Notes on Soft Perfect Sets

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Abstract

The notion of soft sets is introduced as a general mathematical tool for dealing with uncertainty. In this work, we consider the concepts of $\mathcal{G}\mathcal{L}$-soft perfect sets, $\mathcal{G}\mathcal{R}$-soft perfect sets and $\mathcal{G}\bigtriangleup$-soft perfect sets were introduced in the soft topological space $(X, A, \tau)$ with a soft $\mathcal{G}$ which are extensions of the concepts soft $\tau_{\mathcal{G}}$-closed, soft $\tau_{\mathcal{G}}$-dense in itself and soft $\tau_{\mathcal{G}}$-perfect, respectively. Also, we studied a characterization for suitable condition between the soft topology $\tau$ and the soft $\mathcal{G}$ utilizing $\mathcal{G}\mathcal{R}$-soft perfect sets. On a finite soft set a new generalized finite soft topology via $\mathcal{G}\mathcal{R}$-soft perfect sets, which is finer than soft topology $\tau_{\mathcal{G}}$ is obtained.

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1 Introduction

Soft set was introduced by Molodtsov [12] as a mathematical approach to vagueness. The notion of topological space for soft sets was formulated by Shabir et. al. [16]. Of late many authors have studied various properties of soft topological spaces [1, 2, 3, 5, 7, 11, 13, 14, 15, 18].

The current work is aimed to introduce soft grills. The collections of $\mathcal{G}\mathcal{L}$-soft perfect, $\mathcal{G}\mathcal{R}$-soft perfect and $\mathcal{G}\bigtriangleup$-soft perfect sets in soft topological spaces
with soft grill will be presented and some properties of them are investigated.

A characterization for a suitable soft grill via $GR$-soft perfect sets will be studied. Also, a generalized soft topology via $GR$-soft perfect that is finer than soft topology $\tau_G$ on $(X, A)$ where $X$ is a finite set will be obtained.

2 Preliminaries

In this section, we present the basic well-known definitions and results of the soft set theory that are useful for subsequent discussions. These definitions and more detailed explanations related to the soft sets and their properties can be found in [2, 3, 5, 7, 8, 11, 12, 13, 14, 15, 16, 18]. Throughout this work, $X$ refers to an initial universe set, $P(X)$ is the power set of $X$ (that is the set of all subsets of $X$) and $A$ is a set of parameters.

**Definition 2.1** $F_A$ is said to be soft set with support $A$ over the universe $X$, if $F$ is a mapping given by, $F_A : A \rightarrow \mathcal{P}(X)$.

In what follows by $SS(X, A)$, we denote the family of all soft sets over $X$.

**Definition 2.2** A soft set $F_A$ over $X$ is said to be:

$(i)$ A null soft set, denoted by $\varnothing_A$, if $e \in A$, $F(e) = \varnothing$.

$(ii)$ An absolute soft set, denoted by $X_A$, if $e \in A$, $F(e) = X$.

In particular $(X, A)$ will be denoted by $X_A$.

**Definition 2.3** Let $F_A, H_A \in SS(X, A)$. Then,

$(i)$ $F_A$ is called a soft subset of $H_A$, denoted by $F_A \subseteq H_A$, if for all $e \in A$, $F(e) \subseteq H(e)$.

$(ii)$ $F_A$ and $H_A$ are called soft equal, denoted by $F_A = H_A$, if $F_A \subseteq H_A$ and $H_A \subseteq F_A$.

**Definition 2.4** Let $F_A, H_A \in SS(X, A)$. Then,

$(i)$ The soft union $K_A$ of two soft sets $F_A$ and $H_A$ over $X$, denoted by $F_A \sqcup H_A$, is defined as $K(e) = F(e) \cup H(e) \forall e \in A$.

$(ii)$ The soft intersection $K_A$ of two soft sets $F_A$ and $H_A$ over $X$, denoted by $F_A \cap H_A$, is defined as $K(e) = F(e) \cap H(e) \forall e \in A$.

$(iii)$ The soft difference $K_A$ of two soft sets $F_A$ and $H_A$ over $X$, denoted by $F_A - H_A$, is defined as $K(e) = F(e) - H(e) \forall e \in A$.

$(iv)$ The soft complement of soft set $F_A$, denoted by $(F_A)^c = X_A - F_A$, is defined by $(F^c)(e) = X - F(e) \forall e \in A$.

$(v)$ The soft symmetric difference $K_A$ of two soft sets $F_A$ and $H_A$, denoted by $F_A \Delta H_A$, is defined $F_A \Delta H_A \Delta (H_A \Delta F_A)$. Equivalently, $(F_A \cup H_A) \setminus (F_A \cap H_A)$. 

Definition 2.5 Let $F_A$ be a soft set over $X$ and $x \in X$. Then,
(i) $x \in F_A$, if $x = x(e) \in F(e)$ for all $e \in A$.
(ii) $x \notin F_A$, if $x \notin F(e)$ for some $e \in A$.

Definition 2.6 Let $x \in X$ and $\alpha \in A$, then a soft point $x_\alpha$ denotes the soft set over $X$ for which $x(\alpha) = \{x\}$.

Lemma 2.7 $(SS(X, A), \sqcup, \sqcap, c)$ is a De-Morgan algebra.

Lemma 2.8 Every ordinary set is a soft set itself.

In the following, we give some basic results of soft topological spaces.

Definition 2.9 Let $\tau$ be the collection of soft sets over $X$, then $\tau$ is said to be a soft topology on $X$, if it satisfies the following axioms
(i) $\emptyset_A, X_A$ belong to $\tau$.
(ii) The soft union of any number of soft sets in $\tau$ belongs to $\tau$.
(iii) The soft intersection of any two soft sets in $\tau$ belongs to $\tau$.

The triplet $(X, A, \tau)$ is said to be a soft topological space or soft space. Every member of $\tau$ is called a soft open set. A soft set $H_A$ is called soft closed in $(X, A, \tau)$, if $(H_A)^c$ is soft open set. The family of soft closed sets is denoted by $\tau^c$.

Definition 2.10 Let $(X, A, \tau)$ be a soft topological space. A sub-collection $\beta \subseteq \tau$ is said to be a soft base for $\tau$, if every soft open set can be expressed as a soft union of members of $\beta$.

Definition 2.11 A soft set $F_A$ of a soft topological space $(X, A, \tau)$ is said to be a soft neighborhood (abbreviated as a soft nbd.) of the soft set $H_A$ if there exists a soft open set $K_A$ such that $H_A \subseteq K_A \subseteq F_A$. If $H_A = x_\alpha$, then $F_A$ is said to be a soft nbd. of the soft point $x_\alpha$. The soft neighborhood system of a soft point $x_\alpha$, denoted by $N(x_\alpha)$, is the family of all its soft neighborhoods.

Definition 2.12 Let $(X, A, \tau)$ be a soft topological space, $x \in X$ and $\alpha \in A$. A soft point $x_\alpha$ is said to be a closure point of a soft set $F_A$, if for every soft open neighborhood of $x_\alpha$ intersects $F_A$. The set of all closure points of $F_A$ is denoted by $\text{Cl } F_A$.

Definition 2.13 Let $(X, A, \tau_1)$ and $(X, A, \tau_2)$ be two soft topological spaces over the same universe $X$. Then, $\tau_2$ is said to be soft finer than $\tau_1$, if $\tau_1 \subseteq \tau_2$.

Definition 2.14 A non-empty collection $G \subseteq SS(X, A)$ of soft sets over $X$ is called a soft grill, if the following conditions hold:
(i) If $F_A \in G$ and $F_A \subseteq H_A$, then $H_A \in G$.
(ii) If $F_A \sqcup H_A \in G$, then $F_A \in G$ or $H_A \in G$. 
Definition 2.15 Let \( G \) be a soft grill over a soft topological space \((X, A, \tau)\). Consider the soft operator \( \varphi_G: SS(X, A) \rightarrow SS(X, A) \), given by \( \varphi_G(F_A) = \bigcup \{ x_a \mid U_a \cap F_A \in G \} \) for every soft set \( F_A \). Then, the soft operator \( \psi_G: SS(X, A) \rightarrow SS(X, A) \), defined by for every soft set \( F_A, \psi_G(F_A) = F_A \cup \varphi_G(F_A) \) is a kuratowski's closure operator and hence gives rise to a new soft topology \( \tau_G = \{ H_A \mid \psi_G(\bar{X}_A - H_A) = (\bar{X}_A - H_A) \} \) on \((X, A)\) which is a soft finer than \( \tau \) in general. A soft open base \( \beta(G, \tau) \) for the soft topology \( \tau_G \) on \((X, A)\) is given by \( \beta(G, \tau) = \{ (V_A - F_A) \mid V_A \in \tau, F_A \notin G \} \) and \( \tau \subseteq \beta(G, \tau) \subseteq \tau_G \).

Lemma 2.16 Let \( G \) be a soft grill over a soft topological space \((X, A, \tau)\). Then, for every \( F_A \in SS(X, A) \) the following statements hold

(i) If \( F_A \notin G \), then \( \varphi_G(F_A) = \emptyset_A \). Moreover, \( \varphi_G(\emptyset_A) = \emptyset_A \).

(ii) \( \varphi_G(F_A) \subseteq \varphi_G(H_A) \).

(iii) \( \varphi_G(F_A \cup H_A) = \varphi_G(F_A) \cup \varphi_G(H_A) \).

(iv) \( \varphi_G(F_A \cap H_A) \subseteq \varphi_G(F_A) \cap \varphi_G(H_A) \).

(vi) \( \psi_G(F_A) \subseteq F_A \).

(vii) \( F_A \) is a soft closed set if and only if \( \varphi_G(F_A) \subseteq F_A \).

Lemma 2.17 Let \( G \) be a soft grill over a soft topological space \((X, A, \tau)\). Then, for every \( F_A, H_A \in SS(X, A) \) the following statements hold

(i) \( F_A \subseteq H_A \) implies \( \varphi_G(F_A) \subseteq \varphi_G(H_A) \).

(ii) \( \varphi_G(F_A \cup H_A) = \varphi_G(F_A) \cup \varphi_G(H_A) \).

(iii) \( \varphi_G(F_A \cap H_A) \subseteq \varphi_G(F_A) \cap \varphi_G(H_A) \).

(iv) \( \varphi_G(F_A) - \varphi_G(H_A) = \varphi_G(F_A - H_A) - \varphi_G(H_A) \).

(v) \( \psi_G(F_A) \subseteq F_A \).

(vi) \( \psi_G(F_A) \subseteq F_A \).

Lemma 2.18 Let \((X, A, \tau)\) be a soft topological space, \( F_A \in SS(X, A) \) and \( G_1, G_2 \) be two soft grills over \( X \). Then, \( \varphi_G(F_A) \subseteq \varphi_G(F_A) \) if \( G_1 \subseteq G_2 \).

Definition 2.19 Let \((X, A, \tau)\) be a soft topological space and \( F_A \in SS(X, A) \). Then, \( [F_A] = \{ U_A \mid F_A \cap U_A \neq \emptyset_A \} \) is a soft grill. We call this grill the soft principal generated by a soft set \( F_A \).

It is worth to mention that, in the current paper the following formulas will be used, for every \( F_A \in SS(X, A) \), \( F_A = F_A - \varphi_G(F_A) \) and \( F_A = \varphi_G(F_A) - F_A \).

Definition 2.20 Let \( G \) be a soft grill over a soft topological space \((X, A, \tau)\). Then, a soft topology \( \tau \) is said to be suitable for a soft \( G \), if for all \( F_A \in SS(X, A) \), \( F_A \notin G \).

Some equivalent descriptions of the suitability of soft topology with a soft grill will show in the next Theorem.
**Theorem 2.21** Let $\mathcal{G}$ be a soft grill over a soft topological space $(X, A, \tau)$, then the following statements are equivalent

(i) A soft topology $\tau$ is suitable for a soft $\mathcal{G}$.
(ii) For any soft $\tau_\mathcal{G}$-closed set $F_A$, $\overline{F_A} \notin \mathcal{G}$.
(iii) For any soft set $F_A$ and each $x_\alpha \in F_A$ there exist soft open nbd. $U_A$ of $x_\alpha$ with $U_A \cap F_A \notin \mathcal{G}$, it follows that $F_A \notin \mathcal{G}$.
(iv) If $F_A$ is a soft set and $F_A \cap \varphi_\mathcal{G}(F_A) = \emptyset_A$, then $F_A \notin \mathcal{G}$.

## 3 Main Results

In this section, we introduce and study some of properties of the collections of soft subsets $\mathcal{GL}$, $\mathcal{GR}$ and $\mathcal{G}\Delta$-soft perfect sets in a soft topological space $(X, A, \tau)$.

**Definition 3.1** Let $\mathcal{G}$ be a soft grill over a soft topological space $(X, A, \tau)$.

A soft set $F_A$ is said to be:

(i) Soft $\tau_\mathcal{G}$-dense in itself, if $F_A \subseteq \varphi_\mathcal{G}(F_A)$.
(ii) Soft $\mathcal{G}$-dense, if $\varphi_\mathcal{G}(F_A) = \overline{X_A}$.
(iii) Soft $\mathcal{G}$-perfect, if $\varphi_\mathcal{G}(F_A) = F_A$.

**Definition 3.2** Let $\mathcal{G}$ be a soft grill on a soft topological space $(X, A, \tau)$.

A soft set $F_A$ is said to be:

(i) $\mathcal{GL}$-soft perfect, if $\overline{F_A} \notin \mathcal{G}$.
(ii) $\mathcal{GR}$-soft perfect, if $\hat{F_A} \notin \mathcal{G}$.
(iii) $\mathcal{G}\Delta$-soft perfect, if $(F_A \Delta \varphi_\mathcal{G}(F_A)) \notin \mathcal{G}$.

The collection of $\mathcal{GL}$ (resp., $\mathcal{GR}$ and $\mathcal{G}\Delta$)-soft perfect soft sets in $(X, A, \tau)$ are denoted by $\mathcal{GL}$-$\text{PSS}(X, A)$ (resp., $\mathcal{GR}$-$\text{PSS}(X, A)$ and $\mathcal{G}\Delta$-$\text{PSS}(X, A)$).

**Corollary 3.3** A soft set $F_A$ is both $\mathcal{GL}$-soft perfect and $\mathcal{GR}$-soft perfect in $(X, A, \tau)$ if and only if it is $\mathcal{G}\Delta$-soft perfect set.

**Lemma 3.4** Let $(X, A, \tau)$ be a soft topological space with $\mathcal{G} = \mathcal{P}(X)$ - $\{\emptyset_A\}$ and $F_A \in \mathcal{SS}(X, A)$. Then,

(i) $F_A$ is soft $\mathcal{G}$-perfect set and so $\mathcal{GL}$-$\text{PSS}(X, A) = \mathcal{GR}$-$\text{PSS}(X, A) = \mathcal{G}\Delta$-$\text{PSS}(X, A)$.
(ii) Every $\mathcal{GL}$ (resp., $\mathcal{GR}$, $\mathcal{G}\Delta$)-soft perfect sets is a soft $\tau_\mathcal{G}$-dense in itself (resp., soft $\tau_\mathcal{G}$-closed, soft $\tau_\mathcal{G}$-perfect) sets.

**Proof** (i) Obvious.

(ii) Follows directly from the Definitions 3.1, 3.2 and (vi) of Lemma 2.16.
Lemma 3.5 Let \((X, A, \tau)\) be a soft topological space and \(F_A \in \mathcal{SS}(X, A)\) with \(G = \{F_A\}\), then \(\varphi[F_A](F_A) = Cl_F A\) and \(GR - PSS(X, A) = G\Delta - PSS(X, A) = SS(X, A)\).

Proof Obvious from the Definition 2.12.

Lemma 3.6 Let \(G\) be a soft grill on a soft topological space \((X, A, \tau)\). Then, the following statements hold
(i) Every soft \(G\)-dense in itself set is \(GL\)-soft perfect set.
(ii) Every soft \(\tau\)-closed set is \(GR\)-soft perfect set.
(iii) Every soft \(\tau\)-closed set is \(GR\)-soft perfect set.
(iv) If a soft set \(F_A \notin G\), then \(F_A\) is \(GL\)-soft perfect set and \(G\Delta\)-soft perfect set. Moreover, every subset of \(F_A\) is also \(G\Delta\)-soft perfect.

Proof In view of Definitions 3.1, 3.2 and \(\tau \sqsubseteq \tau G\), the Proof follows directly.

Lemma 3.7 Let \(G\) be a soft grill on a soft topological space \((X, A, \tau)\). Then, the following statements hold
(i) \(\emptyset_A\) is \(GL\)-soft perfect, \(GR\)-soft perfect and \(G\Delta\)-soft perfect sets.
(ii) \(X_A\) is \(GR\)-soft perfect set.
(iii) \(\bar{X}_A\) is \(GL\)-soft perfect set, if \(\varphi_G(\bar{X}_A) = \bar{X}_A\).
(iv) \(\varphi_G(F_A), \psi_G(F_A)\) and \(Cl (F_A)\) are \(GR\)-soft perfect set.
(v) Every \(G\)-soft dense set is \(GL\)-soft perfect set.

Proof Obvious by using Lemmas 2.16, 3.6.

Theorem 3.8 Let \(G\) be a soft grill on a soft topological space \((X, A, \tau)\) and a soft set \(F_A \notin G\). Then,
(i) \(\bar{F}_A\) is \(G\Delta\)-soft perfect set.
(ii) \(\bar{F}_A\) is \(G\Delta\)-soft perfect set.
(iii) \((F_A \Delta \varphi_G(F_A))\) is \(G\Delta\)-soft perfect set.

Proof We prove only (i) and the rest of the proofs follows from Lemma 3.6. Since \(F_A \notin G\) and \(\bar{F}_A \subseteq F_A\), then \(\bar{F}_A \notin G\). Hence, by using (i) of Lemma 2.16, \(\varphi_G(\bar{F}_A) = \varphi_G(F_A) = \emptyset_A\). Therefore, \((\bar{F}_A \Delta \varphi_G(\bar{F}_A)) = \bar{F}_A \notin G\). Consequently, \(\bar{F}_A\) is \(G\Delta\)-soft perfect set.

Corollary 3.9 The soft union \(\bar{F}_A \sqcup \bar{F}_A\) is \(G\Delta\)-soft perfect set.

Theorem 3.10 Let \((X, A, \tau)\) be a soft topological space, \(F_A \in \mathcal{SS}(X, A)\) and \(G_1, G_2\) be two soft grills over \(X\) with \(G_1 \subseteq G_2\). Then, \(F_A\) is \(GR\)-soft perfect set with respect to \(G_1\) if it is \(GR\)-soft perfect set with respect to \(G_2\).
**Proof** Since $G_1 \subseteq G_2$ and by using Lemma 2.18, then $\varphi_{G_1}(F_A) \subseteq \varphi_{G_2}(F_A)$. Let $F_A$ be $G\mathcal{R}$-soft perfect set with respect to $G_2$, then $(\varphi_{G_2}(F_A) - F_A) \notin G_2$. Since, $(\varphi_{G_1}(F_A) - F_A) \subseteq (\varphi_{G_2}(F_A) - F_A)$, hence $(\varphi_{G_1}(F_A) - F_A) \notin G_2$. Since $G_1 \subseteq G_2$, thus $(\varphi_{G_1}(F_A) - F_A) \notin G_1$. Consequently, $F_A$ is $G\mathcal{R}$-soft perfect set with respect to $G_1$.

**Lemma 3.11** Let $G$ be a soft grill on a soft topological space $(X, A, \tau)$ and $F_A, H_A$ be soft sets such that $F_A \subseteq H_A \subseteq \varphi_{G}(F_A)$, then $\varphi_{G}(F_A) = \varphi_{G}(H_A)$.

**Proof** Since $F_A \subseteq H_A \subseteq \varphi_{G}(F_A)$, then by using Lemmas 2.16, 2.17 $\varphi_{G}(F_A) \subseteq \varphi_{G}(H_A) \subseteq \varphi_{G}(\varphi_{G}(F_A)) \subseteq \varphi_{G}(F_A)$. Hence, $\varphi_{G}(F_A) = \varphi_{G}(H_A)$.

**Theorem 3.12** Let $G$ be a soft grill on a soft topological space $(X, A, \tau)$ and $F_A, H_A$ be soft sets such that $F_A \subseteq H_A \subseteq \varphi_{G}(F_A)$. Then, the following statements hold

(i) If $H_A$ is $G\mathcal{L}$-soft perfect set, then $F_A$ is $G\mathcal{L}$-soft perfect set.

(ii) If $F_A$ is $G\mathcal{R}$-soft perfect set, then $H_A$ is $G\mathcal{R}$-soft perfect set.

**Proof**
(i) Let $H_A$ be $G\mathcal{L}$-soft perfect set, then $\tilde{H}_A \notin G$. Since, $F_A \subseteq H_A$, then $(F_A - \varphi_{G}(H_A)) \subseteq H_A \notin G$. Hence by using Lemma 3.11 $\varphi_{G}(F_A) = \varphi_{G}(H_A)$ and so $F_A \notin G$. Consequently, $F_A$ is $G\mathcal{L}$-soft perfect set.

(ii) Let $F_A$ be $G\mathcal{R}$-soft perfect set, then $\tilde{F}_A \notin G$. Since $F_A \subseteq H_A$, then $(\varphi_{G}(F_A) - H_A) \subseteq \tilde{F}_A \notin G$. Hence by using Lemma 3.11 $\varphi_{G}(F_A) = \varphi_{G}(H_A)$ and so $H_A \notin G$. Consequently, $H_A$ is $G\mathcal{R}$-soft perfect set.

**Theorem 3.13** Let $G$ be a soft grill on a soft topological space $(X, A, \tau)$ and $F_A, H_A \in \mathbb{SS}(X, A)$. Then,

(i) If $F_A, H_A$ are $G\mathcal{L}$-soft perfect set, then $F_A \cup H_A$ is $G\mathcal{L}$-soft perfect set.

(ii) If $F_A, H_A$ are $G\mathcal{R}$-soft perfect set, then $F_A \cup H_A$ is $G\mathcal{R}$-soft perfect set.

**Proof**
(i) Let $F_A, H_A$ be $G\mathcal{L}$-soft perfect sets. Then, $\tilde{F}_A \notin G$ and $\tilde{H}_A \notin G$. By the definition of soft grill, $\tilde{F}_A \cup \tilde{H}_A \notin G$. Since, $(F_A \cup H_A) - (\varphi_{G}(F_A) \cup \varphi_{G}(H_A)) \subseteq \tilde{F}_A \cup \tilde{H}_A \notin G$. Hence, $F_A \cup H_A$ is $G\mathcal{L}$-soft perfect set.

(ii) Let $F_A, H_A$ be $G\mathcal{R}$-soft perfect sets. Then, $\tilde{F}_A \notin G$ and $\tilde{H}_A \notin G$. By the definition of soft grill, $\tilde{F}_A \cup \tilde{H}_A \notin G$. Since, $(\varphi_{G}(F_A) \cup \varphi_{G}(H_A)) - (F_A \cup H_A) \subseteq \tilde{F}_A \cup \tilde{H}_A$, then $(F_A \cup H_A) - (\varphi_{G}(F_A) \cup \varphi_{G}(H_A)) \subseteq \tilde{F}_A \cup \tilde{H}_A$. Hence, $F_A \cup H_A$ is $G\mathcal{R}$-soft perfect set.

**Theorem 3.14** Let $G$ be a soft grill on a soft topological space $(X, A, \tau)$. If $F_A, H_A$ are $G\Delta$-soft perfect set, then $F_A \cup H_A$ is $G\Delta$-soft perfect set.

**Proof** Follows from Corollary 3.3 and Theorem 3.13.

**Theorem 3.15** Let $G$ be a soft grill on a soft topological space $(X, A, \tau)$. If the soft sets $F_A, H_A$ are $G\mathcal{R}$-soft perfect, then $F_A \cap H_A$ is $G\mathcal{R}$-soft perfect set.
Let $F_A, H_A$ be $GR$-soft perfect sets. Then, $\widehat{F_A} \notin G$ and $\widehat{H_A} \notin G$. By the definition of soft grill, $\widehat{F_A} \sqcup \widehat{H_A} \notin G$. Since, $(\varphi_G(F_A) \cap \varphi_G(H_A)) - (F_A \cap H_A) \subseteq \widehat{F_A} \cap \widehat{H_A} \notin G$. Hence, $(F_A \cap H_A) \notin G$ and so $F_A \cap H_A$ is $GR$-soft perfect set.

Corollary 3.16 Let $G$ be a soft grill on a soft topological space $(X, A, \tau)$ and $F_A, H_A \in SS(X, A)$. Then,
(i) Finite soft union of $GL$-soft perfect sets is an $GL$-soft perfect set.
(ii) Finite soft union of $GR$-soft perfect sets is an $GR$-soft perfect set.
(iii) Finite soft union of $G\Delta$-soft perfect sets is an $G\Delta$-soft perfect set.
(iv) Finite soft intersection of $GR$-soft perfect sets is an $GR$-soft perfect set.

Proof Obvious.

Theorem 3.17 Let $G$ be a soft grill on a soft topological space $(X, A, \tau)$ with $X$ is finite, the collection of soft sets $GR\cdot PSS(X, A) \subseteq SS(X, A)$ is finite soft topology, which is finer than soft topology $\tau_G$.

Proof $GR\cdot PSS(X, A)$ is a soft topology on a finite initial universe set $X$ follows from Lemma 3.7 and Corollary 3.16. A soft topology $\Upsilon$ is finer than a soft topology $\tau_G$ given by Lemma 3.6.

Theorem 3.18 Let $G$ be a soft grill on a soft topological space $(X, A, \tau)$. If the soft topology $\tau$ is suitable for a soft $G$ on $X$, then $GL\cdot PSS(X, A) = SS(X, A)$.

Proof Obvious by Theorem 2.21.

Theorem 3.19 Let $G$ be a soft grill on a soft topological space $(X, A, \tau)$ such that the soft topology $\tau$ is suitable for a soft $G$ and $F_A \in SS(X, A)$. Then,
(i) If $F_A \cap \varphi_G(F_A) = \emptyset$, then $F_A$ is $G\Delta$-soft perfect set.
(ii) $\widehat{F_A}$ is $G\Delta$-soft perfect set.
(iii) $x \in F_A$ there exist soft open nbd $U_A$ of $x$ with $U_A \cap F_A \notin G$, it follows that $F_A$ is $G\Delta$-soft perfect set.

Proof Follows directly from Theorem 2.21 and Lemma 3.6.

Theorem 3.20 Let $G$ be a soft grill on a soft topological space $(X, A, \tau)$ and $F_A \in SS(X, A)$, then the following statements are equivalent
(i) The soft topology $\tau$ is suitable for a soft grill $G$.
(ii) If $F_A$ is $GR$-soft perfect set, then $F_A \Delta \varphi_G(F_A) \notin G$.

Proof (i)$\implies$(ii) Let $F_A$ be $GR$-soft perfect set, then $\widehat{F_A} \notin G$. In view of Theorem 2.21 and (i) of this Theorem $\widehat{F_A} \notin G$ and $\widehat{F_A} \notin G$. Hence, $F_A \Delta \varphi_G(F_A) \notin G$.
(ii)$\implies$(i) Since $\widehat{F_A} \subseteq F_A \Delta \varphi_G(F_A)$, then by (ii) $\widehat{F_A} \notin G$. Hence, The soft topology $\tau$ is suitable for a soft grill $G$. 

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