An Efficient Interval Symmetric Single Step
Procedure ISS1-5D for Simultaneous Bounding
of Real Polynomial Zeros

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Abstract

A new modified interval symmetric single-step procedure ISS1-5D which is the
extension from the previous ISS1 is proposed. In procedure ISS1 we define
informational efficiency of a method as the higher $R$-order of convergence
evaluation. The procedure is tested on five test polynomials and the results are
obtained using MATLAB 2007 software in association with IntLab V5.5 toolbox
to record the CPU times and the number of iterations.

Keywords: Analysis interval, convergence, CPU times, zeros of a polynomial

1 Introduction

Interval iterative procedure for simultaneous inclusion of simple polynomial
zeros determines the bounded closed intervals which contain exact polynomial
zeros. It can be used to determine very narrow computationally rigorous bound on
polynomial zeros. It is a very significant way of obtaining reliable bounds on the
zeros as the intervals sequences generated by the procedures always converge to
the zeros. In this paper, we refer to the methods established by [10], [3], [11], [ 9],
Consider $p : R^1 \to R^1$ a polynomial of degree $n > 1$ defined by

$$p(x) = a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \ldots + a_0 = \prod_{j=1}^{n} (x - x_j^*)$$

(1)

where $a_i \in R(i = 1, \ldots, n)$ are given. Suppose that $p$ has $n$ distinct values $x_j^* \in R(i = 1, \ldots, n)$ and that $X_i^{(0)} \in I(R)$ (set of real intervals) $(i = 1, \ldots, n)$ are such that

$$x_j^* \in X_i^{(0)} (i = 1, \ldots, n)$$

and

$$X_i^{(0)} \cap X_j^{(0)} = \emptyset , (i, j = 1, \ldots, n; i \neq j)$$

(2)

The concept of $R$-order of convergence is discussed in detail in [2], [7], [11] and [12]. The $R$-order of the procedure $I$ which converges to $x^* = (x_1^*, \ldots, x_n^*)^T$ is denoted by $O_R(I, x^*)$ and the $R$-factor of a null sequence $w^{(k)}$ generated from the procedure $I$ is denoted by $R_p(w^{(k)})$, where $p \geq 1$ and $w^{(k)}$ is a null sequence generated from the procedure $I$.

2 The Interval Symmetric Single-Step Procedure ISS1-5D

The interval symmetric single-step procedure ISS1-5D is an extension of the interval single-step procedure IS and ISS1 of [1], [2] and [8]. The interval sequence $X_i^{(k)} (i = 1, \ldots, n)$ are generated as follows.

Step 1: Set $k = 0$,

(4a)

Step 2: For $k \geq 0, x_i^{(k)} = \text{mid} \left( X_i^{(k)} \right), (i = 1, \ldots, n)$;

(4b)

Step 3: Let $\delta_j = \delta_j^{(k)} = -p \left( x_i^{(k)} \right) \prod_{j \neq i}^{n} (x_j^{(k)} - x_i^{(k)})$

(4c)

Step 4:

$$X_i^{(k, b)} = \left\{ x_i^{(k)} - \frac{p(x_i^{(k)})}{\prod_{j=1}^{i-1} (x_i^{(k)} - x_j^{(k)}) \prod_{j=i+1}^{n} (x_j^{(k)} - X_j^{(k, b)}) - 5 \delta_j} \right\} \cap X_i^{(k)}$$

(4d)

$$\left( i = 1, \ldots, n \right)$$

Step 5:
Efficient interval symmetric single step procedure ISS1-5D

\[ X_{j}^{(k,2)} = \left\{ x_{i}^{(k)} \right\} \bigcap \left( \prod_{j=1}^{i-1} \left( x_{i}^{(k)} - X_{j}^{(k,1)} \right) \prod_{j=i+1}^{n} \left( x_{i}^{(k)} - X_{j}^{(k,2)} \right) \right) \quad (i = n, \ldots, 1) \]

Step 6: \( X_{j}^{(k+1)} = X_{j}^{(k,2)} \) \hspace{1cm} (4e)

Step 7: If \( w(X_{i}^{k+1}) < \varepsilon \), then stop, else set \( k = k + 1 \) and go to Step 2. \hspace{1cm} (4f)

The procedure ISS1-5D has the following attractive features:

(a) The use of \( 5\delta \) in (4d) will improve the efficiency of this procedure.
(b) The values \( p(x_{i}^{(k)}) (i = 1, \ldots, n) \) computed for use in (4d) are re-used in (4e).
(c) The products \( \prod_{j=1}^{i-1} (x_{i}^{(k)} - x_{j}^{(k,1)}) \quad (i = 2, \ldots, n) \) computed for use in (4d) are re-used in (4e).
(d) \( x_{n}^{(k,1)} = x_{n}^{(k,2)} \) \( (k \geq 0) \) so that \( x_{n}^{(k,2)} \) need not be computed.

3 Numerical Results and Discussion

Table 1 shows the list of all test polynomials of degree \( n \) while Table 2 shows the comparison of the number of iteration and CPU time in seconds, between procedures ISS1 and ISS1-5D obtained using MATLAB 2007 software in association with IntLab V5.5 toolbox [14].

<table>
<thead>
<tr>
<th>Example</th>
<th>Degree ( n )</th>
<th>Test Polynomial ( p )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4</td>
<td>( \lambda^4 - 10\lambda^3 + 35\lambda^2 - 50\lambda + 24 )</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>( \lambda^5 - 35.6\lambda^4 + 482.86\lambda^3 - 3090.376\lambda^2 + 9197.7665\lambda - 9931.285 )</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
<td>( \lambda^5 - 30\lambda^4 + 311\lambda^3 - 1278\lambda^2 + 1551\lambda + 630 )</td>
</tr>
<tr>
<td>4</td>
<td>6</td>
<td>( \lambda^6 - 44\lambda^5 + 453\lambda^4 - 990 )</td>
</tr>
<tr>
<td>5</td>
<td>6</td>
<td>( \lambda^6 - \frac{61}{9}\lambda^3 + \frac{17}{4}\lambda^4 + \frac{1339}{36}\lambda^3 - \frac{200}{3}\lambda^2 - \frac{625}{18}\lambda - \frac{50}{9} )</td>
</tr>
</tbody>
</table>
Table 2 shows that ISS1-5D converges faster than ISS. Note that the test polynomials 3 and 4 in procedure ISS1-5D require less number of iterations than does procedure ISS1 except for test polynomials 1, 2 and 5. However, for these three test polynomials, the procedure ISS1-5D consumes less CPU times compared to the ISS1 procedure.

<table>
<thead>
<tr>
<th>Test polynomial</th>
<th>Degree $n$</th>
<th>ISS1 No. of iterations</th>
<th>CPU time</th>
<th>ISS1-5D No. of iterations</th>
<th>CPU time</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4</td>
<td>2</td>
<td>0.059375</td>
<td>2</td>
<td>0.056616</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>2</td>
<td>0.101953</td>
<td>2</td>
<td>0.076172</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
<td>3</td>
<td>0.0871094</td>
<td>2</td>
<td>0.086328</td>
</tr>
<tr>
<td>4</td>
<td>6</td>
<td>3</td>
<td>0.099609</td>
<td>2</td>
<td>0.098828</td>
</tr>
<tr>
<td>5</td>
<td>6</td>
<td>2</td>
<td>0.185156</td>
<td>2</td>
<td>0.096875</td>
</tr>
</tbody>
</table>

4 Conclusion

We have developed a new modified method ISS1-5D which is better than ISS1 in terms of the number of iterations and CPU times. From the results, we conclude that the Interval Symmetric Single-Step Procedure ISS1-5D (with the corrector $5\delta$) must have a higher rate of convergence compared to the procedure ISS1 using $w^{(k)} \leq 10^{-12}$ as the stopping criterion.

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References


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