Moroccan Labour Market Dynamics: 
A Markov Chain BFGS Approach

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Abstract

This study uses data from the 2006 Follow-up of Moroccan Vocational Training Graduates – Class of 2002 – to study the labour market transitions between employment, unemployment, and inactivity using a Markov chain BFGS approach. We analyze the way specific characteristics influence labour market transitions. We estimate a multivariate logit three-state model and a labour market transition matrix using Markov Chain BFGS technique. Results show that women are more prone than men to move out of labour force. Single graduates are less likely to move to employment than married graduates. The longer a graduate has been unemployed, the less likely he is to find a job. Furthermore graduates from secondary sector show a high probability to move out of the labor market compared to graduates from primary sector. However, graduates from tertiary sector are more prone to move to employment.

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1 Introduction

Modeling labour market dynamics using Markov chains has received a great deal of attention in several studies. Flinn and Heckman (1982) use Markov chain methods to study labor market structure. Eckstein and van den Berg (2007) present an up-to-date survey of empirical labor market search. Alvarez, Ciocchini and Konwar (2008) use discrete Markov chain method to analyze the dynamics of labour market in Argentina. Bosch and Maloney (2010) estimate a continuous time Markov processes on panel data from Argentina, Brazil and Mexi-
co. In a more recent study, Tansel and Kan (2012) examine the mobility in the Turkish labor market.

In this study, we propose a Markov chain approach allowing the estimation of the transition probabilities given specific characteristics. The Markov chain approach does not require an extensive set of assumptions regarding the distribution, homoscedasticity and serial correlation properties of the time series.

This study uses data from the Follow-up of Moroccan Graduates – Class of 2002 – survey. For each graduate, a retrospective time calendar records the labour market status in each quarter during the four years following graduation, for a total of 16 quarters of observation. In order to examine the nature of labor mobility patterns, we estimate six logit models individually for each labor market transition by adopting a number of characteristics as explanatory variables (Alvarez et al., 2008; Tansel and Kan, 2012).

The remainder of the paper proceeds as follows. In the next section, we describe the methodology of Markov transition analysis. Results of Markov transition analysis are presented in Sections 3. Finally, Section 5 provides the concluding remarks.

2 Methodology

Let \( i \in \Omega = \{E, CH, O\} \) where E, CH, O denote respectively employment, unemployment and inactivity. We assume a stochastic movement over time from one state to another. The graduate’s state location is generated by a stationary time-homogenous Markov process. The Markov chain process is characterized by a transition matrix \( \theta \).

\[
\theta = \begin{bmatrix}
\theta_{EE} & \theta_{ECH} & \theta_{EO} \\
\theta_{CHE} & \theta_{CH} & \theta_{CHO} \\
\theta_{OCH} & \theta_{OCH} & \theta_{OO}
\end{bmatrix}
\]

The stationary vector \( \pi = (\pi_i)_{i \in \Omega} \) satisfies the usual conditions \( \pi = \pi \theta \) and \( \sum_{i=1}^{3} \pi_i = 1 \) and is given by:

\[
\pi_E = \frac{\text{Num}_E(\theta)}{\text{Den}(\theta)} = \frac{\theta_{OE} \theta_{ECH} + \theta_{ECH} \theta_{OCH} + \theta_{OE} \theta_{CHO}}{\text{Den}(\theta)}
\]

\[
\pi_{CH} = \frac{\text{Num}_{CH}(\theta)}{\text{Den}(\theta)} = \frac{\theta_{OE} \theta_{ECH} + \theta_{ECH} \theta_{OCH} + \theta_{EO} \theta_{OCH}}{\text{Den}(\theta)}
\]

\[
\pi_O = \frac{\text{Num}_O(\theta)}{\text{Den}(\theta)} = \frac{\theta_{ECH} \theta_{CHO} + \theta_{EO} \theta_{CHE} + \theta_{EO} \theta_{CHO}}{\text{Den}(\theta)}
\]

When \( \text{Den}(\theta) = \text{Num}_E(\theta) + \text{Num}_{CH}(\theta) + \text{Num}_O(\theta) \)
The transition probabilities are parameterized as a function of individuals' characteristics and labour market conditions using a multivariate logistic link $Y := Y(\theta) :$

$$Y_{\text{ECH}} := \ln\left(\frac{\theta_{\text{ECH}}}{\theta_{\text{EE}}}\right)$$

$$Y_{\text{EO}} := \ln\left(\frac{\theta_{\text{EO}}}{\theta_{\text{EE}}}\right)$$

$$Y_{\text{CHO}} := \ln\left(\frac{\theta_{\text{CHO}}}{\theta_{\text{CCH}}}\right)$$

$$Y_{\text{OCH}} := \ln\left(\frac{\theta_{\text{OCH}}}{\theta_{\text{OO}}}\right)$$

where $Y_{ij} = \beta_{ij}^{(0)} + \sum \beta_{ij}^{(m)} X_m \quad i, j \in \Omega = \{E, CH, O\}$

To deduce transition matrix elements we use the following inverse mapping:

$$\theta_{\text{EE}} = \frac{1}{1 + e^{Y_{\text{ECH}} + e^{Y_{\text{EO}}}}}$$

$$\theta_{\text{ECH}} = \frac{e^{Y_{\text{ECH}}}}{1 + e^{Y_{\text{ECH}} + e^{Y_{\text{EO}}}}}$$

$$\theta_{\text{EO}} = \frac{e^{Y_{\text{EO}}}}{1 + e^{Y_{\text{EO}} + e^{Y_{\text{OCH}}}}}$$

$$\theta_{\text{CHO}} = \frac{e^{Y_{\text{CHO}}}}{1 + e^{Y_{\text{CHO}} + e^{Y_{\text{OCH}}}}}$$

$$\theta_{\text{OCH}} = \frac{e^{Y_{\text{OCH}}}}{1 + e^{Y_{\text{OCH}} + e^{Y_{\text{OEO}}}}}$$

$$\theta_{\text{OO}} = \frac{1}{1 + e^{Y_{\text{EO}} + e^{Y_{\text{OCH}}}}}$$

### Likelihood maximization

The individuals' contributions to the likelihood function represent the product of the probability to occupy the initial state and the transition probabilities corresponding to the individual's history (Ameyma, 1985)

We estimate the parameters by maximizing the likelihood function in two steps: A first step consists in initializing the parameters $\beta$ and a second phase reserved to calculate the parameters using an iterative method.

**Initialization:** We estimate the transition probabilities non-parametrically by calculating the ratio of all transitions from $i$ to $j$ to the number of transitions which start in $i$:

$$\hat{\theta}_{ij}^{(0)} = \# (i \rightarrow j) / (#i)$$

After, we set the initial estimator to be: $\hat{\beta}_{ij}^{(0)} = \ln\left(\frac{\hat{\theta}_{ij}^{(0)}}{\hat{\theta}_{ii}^{(0)}}\right)$. The covariates' coefficients are set to 0: $\beta_{ij} = 0$
Updates: Among a panoply of nonlinear optimization methods, we use a modified quasi-Newton method, called the Broyden-Fletcher-Goldfarb-Shanno algorithm, to calculate the parameters $\beta$. This method is described in detail by Press et al (2002).

3 Results

We report in table 1, maximum likelihood estimators and their standard errors. It is noteworthy that all covariates are significant for all transitions excepted tertiary sector for transition from inactivity to unemployment. Results show that women are more prone that men to move out of labour force.

Table 1: Maximum likelihood estimators:

<table>
<thead>
<tr>
<th></th>
<th>ECH</th>
<th>EO</th>
<th>CHE</th>
<th>CHO</th>
<th>OE</th>
<th>OCH</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>-2.164</td>
<td>-3.895</td>
<td>-1.371</td>
<td>-1.826</td>
<td>-1.035</td>
<td>-0.782</td>
</tr>
<tr>
<td>Gender (Ref.=Male)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Female</td>
<td>0.379</td>
<td>0.738</td>
<td>0.357</td>
<td>0.413</td>
<td>-0.227</td>
<td>-0.243</td>
</tr>
<tr>
<td>Marital status (Ref.=Married)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Single</td>
<td>0.723</td>
<td>1.370</td>
<td>-0.649</td>
<td>0.794</td>
<td>-0.450</td>
<td>0.433</td>
</tr>
<tr>
<td>Unemployment duration by unemployment episode</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.492</td>
<td>0.897</td>
<td>-0.429</td>
<td>0.579</td>
<td>-0.355</td>
<td>0.282</td>
</tr>
<tr>
<td></td>
<td>4.801</td>
<td>6.893</td>
<td>4.275</td>
<td>5.375</td>
<td>3.472</td>
<td>2.882</td>
</tr>
<tr>
<td>Training sector (Ref.=Primary sector)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Secondary sector</td>
<td>0.013</td>
<td>0.017</td>
<td>0.007</td>
<td>0.021</td>
<td>-0.011</td>
<td>-0.005</td>
</tr>
<tr>
<td>Tertiary sector</td>
<td>0.004</td>
<td>-0.004</td>
<td>0.014</td>
<td>-0.017</td>
<td>0.007</td>
<td>-0.002</td>
</tr>
<tr>
<td></td>
<td>2.788</td>
<td>2.373</td>
<td>-9.558</td>
<td>-12.204</td>
<td>-5.194</td>
<td>*-1.446</td>
</tr>
</tbody>
</table>

Note: All coefficients are significant at 5% confidence level except for coefficient or transition from inactivity to unemployment.

Single graduates are less likely to move to employment than married graduates. Unemployment duration by unemployment episode reflects the relationship between past patterns of unemployment and labour market transitions. Estimates show that the longer a graduate has been unemployed, the less likely he is to find a job. Furthermore graduates from secondary sector show a high probability to move out of labour force compared to graduates from primary sector. However, graduates from tertiary sector are more prone to move to employment.
\[
\theta = \begin{bmatrix}
0.42 & 0.28 & 0.30 \\
0.33 & 0.37 & 0.30 \\
0.32 & 0.32 & 0.36
\end{bmatrix}
\]

**Activity rate** \( A = \pi_E + \pi_{CH} \) measures the proportion of unemployed and employed graduates.

**Unemployment rate**: \( CH = \pi_{CH}/(\pi_{CH} + \pi_E) \) measures the proportion of unemployed graduates.

**Inertia**: \( I = 1/3 \text{ trace } (\theta) \) measures the proclivity of the current employment state to perpetuate in time.

**Give-up rate**: \( Ab = \theta_{CHO}/(\theta_{CH} + \theta_{CHO}) \). The probability for the unemployed that a transition to inactivity occurs before a transition to employment.

**Net Outflow**: \( F_S = \pi_E\theta_{EO} - \pi_O\theta_{OE} \). The difference between the proportion of the graduates who leave employment for inactivity and inactive graduates who succeed to get a job.

**Reliability**: \( R = \theta_{EO}/(1 - \theta_{EE}) \). The probability that a graduate move from inactivity to employment before being unemployed.

**Table 2. Labour market indicators:**

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>CH</th>
<th>I</th>
<th>Ab</th>
<th>F_S</th>
<th>R</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.62</td>
<td>0.48</td>
<td>0.37</td>
<td>0.48</td>
<td>0.06</td>
<td>0.5</td>
</tr>
</tbody>
</table>

We report in Table 2. labour market indicators that indicate a high level of activity rate that is about 62%. estimations indicate that unemployment is about 48%. Inertia is about 37% whereas the job reliability is about 50%.

We note that the net Outflow is around 6% and the give-up rate is about 48% . These very low probabilities reflect high persistence in inactivity, which indicate that the high level of inactivity seems to dominate Moroccan vocational training graduates' mobility patterns.
4 Conclusion

This paper characterizes labour market transitions as a recurring Markov chain. It attempts to examine the labour market dynamics in the case of Moroccan vocational training graduates. Our main results show that women are more prone than men to move out of labour force. Single graduates are less likely to move to employment than married graduates. Unemployment duration by unemployment episode reflects the relationship between past patterns of unemployment and labour market transitions. Estimates show that the longer a graduate has been unemployed, the less likely he is to find a job. Furthermore graduates from secondary sector show a high probability to move out of labour force compared to graduates from primary sector. However, graduates from tertiary sector are more prone to move into employment.

Having computed the matrix of transition probabilities, we conclude that the most discernible transition pattern can be observed along the diagonal of the probability matrix, specially the transition from employment to employment. Hence, we can conclude that the high levels of probabilities along the diagonal of the probability matrix imply that most of graduates majority don’t leave initial labour market state.

References


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