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Iteration Schemes for Two Hemicontractive Mappings in Arbitrary Banach Spaces

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Abstract

The purpose of this paper is to characterize conditions for the convergence of the Ishikawa iterative scheme with errors in the sense of Agarwal et al. to the common fixed point of two ϕ-hemicontractive mappings in a nonempty convex subset of an arbitrary Banach space.

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1 Introduction and preliminaries

Let K be a nonempty subset of an arbitrary Banach space X and X^* be its dual space. The symbols $D(T)$ and $F(T)$ stand for the domain and the set of fixed points of T (for a single-valued map $T : X \to X$, $x \in X$ is called a *fixed point* of T iff $T(x) = x$). We denote by J the normalized duality mapping from X to 2^{X^*} defined by

$$J(x) = \{f^* \in X^* : \langle x, f^* \rangle = \|x\|^2 = \|f^*\|^2\}, \quad x \in X.$$

We shall denote the single-valued duality mapping by j.

Let $T : D(T) \subset X \to X$ be a mapping.

Definition 1.1. ([4], [10]) (1) T is said to be *strongly pseudocontractive* if there exists a constant $t > 1$ such that for all x, $y \in D(T)$, there exists $j(x-y) \in J(x-y)$ satisfying

$$\operatorname{Re} \langle Tx - Ty, j(x-y) \rangle \le t^{-1} \|x-y\|^2 .$$

(2) T is said to be *strictly hemicontractive* if $F(T) \neq \emptyset$ and there exists a constant $t > 1$ such that for all $x \in D(T)$ and $q \in F(T)$, there exists $j(x-y) \in J(x-y)$ satisfying

$$\operatorname{Re} \langle Tx - q, j(x-q) \rangle \le t^{-1} \|x-q\|^2 .$$

(3) T is said to be *ϕ-strongly pseudocontractive* if there exists a strictly increasing function $\phi : [0, \infty) \to [0, \infty)$ with $\phi(0) = 0$ such that for all x, $y \in D(T)$, there exists $j(x-y) \in J(x-y)$ satisfying

$$\operatorname{Re} \langle Tx - Ty, j(x-y) \rangle \le \|x-y\|^2 - \phi(\|x-y\|) \|x-y\| .$$

(4) T is said to be *ϕ-hemicontractive* if $F(T) \neq \emptyset$ and if there exists a strictly increasing function $\phi : [0, \infty) \to [0, \infty)$ with $\phi(0) = 0$ such that for all $x \in D(T)$ and $q \in F(T)$, there exists $j(x-y) \in J(x-y)$ satisfying

$$\operatorname{Re} \langle Tx - q, j(x-q) \rangle \le \|x-q\|^2 - \phi(\|x-q\|) \|x-q\| .$$

Clearly, each strictly hemicontractive mapping is ϕ-hemicontractive. It was shown in [4], [10] that the classes of strongly pseudocontractive (respectively, ϕ-strongly pseudocontractive) mappings with fixed points are proper subclasses of the classes of strictly hemicontractive (respectively, ϕ-hemicontractive) mappings.

Chidume [3] established that the Mann iteration sequence converges strongly to the unique fixed point of T in case T is a Lipschitz strongly pseudo-contractive mapping from a bounded closed convex subset of L_p (or l_p) into itself. Afterwards, several authors generalized this result of Chidume in various directions [5]-[9], [11] and [12]-[14].

The purpose of this paper is to characterize conditions for the convergence of the Ishikawa iterative scheme with errors in the sense of Agarwal et al. [1] to the common fixed point of two ϕ-hemicontractive mappings in a nonempty convex subset of an arbitrary Banach space. Our results improve and generalize most results in recent literature [5], [6], [8] and [13].

The following resuls are now well known.

Lemma 1.2. ([2]) *For all $x, y \in X$ and $j(x+y) \in J(x+y)$, we have*

$$\|x+y\|^2 \leq \|x\|^2 + 2\mathrm{Re}\,\langle y, j(x+y)\rangle .$$

Lemma 1.3. *Let $\{\theta_n\}$ be a sequence of nonnegative real numbers and $\{\lambda_n\}$ be a real sequence satisfying $0 \leq \lambda_n \leq 1$ and $\sum_{n=0}^{\infty} \lambda_n = \infty$. Suppose that there exists a strictly increasing function $\phi : [0,\infty) \to [0,\infty)$ with $\phi(0) = 0$. If there exists a positive integer n_0 such that*

$$\theta_{n+1}^2 \leq \theta_n^2 - \lambda_n \phi(\theta_{n+1})\theta_{n+1} + \sigma_n + \gamma_n, \quad \forall n \geq n_0,$$

where $\sigma_n \geq 0$ for all $n \geq 1$, $\sigma_n = \mathrm{o}(\lambda_n)$ and $\sum_{n=0}^{\infty} \gamma_n < \infty$, then $\lim_{n\to\infty} \theta_n = 0$.

2 Main results

Now we prove our main results.

Theorem 2.1. *Let K be a nonempty convex subset of an arbitrary Banach space X and $T, S : K \to K$ be two uniformly continuous and ϕ-hemicontractive mappings. Suppose that $\{u_n\}_{n=0}^{\infty}$ and $\{v_n\}_{n=0}^{\infty}$ are bounded sequences in K and $\{a_n\}_{n=0}^{\infty}$, $\{b_n\}_{n=0}^{\infty}$, $\{c_n\}_{n=0}^{\infty}$, $\{a'_n\}_{n=0}^{\infty}$, $\{b'_n\}_{n=0}^{\infty}$ and $\{c'_n\}_{n=0}^{\infty}$ are sequences in $[0,1]$ satisfying conditions*

(i) $a_n + b_n + c_n = a'_n + b'_n + c'_n = 1$ *for all* $n \geq 0$,

(ii) $\lim_{n\to\infty} b_n = \lim_{n\to\infty} c_n = \lim_{n\to\infty} b'_n = 0$,

(iii) $\sum_{n=0}^{\infty} b'_n = \infty$,

(iv) $c'_n = o(b'_n)$.

For arbitrary $x_0 \in K$, let $\{x_n\}_{n=0}^{\infty}$ be a sequence defined iteratively by

$$\begin{cases} y_n = a_n x_n + b_n S x_n + c_n u_n, \\ x_{n+1} = a'_n x_n + b'_n T y_n + c'_n v_n, \quad n \geq 0. \end{cases} \tag{2.1}$$

Then the following conditions are equivalent:

(a) $\{x_n\}_{n=0}^{\infty}$ *converges strongly to the common fixed point* q *of* T *and* S.

(b) $\{Ty_n\}_{n=0}^{\infty}$ *is bounded.*

Proof. From (iv), we have $c'_n = t_n b'_n$, where $t_n \to 0$ as $n \to \infty$. Since T and S are ϕ-hemicontractive, it follows that $F(T) \cap F(S)$ is a singleton. Let $F(T) \cap F(S) = \{q\}$ for some $q \in K$.

Suppose that $\lim_{n\to\infty} x_n = q$. Then from (ii) and the uniform continuity of T and S, we yield that

$$\lim_{n\to\infty} y_n = \lim_{n\to\infty} [a_n x_n + b_n S x_n + c_n u_n] = q,$$

which implies that $\lim_{n\to\infty} Ty_n = q$. Therefore $\{Ty_n\}_{n=0}^{\infty}$ is bounded.

Put

$$M_1 = \|x_0 - q\| + \sup_{n\geq 0} \|Ty_n - q\| + \sup_{n\geq 0} \|u_n - q\| + \sup_{n\geq 0} \|v_n - q\|.$$

Obviously $M_1 < \infty$. It is clear that $\|x_0 - p\| \leq M_1$. Let $\|x_n - p\| \leq M_1$. Next we will prove that $\|x_{n+1} - p\| \leq M_1$.

Consider

$$\begin{aligned}
&\|x_{n+1} - p\| \\
&= \|a'_n(x_n - q) + b'_n(Ty_n - q) + c'_n(v_n - q)\| \\
&\leq (1 - b'_n) \|x_n - q\| + b'_n \|Ty_n - q\| + c'_n \|v_n - q\| \\
&\leq (1 - b'_n) M_1 + b'_n \|Ty_n - q\| + c'_n \|v_n - q\| \\
&= (1 - b'_n)\Big(\|x_0 - q\| + \sup_{n\geq 0} \|Ty_n - q\| + \sup_{n\geq 0} \|u_n - q\| + \sup_{n\geq 0} \|v_n - p\| \Big) \\
&\quad + b'_n \|Ty_n - q\| + c'_n \|v_n - q\| \\
&\leq \|x_0 - q\| + \Big((1 - b'_n) \sup_{n\geq 0} \|Ty_n - q\| + b'_n \|Ty_n - q\| \Big) \\
&\quad + \sup_{n\geq 0} \|u_n - q\| + \Big((1 - b'_n) \sup_{n\geq 0} \|v_n - q\| + b'_n \|v_n - q\| \Big) \\
&\leq \|x_0 - q\| + \sup_{n\geq 0} \|Ty_n - q\| + \sup_{n\geq 0} \|u_n - q\| + \sup_{n\geq 0} \|v_n - q\| \\
&= M_1.
\end{aligned}$$

So, from the above discussion, we can conclude that the sequence $\{x_n - q\}_{n=0}^{\infty}$ is bounded. Since S is uniformly continuous, so $\{\|Sx_n - q\|\}_{n=0}^{\infty}$ is also bounded. Thus there is a constant $M_2 > 0$ satisfying

$$\begin{aligned}
M_2 = \sup_{n\geq 0} \|x_n - q\| &+ \sup_{n\geq 0} \|Sx_n - q\| + \sup_{n\geq 0} \|y_n - q\| \\
&+ \sup_{n\geq 0} \|Ty_n - q\| + \sup_{n\geq 0} \|u_n - q\| + \sup_{n\geq 0} \|v_n - q\|.
\end{aligned}$$

Denote $M = M_1 + M_2$. Obviously $M < \infty$.

Let $w_n = \|Ty_n - Tx_{n+1}\|$ for each $n \geq 0$. The uniform continuity of T ensures that

$$\lim_{n\to\infty} w_n = 0$$

because

$$\begin{aligned}
&\|y_n - x_{n+1}\| \\
&= \|b_n(Sx_n - x_n) + b'_n(x_n - Ty_n) + c_n(u_n - x_n) - c'_n(v_n - x_n)\| \\
&\leq b_n \|Sx_n - x_n\| + b'_n \|x_n - Ty_n\| + c_n \|u_n - x_n\| + c'_n \|v_n - x_n\| \\
&\leq 2M_2(b_n + c_n + (1 + t_n)b'_n) \\
&\to 0
\end{aligned}$$

as $n \to \infty$.

By virtue of Lemma 1.2 and (2.1), we infer that

$$\begin{aligned}
&\|x_{n+1} - q\|^2 \\
&= \|a'_n(x_n - q) + b'_n(Ty_n - q) + c'_n(v_n - q)\|^2 \\
&\leq (1 - b'_n)^2 \|x_n - q\|^2 + 2b'_n \mathrm{Re}\langle Ty_n - q, j(x_{n+1} - q)\rangle \\
&\quad + 2c'_n \mathrm{Re}\langle v_n - q, j(x_{n+1} - q)\rangle \\
&\leq (1 - b'_n)^2 \|x_n - q\|^2 + 2b'_n \mathrm{Re}\langle Ty_n - Tx_{n+1}, j(x_{n+1} - q)\rangle \\
&\quad + 2b'_n \mathrm{Re}\langle Tx_{n+1} - q, j(x_{n+1} - q)\rangle + 2c'_n \|v_n - q\| \|x_{n+1} - q\| \\
&\leq (1 - b'_n)^2 \|x_n - q\|^2 + 2b'_n \|Ty_n - Tx_{n+1}\| \|x_{n+1} - q\| \\
&\quad + 2b'_n \|x_{n+1} - q\|^2 - 2b'_n\phi(\|x_{n+1} - q\|) \|x_{n+1} - q\| + 2M^2 c'_n \\
&= (1 - b'_n)^2 \|x_n - q\|^2 + 2Mb'_n w_n + 2b'_n \|x_{n+1} - q\|^2 \\
&\quad - 2b'_n\phi(\|x_{n+1} - q\|) \|x_{n+1} - q\| + 2M^2 c'_n.
\end{aligned} \tag{2.2}$$

Consider

$$\begin{aligned}
\|x_{n+1} - p\|^2 &= \|a'_n(x_n - q) + b'_n(Ty_n - q) + c'_n(v_n - q)\|^2 \\
&\leq a'_n \|x_n - q\|^2 + b'_n \|Ty_n - q\|^2 + c'_n \|v_n - q\|^2 \\
&\leq \|x_n - q\|^2 + M^2(b'_n + c'_n),
\end{aligned} \tag{2.3}$$

where the first inequality holds by the convexity of $\|\cdot\|^2$.

Substituting (2.3) in (2.2), we get

$$\begin{aligned}
\|x_{n+1} - q\|^2 &\leq [(1 - b'_n)^2 + 2b'_n] \|x_n - q\|^2 + 2Mb'_n(w_n + M(b'_n + 2t_n)) \\
&\quad - 2b'_n\phi(\|x_{n+1} - q\|) \|x_{n+1} - q\| \\
&\leq \|x_n - q\|^2 + Mb'_n(3Mb'_n + 2(w_n + 2Mt_n)) \\
&\quad - 2b'_n\phi(\|x_{n+1} - q\|) \|x_{n+1} - q\| \\
&= \|x_n - q\|^2 + b'_n l_n - 2b'_n\phi(\|x_{n+1} - q\|) \|x_{n+1} - q\|,
\end{aligned} \tag{2.4}$$

where

$$l_n = M(3Mb'_n + 2(w_n + 2Mt_n)) \to 0 \tag{2.5}$$

as $n \to \infty$.

Let $\delta = \inf\{\|x_{n+1} - q\| : n \geq 0\}$. We claim that $\delta = 0$. Otherwise $\delta > 0$. Thus (2.5) implies that there exists a positive integer $N_1 > N_0$ such that $l_n < \phi(\delta)\delta$ for each $n \geq N_1$. In view of (2.4), we conclude that

$$\|x_{n+1} - q\|^2 \leq \|x_n - q\|^2 - \phi(\delta)\delta b'_n, \quad n \geq N_1,$$

which implies that

$$\phi(\delta)\delta \sum_{n=N_1}^{\infty} b'_n \leq \|x_{N_1} - q\|^2,$$

which contradicts (iii). Therefore $\delta = 0$. Thus there exists a subsequence $\{x_{n_i+1}\}_{i=0}^{\infty}$ of $\{x_{n+1}\}_{n=0}^{\infty}$ such that

$$\lim_{i\to\infty} x_{n_i+1} = q. \tag{2.6}$$

Let $\epsilon > 0$ be a fixed number. By virtue of (2.5)and (2.6), we can select a positive integer $i_0 > N_1$ such that

$$\left\|x_{n_{i_0}+1} - q\right\| < \epsilon, \quad l_n < \phi(\epsilon)\epsilon, \quad n \geq n_{i_0}. \tag{2.7}$$

Let $p = n_{i_0}$. By induction, we show that

$$\|x_{p+m} - q\| < \epsilon, \quad m \geq 1. \tag{2.8}$$

Observe that (2.7) means that (2.8) is true for $m = 1$. Suppose that (2.8) is true for some $m \geq 1$. If $\|x_{p+m+1} - q\| \geq \epsilon$, by (2.4) and (2.7), we know that

$$\begin{aligned}
\epsilon^2 &\leq \|x_{p+m+1} - q\|^2 \\
&\leq \|x_{p+m} - q\|^2 + \frac{b'_{p+m} l_{p+m}}{1 - 2b'_{p+m}} \\
&\quad - \frac{2b'_{p+m}}{1 - 2b'_{p+m}} \phi(\|x_{p+m+1} - q\|) \|x_{p+m+1} - q\| \\
&< \epsilon^2 + \frac{b'_{p+m}\phi(\epsilon)\epsilon}{1 - 2b'_{p+m}} - \frac{2b'_{p+m}\phi(\epsilon)\epsilon}{1 - 2b'_{p+m}} < \epsilon^2,
\end{aligned}$$

which is impossible. Hence $\|x_{p+m+1} - q\| < \epsilon$. That is, (2.8) holds for all $m \geq 1$. Thus (2.8) ensures that $\lim_{n\to\infty} x_n = q$. This completes the proof. $\square$

Using the method of proof in Theorem 2.1, we have the following result.

Theorem 2.2. *Let X, K, T, S, $\{u_n\}_{n=0}^{\infty}$, $\{v_n\}_{n=0}^{\infty}$ be as in Theorem 2.1. Suppose that $\{a_n\}_{n=0}^{\infty}$, $\{b_n\}_{n=0}^{\infty}$, $\{c_n\}_{n=0}^{\infty}$, $\{a'_n\}_{n=0}^{\infty}$, $\{b'_n\}_{n=0}^{\infty}$ and $\{c'_n\}_{n=0}^{\infty}$ are sequences in $[0,1]$ satisfying the conditions* (i)-(iii) *and $\sum_{n=0}^{\infty} c'_n < \infty$.*

For arbitrary $x_0 \in K$, let $\{x_n\}_{n=0}^{\infty}$ be the sequence defined iteratively by (2.1). *Then the conclusions of Theorem 2.1 hold.*

Proof. Substituting (2.3) in (2.2), we get

$$\begin{aligned}
&\|x_{n+1} - q\|^2 \\
&\le [(1-b'_n)^2 + 2b'_n] \|x_n - q\|^2 + 2Mb'_n(w_n + M(b'_n + c'_n)) \\
&\quad - 2b'_n\phi(\|x_{n+1} - q\|) \|x_{n+1} - q\| + 2M^2c'_n \\
&\le \|x_n - q\|^2 + Mb'_n(2w_n + M(3b'_n + 2c'_n)) \\
&\quad - 2b'_n\phi(\|x_{n+1} - q\|) \|x_{n+1} - q\| + 2M^2c'_n \\
&= \|x_n - q\|^2 - 2b'_n\phi(\|x_{n+1} - q\|) \|x_{n+1} - q\| + b'_n l'_n + 2M^2c'_n,
\end{aligned}$$

where

$$l'_n = M(2w_n + M(3b'_n + 2c'_n)) \to 0,$$

as $n \to \infty$.

It follows from Lemma 1.3 that $\lim_{n\to\infty} \|x_n - q\| = 0$. □

Corollary 2.3. *Let K be a nonempty convex subset of an arbitrary Banach space X and $T : K \to K$ be a uniformly continuous and ϕ-hemicontractive mappings. Suppose that $\{u_n\}_{n=0}^{\infty}$ and $\{v_n\}_{n=0}^{\infty}$ are bounded sequences in K and $\{a_n\}_{n=0}^{\infty}$, $\{b_n\}_{n=0}^{\infty}$, $\{c_n\}_{n=0}^{\infty}$, $\{a'_n\}_{n=0}^{\infty}$, $\{b'_n\}_{n=0}^{\infty}$ and $\{c'_n\}_{n=0}^{\infty}$ are sequences in $[0,1]$ satisfying the conditions* (i)-(iv).

For arbitrary $x_0 \in K$, let $\{x_n\}_{n=0}^{\infty}$ be a sequence defined iteratively by

$$\begin{cases} y_n = a_n x_n + b_n T x_n + c_n u_n, \\ x_{n+1} = a'_n x_n + b'_n T y_n + c'_n v_n, \quad n \ge 0. \end{cases} \tag{2.9}$$

Then the following conditions are equivalent:

(*a*) *$\{x_n\}_{n=0}^{\infty}$ converges strongly to the fixed point q of T.*
(*b*) *$\{Ty_n\}_{n=0}^{\infty}$ is bounded.*

Corollary 2.4. *Let X, K, T, $\{u_n\}_{n=0}^{\infty}$ and $\{v_n\}_{n=0}^{\infty}$, be as in Corollary 2.3. Suppose that $\{a_n\}_{n=0}^{\infty}$, $\{b_n\}_{n=0}^{\infty}$, $\{c_n\}_{n=0}^{\infty}$, $\{a'_n\}_{n=0}^{\infty}$, $\{b'_n\}_{n=0}^{\infty}$ and $\{c'_n\}_{n=0}^{\infty}$ are sequences in $[0,1]$ satisfying the conditions* (i)-(iii) *and $\sum_{n=0}^{\infty} c'_n < \infty$.*

For arbitrary $x_0 \in K$, let $\{x_n\}_{n=0}^{\infty}$ be the sequence defined iteratively by (2.9). *Then the conclusions of Corollary 2.3 hold.*

Corollary 2.5. *Let K be a nonempty convex subset of an arbitrary Banach space X and $T, S : K \to K$ be two uniformly continuous and ϕ-hemicontractive* *mappings. Suppose that* $\{\alpha_n\}_{n=0}^{\infty}$ *and* $\{\beta_n\}_{n=0}^{\infty}$ *are sequences in* $[0,1]$ *satisfying conditions*

(v) $\lim_{n\to\infty}\alpha_n = 0 = \lim_{n\to\infty}\beta_n$,

(vi) $\sum_{n=0}^{\infty}\alpha_n = \infty$.

For arbitrary $x_0 \in K$, *let* $\{x_n\}_{n=0}^{\infty}$ *be a sequence defined iteratively by*

$$\begin{cases} y_n = (1-\beta_n)x_n + \beta_n S x_n, \\ x_{n+1} = (1-\alpha_n)x_n + \alpha_n T y_n, \quad n \geq 0. \end{cases}$$

Then the following conditions are equivalent:

(*a*) $\{x_n\}_{n=0}^{\infty}$ *converges strongly to the common fixed point* q *of* T *and* S.

(*b*) $\{Ty_n\}_{n=0}^{\infty}$ *is bounded.*

Corollary 2.6. *Let* X, K, T *be as in Corollary 2.5. Suppose that* $\{\alpha_n\}_{n=0}^{\infty}$ *and* $\{\beta_n\}_{n=0}^{\infty}$ *are sequences in* $[0,1]$ *satisfying the conditions* (v) *and* (vi).

For arbitrary $x_0 \in K$, *let* $\{x_n\}_{n=0}^{\infty}$ *be a sequence defined iteratively by*

$$\begin{cases} y_n = (1-\beta_n)x_n + \beta_n T x_n, \\ x_{n+1} = (1-\alpha_n)x_n + \alpha_n T y_n, \quad n \geq 0. \end{cases}$$

Then the following conditions are equivalent:

(*a*) $\{x_n\}_{n=0}^{\infty}$ *converges strongly to the fixed point* q *of* T.

(*b*) $\{Ty_n\}_{n=0}^{\infty}$ *is bounded.*

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