Some Dynamical Properties of the Family of Tent Maps*

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Abstract

In this paper, we investigated some dynamical behavior of family of tent maps. Such dynamical properties include fixed points and their stability, period orbits, visualize the iterations using a kind of plot called a cobweb plot and demonstrate bifurcation diagram for $T_r$. Furthermore, detecting the presence of chaos in the discrete dynamical system $T_r$ is investigated. Finally, we develop Matlab computer programs that reflect the results interpreting such dynamical behavior.

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1. INTRODUCTION

Consider the parameterized tent map, which can be described piecewise by

$$T_r(x) = \begin{cases} 2rx, & \text{if } 0 \leq x \leq \frac{1}{2} \\ 2r(1-x), & \text{if } \frac{1}{2} < x \leq 1 \end{cases}$$
where $0 < r \leq 2$. This map is continuous, linear on each of the intervals $(-1, 1/2]$ and $[1/2, 1)$ (with respect to slopes $r$ and $-r$) and has the points $(0,0)$ and $(1,0)$ in its graph. This map is known as tent maps and often introduced as one of the first examples of chaotic maps literature for nonlinear discrete dynamical systems. Its dynamics exhibit various features are commonly used to identify chaotic systems (see for instance [5]).

Chaos theory describes complex motion and the dynamics of sensitive systems. Chaotic systems are mathematically deterministic but nearly impossible to be predicted. Chaos is more evident in long-term systems than in short-term systems. Behavior in chaotic systems is a periodic, in other words, no variable describes the state of the system that undergoes a regular repetition of values. A chaotic system can actually evolve in a way that appears to be smooth and ordered; however, chaos refers to the issue of whether or not it is possible to make accurate long-term predictions of any system if the initial conditions are known to an accurate degree [8].

In addition to the sensitivity condition, there is a mixing condition called transitivity and a regularity condition called density of periodic points[2].

Deterministic chaos can be described by deterministic map, such as tent map $T_r$ as follows:

starting from any point between 0 and 1, the trace of the map would still lie in the range $[0, 1]$ of the tent map, which is a absorbing set. Also, each point inside $[0, 1]$ will be often visited arbitrarily closely and infinitely by any ‘typical’ solution, hence it is also an attractor [2].

The purpose of this paper is to investigate the dynamical behavior of the family of tent maps, we will limit ourselves to the case $0 < r \leq 2$ [3]. For certain parameter values, the mapping undergoes stretching and folding transformations and displays sensitivity to initial conditions and periodicity (see [1] and [6]).

The tent map is studied in the mathematics of dynamical systems. Because of its simple shape, the tent map shape under iteration is very well understood. And despite its simple shape, it has several interesting properties [10].

The paper is organized as follows; The first section is the introduction and the second section describes the dynamical behavior for this discrete dynamical system $T_r$, such as fixed points and their stability and period orbits followed by section three, which presents cobweb plots. Section four demonstrates period doubling and obtains the bifurcation diagram of $T_r$ , for the parameter $0 < r \leq 2$. The dramatic bifurcation that occurs at $r = 1$ will be explained. The results will be obtained using Matlab computer programs. One of these programs is to visualize the iterations using a kind of plot called a cobweb plot. Sketching the bifurcation diagram of $T_r$, for the parameter $0 < r \leq 1$ is by using another Matlab program. Fixed points will show up as a single point, a periodic orbit as several points, and a chaotic orbit as a band or several bands of points.

Section five presents the results that are detecting the presence of chaos in the tent maps $T_r$ and examining sensitivity to initial conditions; it is one of the dynamical
properties of strange attractors. Results and discussions will be displayed. Finally, in the last section some concluding remarks will be highlighted.

2. FIXED POINTS AND PERIODIC ORBITS

Consider an iteration

\[ x_{n+1} = f(x_n) \]  (2.1)

A fixed point, or point of period one, of discrete dynamical system (2.1) is a point at which \( x_{n+1} = f(x_n) = x_n \) for all \( n \) [7].

For the tent map, this implies that \( T_r(x_n) = x_n \), for all \( n \). Graphically, the fixed points can be found by identifying intersections of the map \( T_r(x) \) with the line \( y = x \), see [3].

Figures 2.1(a) through 2.1(c) display \( T_r \) for \( r = 2/7, 1/2 \) and \( 5/6 \) by Matlab program 1 in the Appendix. As \( r \) increases, the height of the graph of \( T_r \) rises, because of the factor \( r \) in the formula for \( T_r \). From this observation and the three graphs in Figure 2.1; we deduce that if \( 0 < r < 1/2 \), then \( T_r \) intersects the line \( y = x \) once (at 0), whereas if \( 1/2 < r < 1 \), then there are two points of intersection. We are led to analyze separately the members of \( \{ T_r \} \) for which \( 0 < r < 1/2, r = 1/2 \) and \( 1/2 < r < 1 \). Finally, we will study \( T_1 \), which is the original tent map \( T \) and which has some very interesting features, we shall follow [3].
Graphical iteration of tent map $T_r(x)$

**Case 1.** $0 < r < 1/2$

The graph in Figure 2.1(a) shows that 0 is the only fixed point of $T_r$. Since $0 < r < 1/2$, it follows from the definition of $T_r$ that if $0 < x \leq 1/2$, then

$$0 \leq T_r(x) = 2rx < x$$

and if $1/2 < x \leq 1$, then

$$0 \leq T_r(x) = 2r(1-x) < 1-x < \frac{1}{2} < x$$

Consequently for any $x \in [0,1]$, the sequence $\{T_r^n(x)\}_{n=0}^\infty$ is bounded and decreasing and by the continuity of $T_r$, the sequence converges to the fixed point 0. Therefore, 0 is an attracting fixed point whose basin of attraction is $[0,1]$.

**Case 2.** $r = 1/2$

First we notice that if $0 \leq x \leq 1/2$, then $T_{1/2}(x) = 2(1/2)x = x$, so that $x$ is a fixed point of $T_{1/2}$ (Figure 2.1(b)). Next, we calculate that $1/2 < x \leq 1$, then

$$0 \leq T_r(x) = 2(1/2)(1-x) = 1-x \leq \frac{1}{2}$$

So that $T_{1/2}(x) = x$ is a fixed point of $T_{1/2}$. Consequently, every point in $[0,1]$ either is a fixed point of $T_{1/2}$ or has a fixed point its first iterate.

**Case 3.** $1/2 < r < 1$

In addition to the fixed point 0, there is a second fixed point $p$ that lies in $(1/2,1]$, as one can see in Figure 2.1(c). To evaluate $p$ we solve the equation

$$p = T_r(p) = 2r(1-p)$$

which yields
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\[ p = \frac{2r}{1 + 2r} \]

As \( r \) increases from \( \frac{1}{2} \) toward 1, \( p \) increases from \( 1/2 \) toward 2/3. Because \( |T'_r(x)| = 2r > 1 \) on \([0,1]\) except at \( 1/2 \), both 0 and \( p \) are repelling fixed points. The period-2 points of \( T_r \) are the fixed points of \( T_r^{[2]} \), which is given by

\[
T_r^{[2]}(x) = \begin{cases} 
4r^2x & \text{for } 0 \leq x \leq 1/4r \\
2r(1-2rx) & \text{for } 1/4r < x \leq 1/2 \\
2r(1-2r+2rx) & \text{for } 1/2 < x \leq 1-1/4r \\
4r^2(1-x) & \text{for } 1-1/4r \leq x \leq 1 
\end{cases}
\]

The graph of \( T_8^{[2]} \) appears in Figure 2.2 plotted by Matlab program 2 in the Appendix, and suggests that for \( 1/2 < r < 1 \), \( T_r^{[2]} \) has four fixed points that can be found by solving the four equations \( x = T_r^{[2]}(x) \) arising from the definition of \( T_r^{[2]} \). We find that the fixed points are 0, \( \frac{2r}{1+4r^2} \), \( \frac{2r}{1+2r} \), and \( \frac{4r^2}{1+4r^2} \).

The first and third are the two fixed points of \( T_r \), so it follows that \( \left\{ \frac{2r}{1+4r^2}, \frac{4r^2}{1+4r^2} \right\} \) is a 2-cycle for \( T_r \). This 2-cycle is repelling, because \( \left| T_r^{[2]}(x) \right| = 4r^2 > 1 \) wherever the derivative is defined. Because the graph of \( T_r^{[2]} \) is linear on the \( 2^n \) subintervals \([0,1/2^n], \ldots, [1-1/2^n,1]\), it is possible to describe the various \( n \)-cycles of \( T_r \) - all of which are repelling.
Case 4. \( r = 1 \)

If \( r = 1 \) then \( T_r = T \), the tent map given by \( T(x) = \begin{cases} 2x & \text{for } 0 \leq x \leq 1/2 \\ 2(1-x) & \text{for } 1/2 < x \leq 1 \end{cases} \). The graph of \( T \) appears in Figure 2.3 (a) obtained by Matlab program 1 in the Appendix. The major difference between the graph of \( T \) and the graph of \( T_r \) when \( r < 1 \) is the fact that the range of \( T \) fills out the whole interval \([0,1]\). The map \( T \) stretches the interval \([0,1/2]\) over the entire interval \([0,1]\), and folds the interval \([1/2,1]\) back over the interval \([0,1]\).

As with all members of \( \{T_r\} \), 0 is a fixed point of \( T \). Since \( x = T(x) = 2(1-x) \) if \( x = 2/3 \), we know that 2/3 is the second fixed point of \( T \). Figures 2.3(b)-(c) indicate that \( T^{[2]} \) and \( T^{[3]} \) have, respectively, four and eight fixed points. Thus \( T \) has two period-2 points and six period-3 points, which we could evaluate by solving the equations \( x = T^{[2]}(x) \) and \( x = T^{[3]}(x) \) for \( x \).
Proposition 2.1. A point \( x \in [0,1] \) is periodic under \( T \) iff \( x \) is a rational number of the form \( m / n \) where \( m \) is an even positive integer and \( p \) is an odd positive integer with \( m < p \).

Proof. If \( n \in \mathbb{Z}^{+} \), then \( T^{n}(x) = \begin{cases} \frac{2^{i} \left( x - \frac{i}{2^{n+1}} \right)}{2^{n}} & \text{if } x \in \left[ \frac{2i}{2^{n+1}}, \frac{2i+1}{2^{n}} \right], \quad i=0,1,2,...,2^{n-1} - 1 \[2^{i} \left( x - \frac{i+1}{2^{n+1}} \right) & \text{if } x \in \left[ \frac{2i+1}{2^{n}}, \frac{2i+2}{2^{n}} \right] \end{cases} \)

Therefore, the periodic points under \( T \) are the solutions of \( 2^{i} \left( x - \frac{i}{2^{n+1}} \right) = x \) or \( -2^{i} \left( x - \frac{i+1}{2^{n+1}} \right) = x \), for \( i = 0,1,2,...,2^{n-1} - 1 \).

Solving for \( x \), we get \( x = \frac{2i}{2^{n+1}} \) or \( x = \frac{2i+2}{2^{n+1}} \), \( i = 0,1,2,...,2^{n-1} - 1 \). In either case, \( x \) is of the form \( m/p \), where \( m \) is even and \( p \) odd. Suppose that \( x \) is a rational number in \([0,1]\) of the form \( m/p \), where \( m \) is even and \( p \) is odd. Since \( 2^{\varphi(p)} \equiv 1 \pmod{p} \), where \( \varphi \) is the Euler’s function, \( p | 2^{\varphi(p)} - 1 \); hence there is a natural number \( t \) such that \( 2^{\varphi(p)} - 1 = p \cdot t \). That is, \( x = \frac{m.t}{2^{\varphi(p)} - 1} \). Since \( m \) is even, \( x = \frac{2.k.t}{2^{\varphi(p)} - 1} \) where \( k < \frac{p}{2} \). Therefore, \( k \cdot t = \frac{k}{p} \left( 2^{\varphi(p)} - 1 \right) < \frac{2^{\varphi(p)} - 1}{2} \leq 2^{\varphi(p) - 1} - 1 \). That is, \( x \) is a periodic point.

Now, we will determine the eventually periodic points and periodic points for \( T \).

Proposition 2.2. Let \( x \in [0,1] \). Then \( x \) is rational iff \( x \) is periodic or eventually periodic under \( T \).

Proof[3]. Suppose \( x = \frac{m}{p} \) is a rational number in \([0,1]\), where \( m \) and \( p \) are relatively prime. If \( m \) and \( p \) are both odd, then \( T(x) = \frac{2m}{p} \) or \( T(x) = \frac{2(p-m)}{p} \). In either case, \( T(x) \) is periodic by Proposition 2.1. If \( m \) is odd and \( p \) is even, let \( k \) be the largest positive integer such that \( p = 2^{k} \cdot s \), where \( s \) is odd. A short computation shows that \( T^{k} \) is of the form \( \frac{p}{2^{n}} \), which is periodic. If \( m \) is even and \( p \) is odd, then \( x \) is periodic by Proposition 2.1.

3. COBWEB PLOTS

In this section, we are going to introduce a very helpful graphical view of the iteration process called cobweb plot[9]. The cobweb plot makes use of a plot of the mapping and a
plot of the reference line $y = x$. From these two curves one may construct graphically an iteration sequence. We construct the plot first and then explain it. We do this for the case of three iterations. The Matlab program 3 (with file `functent.m`) which does this is listed in the Appendix. Here the starting value of $x$ is 0.23, the number of iteration is 3, repeat count for mapping (the level of map composition) and at the parameter $r = 1$.

Figure 3.1 shows a cobweb plot of three iterations for the tent map. The starting value is $x = 0.23$. Starting at that value on the x-axis, we move vertically upward until we hit the graph of $\text{cobwebtent}(x)$. The y-value at that point is the value of the first iterate. Now we move horizontally until we hit the reference line $y = x$. The x-coordinate of that point is the value of the iterate. Next we move vertically until we reach the graph of $\text{cobwebtent}(x)$, see program 3. The y-coordinate of that point is the second iterate. We repeat the steps to get the third iterate.

Now let's use program 3 to look at a few typical events. We begin by reducing $r$ to a value of $1/3$, for which we have a single stable equilibrium. We then construct the cobweb plot of the approach to this equilibrium. Figure 3.2 shows a cobweb plot of 10 iterations for the tent map. The starting value is $x = 0.789$. This shows nicely the approach to the origin.

Now let's increase $r$ to 1 (here the starting value of $x = 0.4$ and iterations is 10) and look at a periodic orbit. In a way we are cheated by starting on the orbit (see Figure 3.3). Let's do this again, but this time we start off the orbit (here $r = 1$, the starting value of $x = 0.4$ and the iterations is 10). Now we see the transient approach to the orbit (see Figure 3.4).
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So far we have seen examples of equilibrium and of a periodic orbit. After that, we choose a rational number for the initial $x$ so that all calculations are done exactly, for example $x = \frac{230457}{1000000}$, iterations is 200 and $r = 1$, (see Figure 3.5).

This is a chaotic orbit for the tent map. This is the typical result for the typical initial condition when $r$ exceeds $1/2$. For special choices of initial conditions, we can land on one of the unstable periodic orbits.

4. BIFURCATION DIAGRAM

In this section, we are going to explore the concept of bifurcations and how Matlab can help us visualize these changes. We will begin with some basic definitions, then examine some plots, and finally create a bifurcation diagram.

A bifurcation is a fundamental change in the nature of a solution (see [3] and [10]). When studying dynamical systems, we often want to know when the system undergoes a bifurcation.
A bifurcation diagram is a kind of plot that shows the value of the changing parameter, $r$ in our case, on one axis and the solution to the system on the other axis. Our next goal is to produce the bifurcation diagram of the 'tent' maps $T_r$. 

Figure 4.1

Often we want to know how system behavior depends on parameters. In the case of the tent map $T_r$, we have a single parameter $r$, and we have already known that there is a stable equilibrium for $r < 1/2$. We also know that for $r > 1/2$, there are two unstable equilibria, some unstable periodic solutions, and, as we have just seen, chaotic solutions. We can get an overview of how all of this depends on the parameter $r$ by a bifurcation diagram. That is a plot in which the abscissa is the value of $r$, and on the ordinate we plot all of the $x$-values from an orbit for that value of $r$. Fixed points will show up as a single point, a periodic orbit as several points, and a chaotic orbit as a band or several bands of points. The program which does this is the Matlab Program 4 in the Appendix. Here (see Figure 4.1) the number of values calculated for each parameter value is 1000 points, and the first 100 points are discarded. The values are calculated beginning with initial point $x = 0.23$. The range of parameter variation is 0 to 1. The number of parameter values for which this is done is 1000 points while the number of tent map values is 800 points. Starting with the value 0, and considering 1000 $r$ values in the $r$ range $\{0,1\}$.

5. SENSITIVE DEPENDENCE TO INITIAL CONDITIONS AND CHAOS

Let $I$ be an interval, and suppose that $f : I \rightarrow I$ (signifies that the domain of $f$ is $I$ and the range is contained in $I$). Then $f$ has sensitive dependence on initial conditions at $x$, if there is an $\epsilon > 0$ such that for each $\delta > 0$, there is a $y$ in $I$ and a positive integer $n$ such that

$$|x - y| < \delta \quad \text{and} \quad |f^{[n]}(x) - f^{[n]}(y)| > \epsilon$$

(5.1)

If $f$ has sensitive dependence on initial conditions at each $x \in I$, we say that $f$ has sensitive dependence on initial conditions on $I$ [3].
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The “initial conditions” here refer to the given, or initial, points $x$ and $y$. Sensitive dependence on initial conditions says that $f$ has sensitive dependence if $y$ arbitrarily close to any given point $x$ in the domain of $f$, there is a point and an $nth$ iterate that is farther from the $nth$ iterate of $x$ than a distance $\varepsilon$. This has practical significance because in such instances higher iterates of an approximate value of $x$ may not resemble the true iterates of $x$. Thus computer calculations may be misleading.

According to the well accepted definition of Devaney[2], a one-dimensional map of the form

$$x_{n+1} = f(x_n),$$

is chaotic if it:

(i) $f$ has sensitive dependence on initial conditions (s.d.i.c.), and

(ii) $f$ has a dense set of periodic orbits, and

(iii) $f$ has at least one dense orbit (‘$f$ is topologically transitive’).

The following results detect the presence of chaos in the discrete dynamical system $T$, see [3] and [2].

**Proposition 5.1.** The tent map $T$ has a sensitive dependence on initial conditions on $[0, 1]$ and hence chaotic.

**Proof.** Let $x \in [0, 1]$. First we will show that if $v$ is any dyadic rational number (of the form $j/2^n$, in lowest terms) in $[0, 1]$ and $w$ is any irrational number in $[0, 1]$, then there is a positive integer $n$ such that

$$|T^n(v) - T^n(w)| > \frac{1}{2}$$

Toward that goal, if $v = j/2^m$, then $T^n(v) = 1$ and $T^{n+k}(v) = 0$ for all $k > 0$. By contrast, if $w$ is any irrational number in $[0, 1]$, then since $T$ doubles each number in $(0, 1/2)$, there exists an $n > m$ such that $T^n(w) > 1/2$. Since $n > m$, it follows that $T^n(v) = 0$, so that (5.1) is valid. Next, let $\delta > 0$. Then there exist a dyadic rational $v$ and an irrational number $w$ in $[0, 1]$ such that $|x - v| < \delta$ and $|x - w| < \delta$. Therefore (5.2) implies that

either $$|T^n(x) - T^n(w)| > \frac{1}{4}$$ or $$|T^n(x) - T^n(w)| > \frac{1}{4}$$

Thus if we let $\varepsilon = 1/4$, then sensitive dependence on initial conditions at the arbitrary number $x$, and hence on $[0, 1]$, is proved. Therefore, $T$ is chaotic.

Basically the reason that $T$ has a sensitive dependence on initial conditions is that if $x \neq 1/2$, then $T'(x) \neq 2$, so that distances between pairs of numbers in $(0, 1/2)$ or in $(1/2, 1)$ are doubled by $T$. 
Proposition 5.2. The set of periodic points under $T$ is dense in $[0, 1]$.

**Proof.** Let $(a, b)$ be any open interval in $(0, 1)$. Let $p$ an odd positive integer large enough so that $\frac{1}{2p} < b - a$. There is a least positive integer $k$ such that $\frac{k}{p} \in \left( a, \frac{a+b}{2} \right)$. If $k$ is even, then we are done. If $k$ is odd then $\frac{k+1}{p} \in (a, b)$ will do.

Proposition 5.3. The tent maps is chaotic on $[0, 1]$.

**Proof.** Let $I = (a, b)$ and $J = (c, d)$ be any open intervals in $[0, 1]$. Let $n$ be the least positive integer such that $\frac{1}{2^n} < b - a$. Then $T^n(I) = [0,1], r=1..$. This implies that $(T^n)^{-1}(J)$ is an open interval of $I$. By the density of periodic points in $[0, 1]$, there is a periodic point $x$ in $I$ such that $T^n(x) \in J$.

6. CONCLUSIONS

This paper has focused on discrete-time dynamical system and investigated some dynamical behavior of the parameterized tent maps for some values of its control parameter and has pointed out the Matlab implementations of the dynamical behavior such as fixed points and their stability, period orbits, graphics behavior (cobweb plot) and bifurcation diagrams. The bifurcation analysis was considered and the presence of chaotic behavior of the discrete dynamical system $T_r$ was solved by investigating the sensitive dependence to initial condition. The most complex dynamics-chaos-occur only for large values of the parameter $r$; so the prediction of a chaotic time series could be demonstrated.

APPENDIX

The following Matlab programs (see[4]) interprets the dynamical behavior of the family of tent maps.

**Program 1**

```matlab
function Tparameter(x, maxiter)
% The 'tent' map is given by:
% 2*r*x , if x<0.5
% T(x)={                  ; 'r' is in(0, 2] and 'x' is in [0, 1].
% 2*r*(1-x) , if x>=0.5
% where 'x' can be an element, vector or matrix of initial points.
clc; clf; clear all; close all;
% Default settings.
```
function Titerate(x,maxiter)
% The 'tent' map is given by:
% T(x)={ 2*r*x , if 0 <= x <=0.5
%       2*r*(1-x), if 0.5 < x <= 1
% where 'x' can be an element, vector or matrix of initial points.
clc; clf; clear all; close all;
% Default settings.
if (nargin < 2)
    maxiter = 2;   % maximum number of iterations for T^[1].
    x(1)= 0;       % initial condition.
    r=0.8333;      % parameter value.
    fsize=12;      % font size.
end
% Compute the tent map.
for n = 1:maxiter
    xa(n+1) = 2*r*x(n)*(x(n)<0.5);
    xb(n+1) = 2*(-r*x(n) + r)*(x(n)>=0.5);
    x(n+1) = xa(n+1) + xb(n+1);
end
% Plot tent map.
xc=[0 0.5 1]; yc=[0 r 0];
plot(xc,yc,'b');
% Plot line y=x.
xe = [0 1]; ye = [0 1];
plot(xe,ye,'k');
title('Graphical iteration of tent map: T_r(x)','Fontsize',fsize,'Color','b');
xlabel('x','Fontsize',fsize,'Color','b');
ylabel({'T_r(x), is in (0, 2]';[' r =',num2str(r,'%11.7g'),'.']},'Fontsize',fsize,'Color','b')
\[ x(n+1) = 0.5 \times (xa(n+1) + xb(n+1)) \]

end

\[
\begin{align*}
\text{\% Plot line 'y=x' and 'tent' map} \\
x_e = [0 1]; \quad y_e = [0 1]; \quad \% 'xe', 'ye' values for plotting \\
y = x. \\
\text{\% } x_c = [0 0.5 \times r 1]; \quad \% 'xc' value for plotting \\
T^{[\text{n}]}(x), \text{ n=1.} \\
x_c = [0 0.25 \times r 0.5 0.75 \times r 1]; \quad \% 'xc' value for plotting \\
T^{[\text{2}]}(x), \text{ n=2.} \\
T^{[\text{3}]}(x), \text{ n=3.} \\
\text{\% } x_c = [0 0.1250 \times r 0.25 0.3750 \times r 0.5 0.6250 \times r 0.75 0.8750 \times r 1]; \\
\text{plot(xe, ye, 'k', xc, x, 'b');} \\
\text{title('Graphical iteration of tent map, \ r=0.8333', 'Fontsize', fsize, 'Color', 'b');} \\
\text{xlabel('x', 'Fontsize', fsize, 'Color', 'b');} \\
\text{ylabel('T^{[\text{n}]}(x), n=2', 'Fontsize', fsize, 'Color', 'b');} \\
\end{align*}
\]

**Program 3**

function cobwebtent(x)
% The function cobwebtent(x) successively iterates a given point 'x' 
% with the 'tent' map as a fixed set.
% The 'tent' map is given by:
% \[ T_r(x_n) = \begin{cases} 
2r \times x_n & , \text{ if } 0 \leq x \leq 0.5 \\
2r \times (1-x_n) & , \text{ if } 0.5 \leq x \leq 1 
\end{cases} \]
% 'r' belongs to (0, 2] and 'x' belongs to [0, 1].
% 'nmax' is the maximum number of iterations.

clear all; clc; clf;
global n r
fsize=12; \% font size.
x(1) = input(' Enter the initial condition x_0: '); 
maxiter=input(' Enter the maximum number of iterations: '); 
n=input(' Enter repeat count for mapping: '); 
r=input(' Enter the parameter value: '); 
xn=0:0.0001:1;

y=xn;
plot(xn,functent(xn),'b-') \% Plot function in file functent1.m
hold on
plot(xn,xn,'k') \% Superpose plot of y=x
axis([0 1 0 1]); \% Scale picture
axis('square'); \% Ensure plot is square
fprintf('
    n     Iterate 
') \% Print table header

% Compute the tent map.
for k=1:maxiter
    x(k+1) = functent(x(k)); 
    fprintf(' %3.0f %10.10f \n', k,x(k+1)) \% Print iterate
end

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% Draw cobweb plots of the Tent map
for k=1:maxiter
    xa(2*k-1)=x(k);
    xa(2*k)=x(k);
end
xb(1)=0;xb(2)=x(2);
for k=2:maxiter
    xb(2*k-1)=x(k);
    xb(2*k)=x(k+1);
end
for k=1:maxiter
    if k>1
        xh=[xa(k-1) xa(k)];
        yh=[xb(k) xb(k)];
        plot(xh,yh,'r:');
    end
    xv=[xa(k) xa(k)];
    yv=[xb(k) xb(k+1)];
    plot(xv,yv,'r')
end
fprintf( ' Final iteration is %10.10f 
', x(k+1)) % Has huge magnitude when iterates are unbounded

function y=functent(x)
  global n r
  xn=x;
  for k=1:n
      xn=2*r*(0.5-abs(xn-0.5)); % Mapping is x(n+1)=2*r*(0.5-abs(x_n-0.5))
  end
  y=xn;

Program 4

% Matlab code to produce the bifurcation diagram of the 'tent' map:
% 2*r*x , if x<0.5
% T_r(x)=
% 2*r*(1-x), if x>=0.5
clear all;
fsizes=12; %Font size.
r=linspace(0,1,1000); %This is 1000 points for r.
x=0.23;                     %An initial condition.
fprintf('Going into the initial orbit\n');
for i=1:100
    y=2*r.*(0.5-abs(x-0.5));
    x=y;
end
fprintf('Going into new orbit\n');
% Store the results.
A=zeros(800,1000);
for i=1:800
    y=2*r.*(0.5-abs(x-0.5));
    A(i,:)=y;
    x=y;
end
plot(A','k.','Markersize',4);
plot(A','k.','Markersize',4);
title('Bifurcation diagram for tent map.\n','Fontsize',fsize,'Color','b');
xlabel('r','Fontsize',fsize,'Color','b');
ylabel('T_r^[n](x)\n','Fontsize',fsize,'Color','b');

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REFERENCES


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