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Fixed Point Theorems in Generalized Contraction Mappings in $\mathcal{M}$ – Fuzzy Metric Spaces

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Abstract

In this paper, we introduce generalized contraction mapping in $\mathcal{M}$ - fuzzy metric space. Some fixed point theorems for fuzzy metric space in the sense of George and Veeramani [2].

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1. INTRODUCTION AND PRELIMINARIES

The concept of fuzzy sets was introduced by Zadeh [14] in 1965. Since then, to use this concept in topology and analysis many authors have expansively developed the theory of fuzzy sets and applications. Fixed point theorems in fuzzy mathematics are emerging with vigorous hope and vital trust. It appears that Kramosil and Michalek's study of fuzzy metric spaces paves a way for very soothing machinery to develop fixed point theorems for contractive type maps. Kramosil and Michalek [6] introduced the concept of fuzzy metric space and modified by George and Veeramani [2]. Many authors [4, 7] have proved fixed point theorems in fuzzy metric space. Recently Sedghi and Shobe [12] introduced D* - metric space as a probable modification of the definition of D – metric introduced by Dhage [1], and prove some basic properties in D* - metric spaces. Using D* - metric concepts, Sedghi and Shobe define $\mathcal{M}$ -fuzzy metric space and proved a common fixed point theorem in it. In this paper we introduce fixed point theorems for generalized contraction mapping in $\mathcal{M}$ - fuzzy metric spaces.

Definition: 1.1 Let X be a nonempty set. A generalized metric (or D* - metric) on X is a function $D^* : X^3 \to [0, \infty)$, that satisfies the following conditions for each x, y, z, a $\in$ X

i) $D^*(x, y, z) \geq 0$,

ii) $D^*(x, y, z) = 0$ iff $x = y = z$,

iii) $D^*(x, y, z) = D^*(p\{x, y, z\})$, where p is a permutation function,

iv) $D^*(x, y, z) \leq D^*(x, y, a) + D^*(a, z, z)$.

The pair (X, D*), is called a generalized metric (or D* - metric) space.

Example: 1.2 Examples of D* - metric are

a) $D^*(x, y, z) = \max\{d(x, y), d(y, z), d(z, x)\}$,

b) $D^*(x, y, z) = d(x, y) + d(y, z) + d(z, x)$.

Definition: 1.3 A fuzzy set $\mathcal{M}$ in an arbitrary set X is a function with domain X values in [0, 1].

Definition: 1.4 A binary operation $* : [0, 1] \times [0, 1] \to [0, 1]$ is a continuous t-norm if it satisfies the following conditions

i) * is associative and commutative,

ii) * is continuous,

iii) $a * 1 = a$ for all $a \in [0, 1]$,

iv) $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$, for each a, b, c, d $\in$ [0, 1]

Examples for continuous t-norm are $a * b = \min\{a, b\}$.

Definition: 1.5 A 3-tuple (X, $\mathcal{M}$, *) is called $\mathcal{M}$ – fuzzy metric space if X is an arbitrary non-empty set, * is a continuous t – norm, and $\mathcal{M}$ is a fuzzy set on $X^3 \times (0, \infty)$, satisfying the following conditions for each x, y, z, a $\in$ X and t, s > 0

(FM – 1) $\mathcal{M}(x, y, z, t) > 0$

(FM – 2) $\mathcal{M}(x, y, z, t) = 1$ iff $x = y = z$

(FM – 3) $\mathcal{M}(x, y, z, t) = \mathcal{M}(p\{x, y, z\}, t)$, where p is a permutation function

(FM – 4) $\mathcal{M}(x, y, a, t) * \mathcal{M}(a, z, z, s) \leq \mathcal{M}(x, y, z, t + s)$

(FM – 5) $\mathcal{M}(x, y, z, .) : (0, \infty) \to [0, 1]$ is continuous

(FM – 6) $\lim_{t \to \infty} \mathcal{M}(x, y, z, t) = 1$.

If, in the above definition , the triangular inequality (FM– 4) is replaced by (NAF) $\mathcal{M}(x, y, a, t) * \mathcal{M}(a, z, z, t) \leq \mathcal{M}(x, y, z, t)$ for all x, y, z, a $\in$ X and t > 0, then the triple (X, $\mathcal{M}$, *) is called non- Archimedean $\mathcal{M}$ – fuzzy metric space.

Example: 1.6 Let X be a nonempty set and D* is the D* - metric on X. Denote a * b = a , b for all a, b ϵ [0, 1]. For each t ϵ (0, ∞) , define

$$\mathcal{M}(x, y, z, t) = \frac{t}{t + D^*(x, y, z)}$$

for all x, y, z ϵ X, then (X, $\mathcal{M}$, *) is a $\mathcal{M}$ - fuzzy metric space. We call this $\mathcal{M}$ - fuzzy metric induced by D* - metric space. Thus every D* - metric induces a $\mathcal{M}$ - fuzzy metric.

Lemma: 1.7 [12] Let (X, $\mathcal{M}$, *) be a $\mathcal{M}$ -fuzzy metric space. Then for every t > 0 and for every x, y ϵ X. We have $\mathcal{M}$ (x, x, y, t) = $\mathcal{M}$ (x, y, y, t).

Lemma: 1.8 [12] Let (X, $\mathcal{M}$, *) be a $\mathcal{M}$ -fuzzy metric space. Then $\mathcal{M}$ (x, y, z, t) is non-decreasing with respect to t, for all x, y, z in X.

Definition: 1.9 Let (X, $\mathcal{M}$, *) be a $\mathcal{M}$ - fuzzy metric space. For t > 0, the open ball $B_{\mathcal{M}}$ (x, r, t) with center x ϵ X and radius 0 < r < 1 is defined by $B_{\mathcal{M}}$ (x, r, t) = {y ϵ X : $\mathcal{M}$ (x, y, y, t) > 1 – r}. A subset A of X is called open set if for each x ϵ A there exist t > 0 and 0 < r < 1 such that $B_{\mathcal{M}}$ (x, r, t) ⊆ A.

Definition: 1.10 Let (X, $\mathcal{M}$, *) be a $\mathcal{M}$-fuzzy metric space and {x_n} be a sequence in X

a) {x_n} is said to be converges to a point x ϵX if $\lim_{n\to\infty} \mathcal{M}(x, x, x_n, t)=1$ for all t >0
b) {x_n} is called Cauchy sequence if $\lim_{n\to\infty} \mathcal{M}(x_{n+p}, x_{n+p}, x_n, t)=1$ for all t>0 and p>0
c) A $\mathcal{M}$-fuzzy metric space in which every Cauchy sequence is convergent is said to be complete.

Example: 1.11 Let (X, d) be a ordinary metric space and * be a t- norm.

(1) Let $\mathcal{M}_d$ be a fuzzy set on X^3 x (0, 1), define as follow :

$$\mathcal{M}_d(x, y, z, t) = \frac{ht^n}{ht^n + m\,D^*(x, y, z)} \qquad (1.1)$$

for all t, h, m, n ϵ $\mathbb{R}^+$. Then (X, $\mathcal{M}_d$, *) is a fuzzy metric space and called induced $\mathcal{M}$- fuzzy metric space. If in equation (1.1) we take h = m = n = 1 then we have

$$\mathcal{M}_d(x, y, z, t) = \frac{t}{t + D^*(x, y, z)}$$

This fuzzy metric space (X, $\mathcal{M}_d$, *) is called standard $\mathcal{M}$ - fuzzy metric space.

(2) It is immediate to show that (X, d) is a non –Archimedean metric space if and only
if (X, $\mathcal{M}_d$, min) is non –Archimedean $\mathcal{M}$- fuzzy metric space.

MAIN RESULTS

Definition: 2.1 Let (X, $\mathcal{M}$, *) be a fuzzy metric space. We call the mapping T: X→X is a fuzzy contractive mapping, if there exists $\lambda \in (0, 1)$ such that

$$\frac{1}{\mathcal{M}\ (Tx,\ Ty,\ Tz, t)} - 1 \leq \lambda \left(\frac{1}{\mathcal{M}\ (x, y, z, t)} - 1\right)$$

for each x, y, z $\in$ X and t > 0. (λ is called the contractive constant of T).

Definition: 2.2 Let (X, $\mathcal{M}$, *) be a fuzzy metric space. A sequence x_n is called fuzzy contractive if there exists $\lambda \in (0, 1)$ such that

$$\frac{1}{\mathcal{M}\ (x_n,\ x_{n+1},\ x_{n+1},\ t)} - 1 \leq \lambda \left(\frac{1}{\mathcal{M}\ (x, y, z, t)} - 1\right)$$

for every t > 0, n $\in$ N.

Definition: 2.3 Let (X, $\mathcal{M}$, *) be a fuzzy metric space. A sequence is called x_n G - Cauchy iff for each t > 0 and p $\in$ N, $\lim_{n\to\infty}$ $(x_{n+p}, x_{n+p}, x_n, t) = 1$.

A fuzzy metric space in which every G - Cauchy sequence is convergent is called G - Complete.

Proposition: 2.4

(a) The sequence x_n in the metric space (X, d) is contractive iff x_n is fuzzy contractive in the induced fuzzy metric space (X, $\mathcal{M}_d$, *) .

(b) The standard fuzzy metric space (X, $\mathcal{M}_d$, *) is complete iff the metric space (X, d) is complete.

(c) If a sequence x_n is fuzzy contractive in (X, $\mathcal{M}_d$, *) then it is G - Cauchy.

A continuous t-norm * is of Hadzic - type, if there exists a strictly increasing sequence $b_n \in (0, 1)$ such that $b_n * b_n = b_n$ for all n $\in$ N, * = min is an example of such t-norm.

Theorem 2.5 Let (X, $\mathcal{M}$, *) be a complete non - Archimedean $\mathcal{M}$ -fuzzy metric space where the continuous t - norm * is defined as min and T: X→X be a self mapping on X such that for each x, y, z $\in$ X, t > 0,

$$\frac{1}{\mathcal{M}\ (Tx,Ty,Tz,t)} - 1 \leq \alpha \left(\frac{1}{\mathcal{M}\ (x,y,z,t)} - 1\right) + \beta \left(\frac{1}{\mathcal{M}\ (x,Tx,z,t)} - 1\right) +$$

$$\gamma \left(\frac{1}{\mathcal{M}\ (x,Ty,Tz,t)} - 1\right) + \delta \left(\frac{1}{\mathcal{M}\ (y,Ty,Tz,t)} - 1\right) + \eta \left(\frac{1}{\mathcal{M}\ (Tx,Ty,z,t)} - 1\right) \qquad (2.5.1)$$

Where α, β, γ, δ, η $\in$ [0,1] and λ = α + β + γ + δ + η < 1, Then T has a unique fixed point.

Proof: Let $x \epsilon X$ and $t > 0$ be arbitrary and consider a sequence pickard iterations x_n, defined inductively by $x_0 = x$, $x_1 = x_0, . . . , x_{n+1} = T(x_n)$ for each $n \epsilon N$, we will show that x_n is fuzzy contractive.
from (2.5.1) by replacing $x = x_n$, $y = x_{n+1}$, $z = x_{n+1}$, we get

$$\frac{1}{\mathcal{M}(x_n, x_{n+1}, x_{n+1}, t)} - 1 = \frac{1}{\mathcal{M}(Tx_{n-1}, Tx_n, Tx_n, t)} - 1$$

$$\leq \alpha\left(\frac{1}{\mathcal{M}(x_{n-1}, x_n, x_n, t)} - 1\right) + \beta\left(\frac{1}{\mathcal{M}(x_{n-1}, Tx_{n-1}, x_n, t)} - 1\right) +$$

$$\gamma\left(\frac{1}{\mathcal{M}(x_{n-1}, Tx_n, Tx_n, t)} - 1\right) + \delta\left(\frac{1}{\mathcal{M}(x_n, Tx_n, Tx_n, t)} - 1\right) +$$

$$\eta\left(\frac{1}{\mathcal{M}(Tx_{n-1}, Tx_n, x_n, t)} - 1\right)$$

$$\leq \alpha\left(\frac{1}{\mathcal{M}(x_{n-1}, x_n, x_n, t)} - 1\right) + \beta\left(\frac{1}{\mathcal{M}(x_{n-1}, x_n, x_n, t)} - 1\right) +$$

$$\gamma\left(\frac{1}{\mathcal{M}(x_{n-1}, x_{n+1}, x_{n+1}, t)} - 1\right) + \delta\left(\frac{1}{\mathcal{M}(x_n, x_{n+1}, x_{n+1}, t)} - 1\right) +$$

$$\eta\left(\frac{1}{\mathcal{M}(x_n, x_{n+1}, x_n, t)} - 1\right) \qquad (2.5.2)$$

By our Choice of t-norm * and triangular inequality for the last parenthesis in (2.5.2) , we have

$$\left(\frac{1}{\mathcal{M}(x_{n-1}, x_{n+1}, x_{n+1} t)} - 1\right) \leq \left(\frac{1}{\min\{\mathcal{M}(x_{n+1}, x_{n+1}, x_n, t), \mathcal{M}(x_n, x_{n-1}, x_{n-1}, t)\}} - 1\right)$$

$$= \max\left(\frac{1}{\mathcal{M}(x_n, x_{n+1}, x_{n+1}, t)} - 1, \frac{1}{\mathcal{M}(x_n, x_{n-1}, x_{n-1}, t)} - 1\right) \quad (2.5.3)$$

From 2.5.2

$$\frac{1}{\mathcal{M}(x_n, x_{n+1}, x_{n+1}, t)} - 1 \leq (\alpha + \beta + \delta + \eta)\max\left(\frac{1}{\mathcal{M}(x_{n-1}, x_n, x_n, t)} - 1, \frac{1}{\mathcal{M}(x_n, x_{n+1}, x_{n+1}, t)} - 1\right)$$

$$+ \gamma \max\left(\frac{1}{\mathcal{M}(x_n, x_n, x_{n+1}, t)} - 1, \frac{1}{\mathcal{M}(x_n, x_{n-1}, x_{n-1}, t)} - 1\right)$$

hence,

$$\frac{1}{\mathcal{M}(x_n, x_{n+1}, x_{n+1}, t)} - 1 \leq \lambda \max\left(\frac{1}{\mathcal{M}(x_{n-1}, x_n, x_n, t)} - 1, \frac{1}{\mathcal{M}(x_n, x_{n+1}, x_{n+1}, t)} - 1\right)$$

Where $\lambda < 1$, this implies

$$\frac{1}{\mathcal{M}(x_n, x_{n+1}, x_{n+1}, t)} - 1 \leq \lambda\left(\frac{1}{\mathcal{M}(x_{n-1}, x_n, x_n, t)} - 1\right)$$

So sequence x_n is fuzzy contractive sequence.

Since (X, $\mathcal{M}$, *) is a complete FMS so by proposition 2.4., sequence x_n converges to u for some u ϵ X, Now we show u is fixed point of T, from (2.5.1) we have

$$\frac{1}{\mathcal{M}(Tu,Tx_n,Tx_n,t)} - 1 \leq \alpha\left(\frac{1}{\mathcal{M}(u,x_n,x_n,t)} - 1\right) + \beta\left(\frac{1}{\mathcal{M}(u,Tu,x_n,t)} - 1\right) +$$

$$\gamma\left(\frac{1}{\mathcal{M}(u,Tx_n,Tx_n,t)} - 1\right) + \delta\left(\frac{1}{\mathcal{M}(x_n,Tx_n,Tx_n,t)} - 1\right) + \eta\left(\frac{1}{\mathcal{M}(Tu,Tx_n,x_n,t)} - 1\right) \quad (2.5.1)$$

Taking the limit as $n \to \infty$ we obtain,

$$\frac{1}{\mathcal{M}(Tu,u,u,t)} - 1 \leq \beta\left(\frac{1}{\mathcal{M}(u,Tu,u,t)} - 1\right) + \eta\left(\frac{1}{\mathcal{M}(Tu,u,u,t)} - 1\right) \leq \lambda\left(\frac{1}{\mathcal{M}(u,u,Tu,t)} - 1\right)$$

Since $\lambda < 1$, We have $\mathcal{M}$ (u, u, Tu, t) = 1.
Thus Tu = u by 2.5.1

Uniqueness:

Suppose there exist v ϵ X such that Tv = v and v $\neq$ u. Now consider
$\mathcal{M}$ (u, v, v, t) = $\mathcal{M}$ (Tu, Tv, Tv, t)

$$\leq \alpha\left(\frac{1}{\mathcal{M}(u,v,v,t)} - 1\right) + \beta\left(\frac{1}{\mathcal{M}(u,Tu,v,t)} - 1\right) + \gamma\left(\frac{1}{\mathcal{M}(u,Tv,Tv,t)} - 1\right) +$$

$$\delta\left(\frac{1}{\mathcal{M}(v,Tv,Tv,t)} - 1\right) + \eta\left(\frac{1}{\mathcal{M}(Tu,Tv,v,t)} - 1\right)$$

$$\leq \alpha\left(\frac{1}{\mathcal{M}(u,v,v,t)} - 1\right) + \beta\left(\frac{1}{\mathcal{M}(u,u,v,t)} - 1\right) + \gamma\left(\frac{1}{\mathcal{M}(u,v,v,t)} - 1\right) +$$

$$\delta\left(\frac{1}{\mathcal{M}(v,v,v,t)} - 1\right) + \eta\left(\frac{1}{\mathcal{M}(u,v,v,t)} - 1\right)$$

$$\leq (\alpha + \beta + \gamma + \eta)\left(\frac{1}{\mathcal{M}(u,v,v,t)} - 1\right) \quad \leq \lambda\left(\frac{1}{\mathcal{M}(u,v,v,t)} - 1\right)$$

Since $\lambda < 1$, we have $\mathcal{M}$ (u, v, v, t) = 1 Then u = v.

Therefore u is fixed point of T.

Theorem 2.6 Let (X, $\mathcal{M}$, *) be a G - complete $\mathcal{M}$ - fuzzy metric space where the continuous t - norm * is defined as min and T : X $\to$ X be self mapping on X such that for each x, y, z ϵ X, t > 0, λ ϵ (0,1)

$$\frac{1}{\mathcal{M}(Tx,Ty,Tz,t)} - 1 \leq \alpha\left(\frac{1}{\mathcal{M}(x,y,z,t)} - 1\right) + \beta\left(\frac{1}{\mathcal{M}(x,Tx,z,t)} - 1\right) + \gamma\left(\frac{1}{\mathcal{M}(x,Ty,Tz,t)} - 1\right) +$$

$$\delta\left(\frac{1}{\mathcal{M}(y,Ty,Tz,2t)} + \frac{1}{\mathcal{M}(Tx,Ty,z,2t)} - 2\right) \quad (2.6.1)$$

Where $\alpha, \beta, \gamma, \delta, \epsilon$ [0, 1] and $\lambda = \alpha + \beta + \gamma + \delta < 1$, Then T has a unique fixed point.

Proof: The proof is very similar as the theorem (2.5), Instead of this equation (2.5.2.), we have

$$\frac{1}{\mathcal{M}(x_{n-1},x_{n+1},x_{n+1},2t)}-1 \leq \left(\frac{1}{\min\{\mathcal{M}(x_{n+1},x_{n+1},x_n,t),\ \mathcal{M}(x_n,x_{n-1},x_{n-1},t)\}}-1\right)$$

$$= \max\left(\frac{1}{\mathcal{M}(x_n,x_{n+1},x_{n+1},t)}-1, \frac{1}{\mathcal{M}(x_n,x_{n-1},x_{n-1},t)}-1\right)$$

Proceed as the proof of the theorem (2.5),
Then we find Sequence x_n is fuzzy contractive, Thus by proposition (2.4) is G - Cauchy. Since X is G - Complete, x_n converges to u for some $u \epsilon X$.
Instead of 2.6. , we find

$$\frac{1}{\mathcal{M}(Tu,Tx_n,Tx_n,t)}-1 \leq \alpha\left(\frac{1}{\mathcal{M}(u,x_n,x_n,t)}-1\right)+\beta\left(\frac{1}{\mathcal{M}(u,Tu,x_n,t)}-1\right)$$

$$\gamma\left(\frac{1}{\mathcal{M}(u,Tx_n,Tx_n,t)}-1\right)+\delta\left(\frac{1}{\mathcal{M}(x_n,Tx_n,Tx_n,2t)}+\frac{1}{\mathcal{M}(Tu,Tx_n,x_n,2t)}-2\right)$$

Taking the limit as $n \to \infty$, we obtain

$$\frac{1}{\mathcal{M}(Tu,u,u,t)}-1 \leq \beta\left(\frac{1}{\mathcal{M}(u,Tu,u,t)}-1\right)+\delta\left(\frac{1}{\mathcal{M}(Tu,Tu,u,2t)}-1\right) \quad \leq \lambda\left(\frac{1}{\mathcal{M}(u,u,Tu,t)}-1\right)$$

Since $\lambda < 1$, We have $\mathcal{M}$ (u, u, Tu, t) = 1 .Thus Tu = u.
We find fixed point is unique.

Remark: 2.7 A similar proof we can find that the generalized contraction conditions (2.5.1) and (2.6.1) are equivalent to following:

$$\frac{1}{\mathcal{M}(Tx,Ty,Tz,t)}-1 \leq \lambda \max\left\{\left(\frac{1}{\mathcal{M}(x,y,z,t)}-1\right),\ \left(\frac{1}{\mathcal{M}(x,Tx,z,t)}-1\right)\right.$$

$$\left.\left(\frac{1}{\mathcal{M}(x,Ty,Tz,t)}-1\right),\ \left(\frac{1}{\mathcal{M}(y,Ty,Tz,t)}-1\right),\ \left(\frac{1}{\mathcal{M}(Tx,Ty,z,t)}-1\right)\right\}$$

$$\frac{1}{\mathcal{M}(Tx,Ty,Tz,t)}-1 \leq \lambda \max\left\{\left(\frac{1}{\mathcal{M}(x,y,z,t)}-1\right),\ \left(\frac{1}{\mathcal{M}(x,Tx,z,t)}-1\right)\right.$$

$$\left.\left(\frac{1}{\mathcal{M}(x,Ty,Tz,t)}-1\right),\left(\frac{1}{\mathcal{M}(y,Ty,Tz,2t)}-1\right),\left(\frac{1}{\mathcal{M}(Tx,Ty,z,2t)}-1\right)\right\}$$

respectively, where $\lambda \epsilon [0, 1]$.

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