On Modified Interval Symmetric Single-Step Procedure ISS2-5D for the Simultaneous Inclusion of Polynomial Zeros

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Abstract

In this paper, we present a new modified interval symmetric single-step procedure ISS2-5D which is the extension from the previous procedure ISS2. The algorithm of ISS2-5D includes the introduction of reusable correctors $\delta^{(k)}$ for $k \geq 0$. The procedure is tested on five test polynomials and the results are obtained using MATLAB 2007 software in association with IntLab V5.5 toolbox to record the CPU times and the number of iterations.

Keywords: interval procedure, polynomial zeros, symmetric single-step, simultaneous inclusion
1 Introduction

Interval iterative procedure for simultaneous inclusion of simple polynomial zeros were discussed in [1,3,5,8,11] In this paper, we consider the procedures developed by [4,6,7,9,13] in order to describe the algorithm of the interval symmetric single-step procedure ISS2-5D. This procedure needs some pre-conditions for initial intervals \( X_i^{(0)}(i = 1,...,n) \) to converge to the zeros \( x_i^*(i = 1,...,n) \) respectively, starting with some disjoint intervals \( X_i^{(0)}(i = 1,...,n) \) each of which contains a polynomial zero. It will produce bounded closed intervals which will trap the required zero.

The forward step [13] is modified by adding a \( \delta = \delta_i(i = 1,...,n)(k \geq 0)(1(c)) \) on the second part of the summation of the denominator (see (1(d)). The backward step of this procedure comes from [8]. The interval analysis is very straightforward compared to the analysis of the point procedures [6,9]. The programming language used is Matlab 2007a with the Intlab V5.5 toolbox [12]. The effectiveness of our procedure is measured numerically using CPU time and the number of iterations.

2 The Interval Symmetric Single-Step Procedure ISS2-5D

The interval symmetric single-step procedure ISS2-5D is an extension of the interval single-step procedure ISS2 [13] based on [2,3,6,8,9,10]. The sequences \( X_i^{(k)}(i = 1,...,n) \) are generated as follows.

Step 1: \( X_i^{(0)} = X_i^{(0)} \) (Initial intervals) \hspace{1cm} (1a)
Step 2: For \( k \geq 0, x_i^{(k)} = \text{mid}(X_i^{(k)}), (i = 1,...,n) \) \hspace{1cm} (1b)
Step 3: Let \( \delta_i^{(k)} = -\frac{p(x_i^{(k)})}{p'(x_i^{(k)})}(i = 1,...,n) \) \hspace{1cm} (1c)
Step 4:

\[
X_i^{(k,i)} = \left\{ x_i^{(k)} + \frac{\delta_i^{(k)}}{1 + \delta_i^{(k)} \left( \frac{1}{\sum_{j=1}^n x_i^{(k)} - x_j^{(k)}} + \sum_{j=1}^n x_i^{(k)} - x_j^{(k)} - 5\delta_j^{(k)} \right)} \right\} \cap X_i^{(k)} \hspace{1cm} (1d)
\]

\((i = 1,...,n)\)

Step 5:
Step 6: \( X_{i}^{(k,2)} = X_{i}^{(k,2)} \ (i = 1, \ldots, n) \)  

Step 7: If \( w(X_{i}^{k+1}) < \varepsilon \), then stop. Else set \( k = k + 1 \) and go to Step 2. 

Step 4 is from [6] and pointed out without \( \delta \) by [9], while Step 5 is from [8].

The procedure ISS2-5D has the following attractive features:

(a) The use of \( 5\delta_j \) instead of \( \delta_j^{(k)} \) as in [6].

(b) The values \( \delta_j^{(k)} \) computed for use in Step 4 are reused in Step 5.

(c) The summations \( \sum_{j=1}^{n} \frac{1}{x_j^{(k+1)} - x_j^{(k)}} \) used in Step 4 are reused in Step 5.

(d) \( X_{n}^{(k,1)} = X_{n}^{(k,2)} \ (k \geq 0) \) so that \( x_{n}^{(k,2)} \) need not be computed.

3 Numerical Results and Discussion

We used the Intlab V5.5 toolbox [12] for MATLAB R2007 to get the following results below. The algorithms ISS and ISS2-5D are run on five test polynomials where the stopping criterion used is \( w^{(i)} \leq 10^{-10} \).

Consider the following polynomial [3]

\[
p(\lambda) = \det(\lambda I - A),
\]

where

\[
A = \begin{pmatrix}
  a_1 & b_1 \\
  b_1 & a_2 & \cdots & 0 \\
  \vdots & \ddots & \ddots & \ddots \\
  0 & \cdots & a_{n-1} & b_{n-1} \\
  b_{n-1} & a_n
\end{pmatrix}
\]

and
\[ f^{(0)}(\lambda) = 1, \]
\[ f^{(1)}(\lambda) = (\lambda - a_1), \]
\[ f^{(k)}(\lambda) = (\lambda - a_k) f^{(k-1)}(\lambda)(b_{k-1})^2 f^{(k-2)}(\lambda) \quad (2 \leq k \leq n), \]
\[ p(\lambda) = f^{(n)}(\lambda). \quad (4) \]

**Test Polynomial 1:** Salim [13]
For this example from [6]

\( n = 5, \)
\[ a_i = -1; a_2 = 1; a_3 = 2; a_4 = -0.5; a_5 = 0.5 \]
\[ b_i = 1(i = 1, \ldots, n-1), \]

Initial Intervals:
\[ X^{(0)}_1 = [-1.55, 0.35], X^{(0)}_2 = [0.89, 1.59], X^{(0)}_3 = [1.60, 2.59], X^{(0)}_4 = [-0.99, -0.25], X^{(0)}_5 = [0.39, 0.80] \]

**Test Polynomial 2:** Salim [13]
The polynomial is given by (3) with

\( n = 9, \)
\[ a_1 = 15; a_2 = -15; a_3 = 7; a_4 = 4; a_5 = -10a_6 = -7; a_7 = -4 \]
\[ b_i = 1(i = 1, \ldots, n) \]

Initial intervals:
\[ X^{(0)}_1 = [12.0, 17.0]; X^{(0)}_2 = [8.6, 11.2]; X^{(0)}_3 = [5.2, 8.4]; X^{(0)}_4 = [2.4, 5.0]; X^{(0)}_5 = [-2.0, 2.2]; \]
\[ X^{(0)}_6 = [6.4, -2.9]; X^{(0)}_7 = [-8.2, -6.5]; X^{(0)}_8 = [-11.8, -8.0]; X^{(0)}_9 = [-17.2, -13.5] \]

**Test Polynomial 3:** Monsi [8]
The polynomial is given by (3) with

\( n = 9, \)
\[ a_1 = -2.4519; a_2 = -1.9021; a_3 = 1.9275; a_4 = 1.5765; a_5 = 1.1867; a_6 = 0.3210; \]
\[ a_7 = 0.2674; a_8 = -0.1254; a_9 = 0.0435 \]
\[ b_i = 1(i = 1, \ldots, n-1) \]

Initial intervals:
\[ X^{(0)}_1 = [-2.5, -2.0]; X^{(0)}_2 = [-2.0, -1.89]; X^{(0)}_3 = [-0.13, -0.10]; X^{(0)}_4 = [0.03, 0.05]; \]
\[ X^{(0)}_5 = [0.25, 0.28]; X^{(0)}_6 = [0.30, 0.39]; X^{(0)}_7 = [1.09, 1.31]; X^{(0)}_8 = [1.49, 1.70]; \]
\[ X^{(0)}_9 = [1.80, 2.10] \]

**Test Polynomial 4:** Monsi [8]
The polynomial is given by (2) with
\( n = 6, \)
\( a_1 = -2; a_2 = 0.3; a_3 = 0.4; a_4 = 1; a_5 = 2; a_6 = 4 \)
\( b_i = 1(i = 1, \ldots, n) \)

Initial intervals:
\( X_1^{(0)} = [-3.9, -1.5]; X_2^{(0)} = [-1.2, 0.39]; X_3^{(0)} = [0.4, 0.99]; X_4^{(0)} = [1.01, 1.99]; \)
\( X_5^{(0)} = [2.09, 3.1]; X_6^{(0)} = [3.19, 5.3] \)

Test Polynomial 5: Monsi [8]
The polynomial is given by (3) with
\( n = 10, \)
\( a_1 = -33; a_2 = -21; a_3 = -13; a_4 = -7; a_5 = -1; a_6 = 1; a_7 = 7; a_8 = 13; a_9 = 21; a_{10} = 33 \)
\( b_i = 1(i = 1, \ldots, n) \)

Initial intervals:
\( X_1^{(0)} = [-35, -30]; X_2^{(0)} = [-25, -19]; X_3^{(0)} = [-15, -11]; X_4^{(0)} = [-9, -5]; X_5^{(0)} = [-2, -0.5]; X_6^{(0)} = [0.4, 2]; \)
\( X_7^{(0)} = [5, 9]; X_8^{(0)} = [11, 15]; X_9^{(0)} = [19, 25]; X_{10}^{(0)} = [30, 35]; \)

<table>
<thead>
<tr>
<th>Polynomial</th>
<th>Degree</th>
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<th>ISS2-5D</th>
</tr>
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<tr>
<td></td>
<td></td>
<td>No. of iterations</td>
<td>CPU time</td>
</tr>
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<td>2</td>
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<td>1</td>
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<td>4</td>
<td>6</td>
<td>2</td>
<td>0.099609</td>
</tr>
<tr>
<td>5</td>
<td>10</td>
<td>2</td>
<td>0.134375</td>
</tr>
</tbody>
</table>

Table 1 shows that the procedure ISS2-5D required less CPU times than the procedure ISS2 for all 5 test polynomials, and required less number of iterations meaning ISS2-5D converges faster than ISS2. However, for test polynomials 2, 3 and 5, the number of iterations for both procedures is the same, but the time consumed for procedure ISS2-5D is still less than the ISS2 procedure.

4 Conclusion

The above results have shown numerically that the procedure ISS2-5D performs better than ISS2 in terms of CPU times and number of iterations. The attractive features of our procedure mentioned in Section 2 contribute to these performances.
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References


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