On the Interval Zoro Symmetric Single Step Procedure IZSS1-5D for the Simultaneous Bounding of Real Polynomial Zeros

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Abstract

A new modified method IZSS1-5D for the simultaneously bounding all the zeros of a polynomial is formulated in this paper. The efficiency of this method is measured on the CPU times and the number of iterations after satisfying the convergence criteria where the results are obtained using five test polynomials. The R-order of convergence of this method is at least five.

Keywords: CPU times, initial disjoint intervals, interval, number of iteration.

1. Introduction

In this paper, we refer to the methods established by Alefeld and Herzberger [1, 2], Bakar et.al [3], Monsi [4, 5], Monsi et.al [6], Monsi and Wolfe [7], Petkovic [8], Jamaluddin et.al [9, 10, 11] and Sham et.al [12, 13] in order to increase the rate of
convergence of the interval symmetric single-step method ISS1-5D. The aim of this paper is to present the interval zoro symmetric single-step procedure, IZSS1-5D which is the modification of interval zoro symmetric single-step procedure, IZSS1 Rusli et. al [14]. IZSS1-5D method has been modified in order to increase the efficiency of the method. We repeat the same inner looping and add up another step to test the accuracy. The efficiency of the algorithm is measured numerically by taking the CPU time and also the number of iterations of the algorithm.

Consider \( p : R^1 \rightarrow R^1 \) a polynomial of degree \( n > 1 \) defined by

\[
p(x) = a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \ldots + a_0 = \prod_{j=1}^{n} (x - x_j^*)
\]

(1)

where \( a_i \in R^1 (i = 1, \ldots, n) \) are given. Suppose that \( p \) has \( n \) distinct values \( x_j^* \in R (i = 1, \ldots, n) \) and that \( X_i^{(0)} \in I(R) \) (set of real intervals) \( (i = 1, \ldots, n) \) are such that

\[
x_j^* \in X_i^{(0)}, \quad (i = 1, \ldots, n)
\]

(2) and

\[
X_j^{(0)} \cap X_i^{(0)} = \emptyset, (i, j = 1, \ldots, n; i \neq j).
\]

(3)

2. The Interval Zoro Symmetric Single-Step Procedure IZSS1-5D

The interval symmetric single-step procedure IZSS1-5D is an extension of the interval single-step procedure IS and ISS1 of [2] and [7]. The interval sequence \( X_i^{(k)} (i = 1, \ldots, n) \) of IZSS1-5D are generated as follows.

Step 1: Set \( k = 0 \),

(4a)

Step 2: For \( k \geq 0 \), \( x_j^{(k)} = \text{mid}(X_i^{(k)}), (i = 1, \ldots, n); \)

(4b)

Step 3: Let \( \delta_j = \delta_j^{(k)} = -p(x_j^{(k)})/\prod_{j=1}^{n} (x_i^{(k)} - x_j^{(k)}) \)

(4c)

Step 4:

\[
X_j^{(k,1)} = x_j^{(k)} - \frac{p(x_j^{(k)})}{\prod_{j=1}^{n}(x_j^{(k)} - X_j^{(k,0)})\prod_{j=1}^{n}(x_i^{(k)} - X_i^{(k,0)} - 5\delta)} \cap X_i^{(k)}
\]

(4d)

\( (i = 1, \ldots, n) \)

Step 5:

\[
X_i^{(k,2)} = x_i^{(k)} - \frac{p(x_i^{(k)})}{\prod_{j=1}^{n}(x_j^{(k)} - X_j^{(k,1)})\prod_{j=1}^{n}(x_i^{(k)} - X_i^{(k,2)})} \cap X_i^{(k,1)}
\]

(4e)

\( (i = n, \ldots, 1) \)
Interval zoro symmetric single step procedure IZSS1-5D

Step 6:

\[ X_i^{(k,2)} = \left\{ x_i^{(k)} - \frac{p(x_i^{(k)})}{\prod_{j=1}^{k-1} (x_i^{(k)} - x_j^{(k,1)}) \prod_{j=k+1}^{n} (x_i^{(k)} - x_j^{(k,2)})} \right\} \cap X_i^{(k,1)} \]  

\[(i = n, ..., 1) \]  

(4f)

Step 7:

\[ X_i^{(k+1)} = X_i^{(k,3)} \]  

(4g)

Step 8: If \( w(X_i^{k+1}) < \varepsilon \), then stop, else set \( k = k + 1 \) and go to Step 2.  

(4h)

3. Numerical Results and Discussion

Table 1 shows the comparison of the number of iteration and CPU time in seconds, between procedures IZSS1 and IZSS1-5D obtained using MATLAB 2007 software in association with IntLab V5.5 toolbox [15].

Test Polynomial 1: [13]
\[ p = \lambda^6 - 44\lambda^4 + 453\lambda^2 - 990 \] where \( n = 6 \),
\[ a_1 = 5.4772, a_2 = -5.4772, a_3 = 3.3166, a_4 = 1.7321, a_5 = -3.3166, a_6 = -1.7321, b_i = 1 (i = 1, ..., 5), \] with initial intervals, \( X_1^{(0)} = [1, 2], X_2^{(0)} = [3, 4], X_3^{(0)} = [5, 6], X_4^{(0)} = [-2, -1], X_5^{(0)} = [-4, -3], X_6^{(0)} = [-6, -5] \).

Test Polynomial 2: [2]
\[ p = \lambda^5 - 35.61\lambda^3 - 3090.376\lambda^2 + 9197.7665\lambda - 9931.285 \] where \( n = 5 \),
\[ a_1 = 11.5, a_2 = 9.1, a_3 = 7.3, a_4 = 5.2, a_5 = 2.5, b_i = 1 (i = 1, ..., 4), \] with initial intervals, \( X_1^{(0)} = [-2.5, 2.1], X_2^{(0)} = [2.2, 4.5], X_3^{(0)} = [4.6, 7.9], X_4^{(0)} = [8.0, 10.8], X_5^{(0)} = [10.9, 13.1] \).

Test Polynomial 3: [13]
\[ p = \lambda^4 - 10\lambda^3 + 35\lambda^2 - 50\lambda + 24 \] where \( n = 4 \), \( a_1 = 1, a_2 = 2, a_3 = 3, a_4 = 4, b_i = 1 (i = 1, 2, 3), \) with initial intervals, \( X_1^{(0)} = [0.6, 1.3], X_2^{(0)} = [1.6, 2.3], X_3^{(0)} = [2.6, 3.3], X_4^{(0)} = [3.6, 4.3] \).

Test Polynomial 4: [2]
\[ p = \lambda^9 - 398\lambda^7 + 45944\lambda^5 - 1778055\lambda^3 + 1786379\lambda \] where \( n = 9 \),
\[ a_1 = 15, a_2 = 10, a_3 = 7, a_4 = 4, a_5 = 0, a_6 = -4, a_7 = -7, a_8 = -10, a_9 = -15, b_i = 1 (i = 1, ..., 8), \] with initial intervals, \( X_1^{(0)} = [12.0, 17.0], X_2^{(0)} = [8.6, 11.2], X_3^{(0)} = [5.2, 8.4], X_4^{(0)} = [2.4, 5.0], X_5^{(0)} = [-2.0, 2.2], X_6^{(0)} = [-6.4, -2.9], \)
\[ x_7^{(0)} = [-8.2, -6.5], x_8^{(0)} = [-11.8, -8.0], x_9^{(0)} = [-17.2, -13.5]. \]

**Test Polynomial 5:** [2]
\[ p = \lambda^5 - 30\lambda^4 + 311\lambda^3 - 1278\lambda^2 + 1551\lambda = 630 \text{ where } n = 5, \]
\[ a_1 = 0, a_2 = 3, a_3 = 6, a_4 = 9, a_5 = 12, b_i = 1 \text{ (i = 1, \ldots, 4)}, \] with initial intervals,
\[ x_1^{(0)} = [1.9, 3.4], x_2^{(0)} = [4.8, 5.9], x_3^{(0)} = [6.5, 8.1], x_4^{(0)} = [8.3, 9.8], x_5^{(0)} = [10.7, 11.9]. \]

<table>
<thead>
<tr>
<th>Polynomial</th>
<th>Degree ( n )</th>
<th>IZSS1</th>
<th>CPU time</th>
<th>IZSS1-5D</th>
<th>CPU time</th>
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</tbody>
</table>

4. **Conclusion**

We have developed a new modified method IZSS1-5D which is better than IZSS1 in terms of the number of iterations and CPU times. From the results, we conclude that the Interval Zoro Symmetric Single-Step Procedure IZSS1-5D (with the corrector 5\( \delta \)) must have a higher rate of convergence compared to the procedure IZSS1 using \( w^{(k)} \leq 10^{-12} \) as the stopping criterion.

**Acknowledgement.** We are indebted to Universiti Kebangsaan Malaysia for funding this research under the grant BKBP-FST-K005560.

**References**


Received: October 28, 2013