constant, n is exponent, and x is measured from the leading edge of the surface.

The momentum and energy equations (2) – (3) after some simplifications, reduce to the following forms:

$$f''' - \left(\frac{1}{\theta-\theta_r}\right)\theta' f'' - \left(\frac{\theta-\theta_r}{2\theta_r}\right) f f'' + \mathrm{Re}_m{}^2 \left(\frac{\theta-\theta_r}{\theta_r}\right) f' = 0$$ …………….(10)

$$\theta'' - n \Pr f' \theta + \frac{\Pr}{2} f \theta' - \Pr Ec \left(\frac{\theta_r}{\theta-\theta_r}\right) f''^2 + \mathrm{Re}_m{}^2 \Pr Ecf'^2 = 0$$ …………...(11)

The corresponding boundary conditions are

$\eta = 0$; $f = 0$; $f' = 1$; $\theta = 1$

$\eta = \infty$; $f' = 0$; $\theta = 0$ ………………………………..(12)

Where the prime (′) denotes differentiation with respect to η and

$$\mathrm{Re}_m = B_0 \sqrt{\frac{\sigma_e x}{\rho U}}$$ (Magnetic parameter)

$$Ec = \frac{U^2}{c_p (T_w - T_\infty)}$$

(Eckert number)……………………………………..(13)

and $\theta_r = \frac{T_r - T_\infty}{T_w - T_\infty}$ (Dimensionless reference temperature, constant)

It is important to note that θ_r is negative for liquids (Pr > 1.0) and positive for gases (Pr < 1.0).
The equations (10) and (11) constitute a non-linear coupled boundary value problem prescribed at two boundaries, the analytical solution of which is not feasible. Therefore, these equations have been solved numerically on computer using Newton's shooting techniques with the Runge-kutta Gill method with a step size of 0.01. The corresponding velocity and temperature profiles are shown in fig.2 to fig.9.

RESULTS AND DISCUSSION

The profiles for velocity and temperature are shown in fig. 2 to fig.9. It is seen from fig.2 that the velocity decreases as n and Re_m, the power-law index of the surface temperature variation (exponent) and the magnetic parameter increases with Prandtl number Pr = 0.02 and the variable viscosity parameter $\theta_r = 2$ respectively. From fig.3 and fig. 4, it is understood that the velocity profiles are almost identical for different values of n and Re_m. From fig.5 the velocity decreases as Magnetic parameter Re_m increases.

Figs. 6,7, 8 and 9 show the temperature profiles for different values of n and Re_m. It is seen from fig.6 that as n increases the temperature decreases for fixed value of Re_m. Fig.7 and fig.8 reveal that the temperature decreases as n and Re_m increases. Fig. 9 shows that the temperature decreases as the magnetic parameter increases with Pr=0.02, n =0.3 and Ec =0.

Fig. 10 gives the values of $-\theta'(0)$, the heat transfer rate for various values of n and Re_m. From the figure it is observed that as Re_m increases the heat transfer rate $-\theta'(0)$ decreases but as n increases the heat transfer rate increases. So the parameters n and Re_m have considerable influence on the heat transfer rate $-\theta'(0)$.

CONCLUSION

The effects of power law index of the surface temperature variation (exponent), magnetic parameter and variable parameter on a hydromagnetic flow and heat transfer on a continuously moving surface have been studied numerically using the RK Gill method together with the shooting technique. From the previous results and discussion, we conclude the following

(1) The velocity decreases with the increase of power law index of the surface temperature variation (exponent) and the magnetic parameter
(2) The temperature decreases with the increase of the power law index of the surface temperature variation (exponent) and the magnetic parameter .
(3) The heat transfer rate increases rapidly with the increase of power law index of the surface temperature variation (exponent) whereas when the magnetic parameter increases the heat transfer rate decreases.

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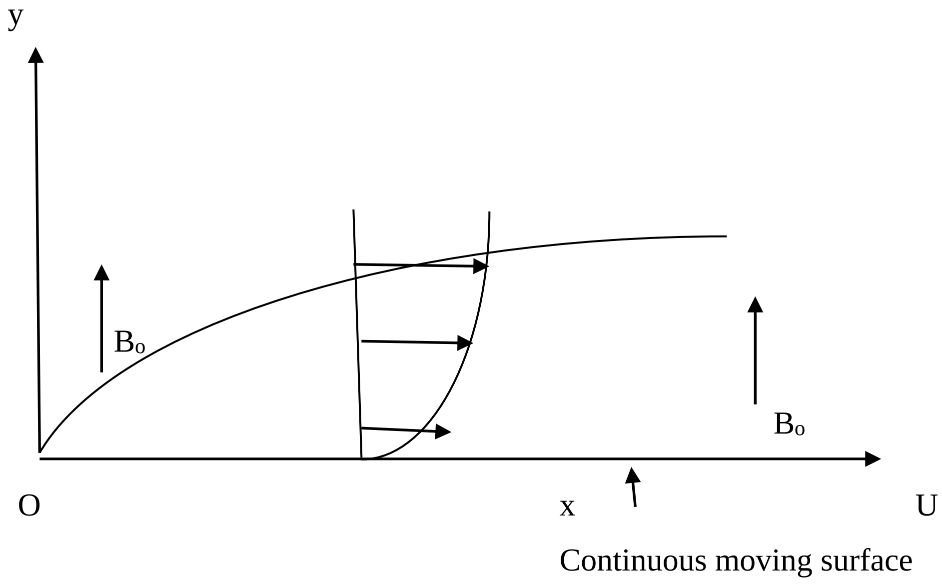

Fig.1. Coordinate system for continuously moving surface

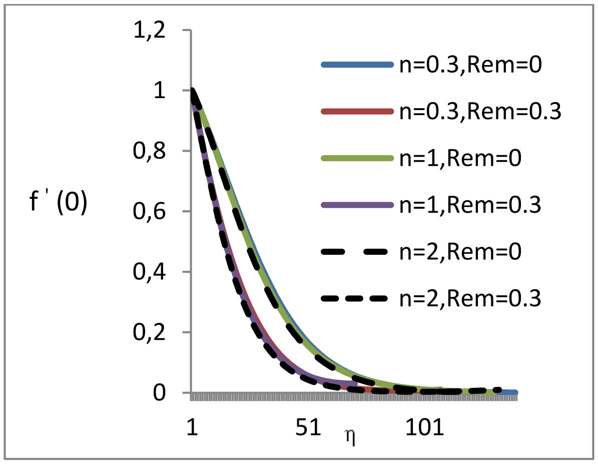

Fig.2 velocity profile against η for different values of n and Re_m with Pr=0.02 and θ_r = 2.0

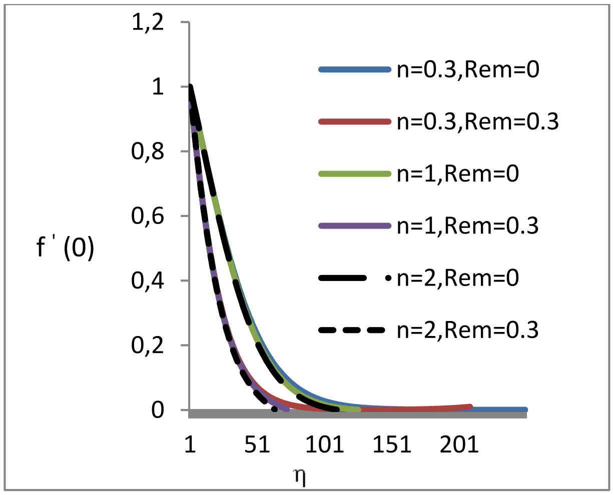

Fig.3 velocity profile against η for different values of n and Re_m with Pr=0.02, Ec= 0 and θ_r =2.0

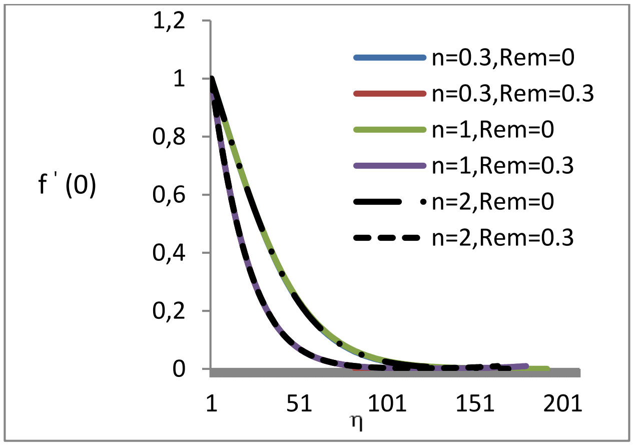

Fig.4 velocity profile against η for different values of n and Re_m with Pr=0.02,Ec=0.5 & θ_r =2.0

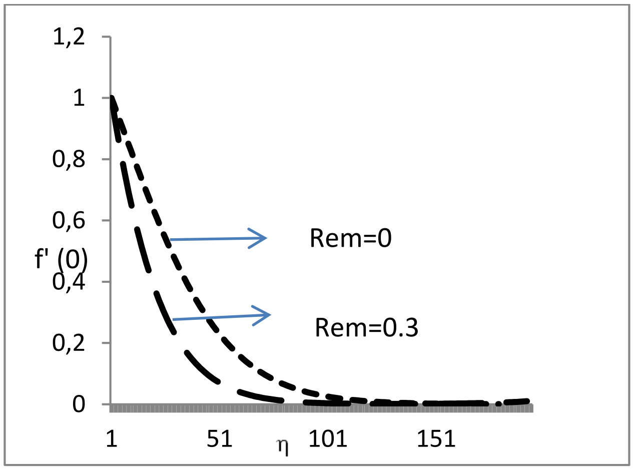

Fig.5 velocity profile against η for different values of n and Re_m with Pr=0.02 , n =0.3 & Ec =0.0

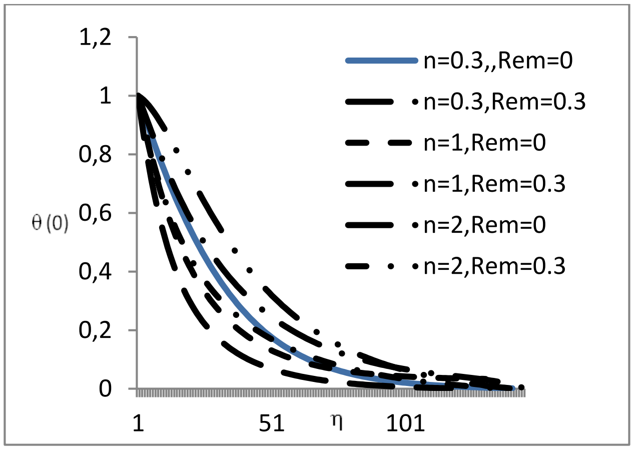

Fig.6 Temperature profile against η for different values of n and Re_m with Pr=0.02 and $\theta_r = 2.0$

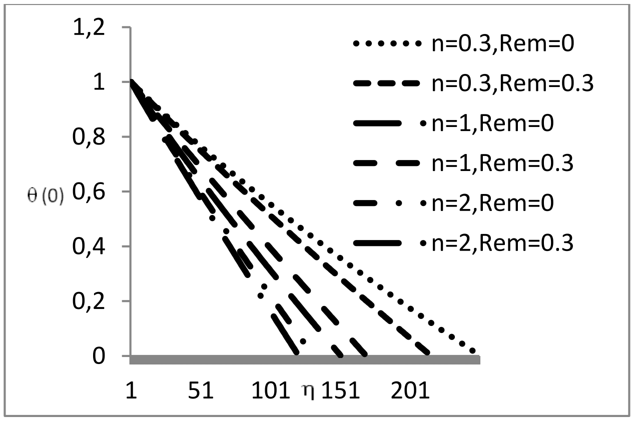

Fig.7 Temperature profile against η for different values of n & Re_m with Pr=0.02 ,Ec = 0.0 & $\theta_r = 2.0$

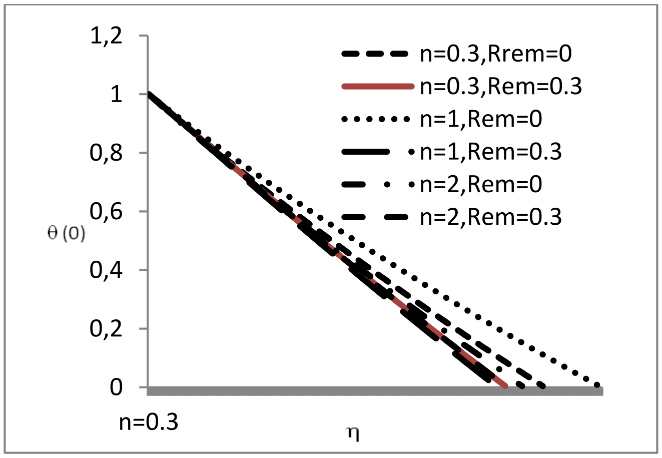

Fig.. 8 Temperature profile against η for different values of n and Re_m with Pr=0.02, Ec= 0.5& $\theta_r = 2.0$

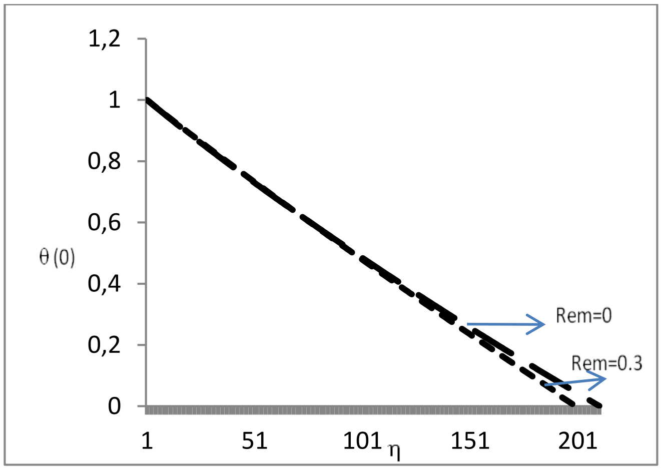

Fig. 9 Temperature profile against η for different values of n & Re_m with Pr=0.02,n =0.3&Ec = 0

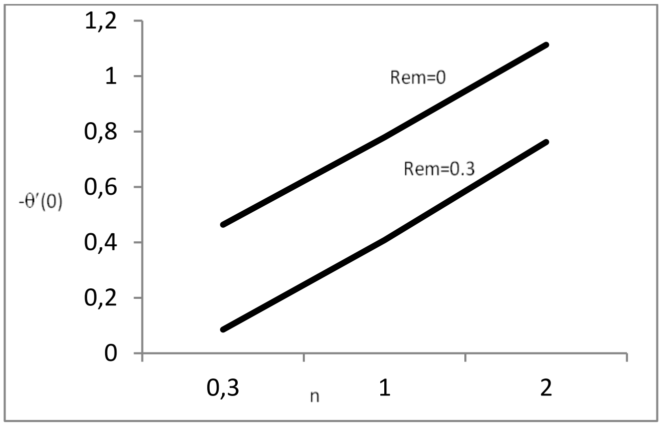

Fig. 10 Heat transfer rate

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