On Regular Generalized b-Closed Set

K. Mariappa
Department of Mathematics
King College of Technology
Namakkal– 637 020, India
kmarichand@yahoo.co.in

S. Sekar
Department of Mathematics
Government Arts College (Autonomous)
Salem– 636 007, India
Sekar_nitt@rediffmail.com

Abstract
In this paper, the authors introduce a new class of sets called regular generalized b-closed sets in topological spaces (briefly rgb-closed set). Also we discuss some of their properties and investigate the relations between the associated topology.

Mathematics Subject Classification: 54A05

Keywords: rgb- closed set, b-closed set and gb closed set.

1. Introduction

In 1970, Levine introduced the concept of generalized closed set and discussed the properties of sets, closed and open maps, compactness, normal and separation axioms. Later in 1996 Andrijvic gave a new type of generalized closed set in topological space called b closed sets. The investigation on generalization of closed set has lead to significant contribution to the theory of separation axiom, generalization of continuity and covering properties. A.A. Omari and M.S.M. Noorani made an analytical study and gave the concepts of generalized b closed sets in topological spaces

In this paper, a new class of closed set called regular generalized b-closed set is introduced to prove that the class forms a topology. The notion of regular generalized b-closed set and its different characterizations are given in this paper.
It has been proved that the class of regular generalized b-closed set lies between the class of b-closed set and rg-closed set. Throughout this paper \((X, \tau)\) and \((Y, \sigma)\) represent the non-empty topological spaces on which no separation axioms are assumed, unless otherwise mentioned.

Let \(A \subseteq X\), the closure of \(A\) and interior of \(A\) will be denoted by \(\text{cl} (A)\) and \(\text{int} (A)\) respectively, union of all b-open sets \(X\) contained in \(A\) is called b-interior of \(A\) and it is denoted by \(\text{bint} (A)\), the intersection of all b-closed sets of \(X\) containing \(A\) is called b-closure of \(A\) and it is denoted by \(\text{bcl} (A)\).

**Definition 2.1:** Let \(A\) subset \(A\) of a topological space \((X, \tau)\),is called

1) a **pre-open set** [17] if \(A \subseteq \text{int} (\text{cl} (A))\).
2) a **semi-open set**[13] if \(A \subseteq \text{cl} (\text{int} (A))\).
3) a **\(\alpha\)**-open set [18] if \(A \subseteq \text{int} (\text{cl} (\text{int} (A)))\).
4) a **b-open set** [2] if \(A \subseteq \text{cl} (\text{int} (A)) \cup \text{int} (\text{cl} (A))\).
5) a **generalized closed set** (briefly g-closed) [15] if \(\text{cl} (A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is open in \(X\).
6) a **generalized \(\alpha\)** closed set (briefly g\(\alpha\) - closed) [14]: if \(\alpha \text{ cl}(A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is \(\alpha\) open in \(X\).
7) a **generalized b- closed set** (briefly gb- closed) [2] if \(\text{bcl} (A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is open in \(X\).
8) a **weakly generalized closed set** (briefly wg- closed) [19] if \(\text{cl} (\text{int} (A)) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is open in \(X\).
9) a **generalized semi-pre closed set** (briefly gsp- closed) [16] if \(\text{spcl} (A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is open in \(X\).
10) a **generalized pre- closed set** (briefly gp- closed) [18] if \(\text{pcl} (A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is open in \(X\).
11) a **generalized semi- closed set** (briefly gs- closed) [5]: if \(\text{scl} (A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is open in \(X\).
12) a **semi generalized closed set** (briefly sg- closed) [6] if \(\text{scl} (A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is semi open in \(X\).
13) a **generalized \(\alpha\)** b- closed set (briefly g\(\alpha\) b- closed) [20] if \(\text{bcl} (A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is \(\alpha\) open in \(X\).
14) a **generalized pre regular closed set** (briefly gpr-closed) [11] if \(\text{pcl} (A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is regular open in \(X\).
15) a **semi generalized b- closed set** (briefly sgb- closed) [12] if \(\text{bcl} (A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is semi open in \(X\).

### 3. Regular generalized b-closed sets

In this section we introduce regular generalized b-closed set and investigate some of their properties.
Definition 3.1 A subset A of a topological space \((X, \tau)\), is called regular generalized b- closed set (briefly rgb-closed set) if \(bcl(A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is regular open in \(X\).

Theorem 3.2. Every closed set is rgb-closed.

Proof. Let \(A\) be any closed set in \(X\) such that \(A \subseteq U\), where \(U\) is regular open. Since \(bcl(A) \subseteq cl(A) = A\). Therefore \(bcl(A) \subseteq U\). Hence \(A\) is rgb-closed set in \(X\).

The converse of above theorem need not be true as seen from the following example.

Example 3.3. Let \(X = \{a,b,c\}\) with \(\tau = \{X, \emptyset, \{a\}, \{b\}, \{a,b\}, \{a,c\}\}\). The set \(\{a,b\}\) is rgb-closed set but not a closed set.

Theorem 3.4. Every b-closed set is rgb-closed set.

Proof. Let \(A\) be any b-closed set in \(X\) such that \(U\) be any regular open set containing \(A\). Since \(A\) is b-closed, \(bcl(A) = A\). Therefore \(bcl(A) \subseteq U\). Hence \(A\) is rgb-closed set.

The converse of above theorem need not be true as seen from the following example.

Example 3.5. Let \(X = \{a,b,c\}\) with \(\tau = \{X, \emptyset, \{a\}, \{a,b\}, \{a,c\}\}\). The set \(\{a,b\}\) is rgb-closed set but not a b-closed set.

Theorem 3.6. Every \(\alpha\) - closed set is rgb-closed set.

Proof. Let \(A\) be any \(\alpha\) - closed set in \(X\) and \(U\) be any regular open set containing \(A\). Since \(A\) is \(\alpha\) closed, \(bcl(A) \subseteq \alpha cl(A) \subseteq U\). Therefore \(bcl(A) \subseteq U\). Hence \(A\) is rgb-closed set.

The converse of above theorem need not be true as seen from the following example.

Example 3.7. Let \(X = \{a,b,c\}\) with \(\tau = \{X, \emptyset, \{b\}, \{c\}, \{a,b\}, \{b,c\}\}\). The set \(\{b,c\}\) is rgb-closed set but not a \(\alpha\) - closed set.

Theorem 3.8. Every semi-closed set is rgb-closed set.

Proof. Let \(A\) be any semi-closed set in \(X\) and \(U\) be any regular open set containing \(A\). Since \(A\) is semi-closed, \(bcl(A) \subseteq scl(A) \subseteq U\). Therefore \(bcl(A) \subseteq U\). Hence \(A\) is rgb-closed set.

The converse of above theorem need not be true as seen from the following example.
Example 3.9 Let \( X = \{a, b, c\} \) with \( \tau = \{X, \varnothing, \{a\}, \{a,c\}\} \). The set \( \{a,c\} \) is rgb-closed set but not a semi-closed set.

**Theorem 3.10.** Every pre-closed set is rgb-closed set.

**Proof.** Let \( A \) be any pre-closed set in \( X \) and \( U \) be any regular open set containing \( A \). Since \( A \) is semi closed, \( \text{pcl}(A) \subseteq \text{bcl}(A) \subseteq U \). Therefore \( \text{bcl}(A) \subseteq U \). Hence \( A \) is rgb-closed set.

The converse of above theorem need not be true as seen from the following example.

Example 3.11 Let \( X = \{a, b, c\} \) with \( \tau = \{X, \varnothing, \{a\}\} \). The set \( \{a,b\} \) is rgb-closed set but not a pre-closed set.

**Theorem 3.12.** Every \( g^* \)-closed set is rgb-closed set.

**Proof.** Let \( A \) be any \( g^* \)-closed set in \( X \) and \( U \) be any regular open set containing \( A \). Since \( A \) is \( g^* \)-closed, \( \text{bcl}(A) \subseteq \text{cl}(A) \subseteq U \). Therefore \( \text{bcl}(A) \subseteq U \). Hence \( A \) is rgb-closed set.

The converse of above theorem need not be true as seen from the following example.

Example 3.13 Let \( X = \{a, b, c\} \) with \( \tau = \{X, \varnothing, \{a,b\}\} \). The set \( \{a\} \) is rgb-closed set but not a \( g^* \)-closed set.

**Theorem 3.14.** Every gpr-closed set is rgb-closed set.

**Proof.** Let \( A \) be any closed set in \( X \) and \( U \) be any regular open set containing \( A \). Since every pre open sets are b-open sets, \( \text{pcl}(A) \subseteq \text{bcl}(A) \subseteq U \). Therefore \( \text{bcl}(A) \subseteq U \). Hence \( A \) is rgb-closed set.

The converse of above theorem need not be true as seen from the following example.

Example 3.15 Let \( X = \{a,b,c\} \) with \( \tau = \{X, \varnothing, \{c\}, \{b\}, \{b,c\}\} \). The set \( \{c\} \) is rgb-closed set but not a gpr-closed set.

**Theorem 3.16.** Every rgb-closed set is gb-closed set.

**Proof.** Let \( A \) be rgb-closed set in \( X \) such that \( U \) be any regular open set containing \( A \). Since every regular open set are open sets, \( \text{bcl}(A) \subseteq U \). Hence \( A \) is gb-closed set.
The converse of above theorem need not be true as seen from the following example.

**Example 3.17.** Let $X = \{a, b, c\}$ with $\tau = \{X, \varnothing, \{c\}, \{b\}, \{b,c\}\}$. The set $\{b\}$ is gb-closed set but not a rgb-closed set.

**Theorem 3.18.** Every rgb-closed set is gsp-closed set.

**Proof.** Let $A$ be rgb-closed set in $X$ such that $U$ be any regular open set containing $A$. Since every regular open set are open sets, $bcl(A) \subseteq spcl(A) \subseteq U$. Therefore $spcl(A) \subseteq U$. Hence $A$ is gsp-closed set.

The converse of above theorem need not be true as seen from the following example.

**Example 3.19.** Let $X = \{a, b, c\}$ with $\tau = \{X, \varnothing, \{a\}, \{b\}, \{a,b\}\}$. The set $\{a\}$ is gsp-closed set but not a rgb-closed set.

**Theorem 3.20.** Every $g\alpha$-closed set is rgb-closed set.

**Proof.** Let $A$ be $g\alpha$-closed set in $X$ such that $U$ be any regular open set containing $A$. Since every regular open set are open sets, $bcl(A) \subseteq \alpha cl(A) \subseteq U$. Therefore $bcl(A) \subseteq U$. Hence $A$ is rgb-closed set.

The converse of above theorem need not be true as seen from the following example.

**Example 3.21.** Let $X = \{a, b, c\}$ with $\tau = \{X, \varnothing, \{a\}, \{b\}, \{a,b\}\}$. The set $\{a, b\}$ is rgb-closed set but not a $g\alpha$-closed set.

**Theorem 3.21.** Every rgb-closed set is $g\alpha$-closed set.

**Proof.** Let $A$ be rgb-closed set in $X$ such that $U$ be any regular open set containing $A$. Since every regular open set are $\alpha$ open sets, Therefore $bcl (A) \subseteq U & A \subseteq U$, $U$ is $\alpha$ open. Hence $A$ is rgb-closed set.

The converse of above theorem need not be true as seen from the following example.

**Example 3.22.** Let $X = \{a, b, c\}$ with $\tau = \{X, \varnothing, \{a\}, \{b\}, \{a,b\}\}$. The set $\{a\}$ is $g\alpha$ b-closed set but not a rgb-closed set.

**Theorem 3.23.** Every sgb-closed set is rgb-closed set.
Proof. Let $A$ be sgb-closed set in $X$ such that $U$ be any semi open set containing $A$. Since every semi open set are regular open sets, Therefore $\text{bcl}(A) \subseteq U$ & $A \subseteq U$, $U$ is regular open. Hence $A$ is rgb-closed set.

The converse of above theorem need not be true as seen from the following example.

Example 3.24. Let $X = \{a, b, c\}$ with $\tau = \{X, \varnothing, \{a\}, \{b\}, \{a,b\}\}$. The set $\{a,b\}$ is rgb-closed set but not a sgb-closed set.

Remark 3.25. rgb-closed set and rg-closed set are independent to each other as seen from the following examples.

Example 3.25. (a) Let $X = \{a, b, c\}$ with $\tau = \{X, \varnothing, \{a\}, \{b\}, \{a,b\}\}$. The set $\{b\}$ is rgb-closed set but not a rg-closed set.

Example 3.25.(b) Let $X = \{a, b, c\}$ with $\tau = \{X, \varnothing, \{a\}, \{a,c\}, \{a,b\}\}$. The set $\{a, c\}$ is rg- closed set but not a rgb- closed set.

Remark 3.26. rgb-closed set and gp-closed set are independent to each other as seen from the following examples.

Example 3.26(a). Let $X = \{a, b, c\}$ with $\tau = \{X, \varnothing, \{c\}\}$. The set $\{b\}$ is rgb- closed set but not a gp- closed set.

Example 3.26(b). Let $X = \{a, b, c\}$ with $\tau = \{X, \varnothing, \{a\}, \{a,c\}, \{a,b\}\}$. The set $\{a, c\}$ is gp- closed set but not a rgb- closed set.

Remark 3.27. rgb-closed set and sg-closed set are independent to each other as seen from the following examples.

Example 3.27(a) Let $X = \{a, b, c\}$ with $\tau = \{X, \varnothing, \{a\}\}$. The set $\{a, b\}$ is rgb-closed set but not a sg- closed set.

Example 3.27(b) Let $X = \{a, b, c\}$ with $\tau = \{X, \varnothing, \{a\}, \{b\}, \{a,b\}\}$. The set $\{a\}$ is sg-closed set but not a rgb-closed set.

Remark 3.28. rgb-closed set and gs-closed set are independent to each other as seen from the following examples.

Example 3.28(a) Let $X = \{a, b, c\}$ with $\tau = \{X, \varnothing, \{a\}, \{b\}, \{a,b\}\}$. The set $\{a, b\}$ is rgb-closed set but not a gs-closed set.
**Example 3.28(b)** Let \( X = \{a, b, c\} \) with \( \tau = \{X, \emptyset, \{c\},\{b\},\{b,c\}\} \). The set \( \{b\} \) is gs closed set but not a rgb closed set.

**Remark: 3.29:** By the above results we have the following diagram:

4. Characteristics of rgb-closed sets

**Theorem 4.1** If \( A \) and \( B \) are rgb-closed sets in \( X \) then \( A \cup B \) is rgb-closed set in \( X \).

**Proof.** Let \( A \) and \( B \) are rgb-closed sets in \( X \) and \( U \) be any regular open set containing \( A \) and \( B \). Therefore \( \text{cl}(A) \subseteq U \), \( \text{cl}(B) \subseteq U \). Since \( A \subseteq U \), \( B \subseteq U \) then
A ∪ B ⊆ U. Hence cl (A ∪ B) = cl (A) ∪ cl (B) ⊆ U. Therefore A ∪ B is rgb-closed set in X.

**Theorem 4.2.** If a set A is rgb-closed set iff bcl (A)-A contains no non empty regular closed set.

**Proof: Necessity:** Let F be a regular closed set in X such that F ⊆ bcl (A) – A. Then A ⊆ X – F. Since A is rgb closed set and X - F is regular open then bcl (A) ⊆ X – F. (i.e.) F ⊆ X – bcl (A). So F ⊆ (X – bcl (A)) ∩ (bcl (A) – A). Therefore F = ϕ

**Sufficiency:** Let us assume that bcl(A)-A contains no non empty regular closed set. Let A ⊆ U, U is regular open. Suppose that bcl(A) is not contained in U, bcl(A) ∩ U^c is non-empty regular closed set of bcl(A)-A which is contradiction. Therefore bcl(A) ⊆ U. Hence A is rgb-closed.

**Theorem 4.3.** The intersection of any two subsets of rgb-closed sets in X is rgb-closed set in X.

**Proof.** Let A and B are any two sub sets of rgb-closed sets. A ⊆ U, U is any regular open and B ⊆ U, U is regular open. Then bcl (A) ⊆ U, bcl (B) ⊆ U, therefore bcl (A ∩ B) ⊆ U, U is regular open in X. Since A and B are rgb-closed set, Hence A ∩ B is a rgb-closed set.

**Theorem 4.4** If A is rgb-closed set in X and A ⊆ B ⊆ bcl (A), Then B is rgb-closed set in X.

**Proof.** Since B ⊆ bcl(A), we have bcl(B) ⊆ bcl(A) then bcl(B) - B ⊆ bcl(A) – A. By theorem 4.2, bcl (A)-A contains no non empty regular closed set. Hence bcl (B) - B contains no non empty regular closed set. Therefore B is rgb-closed set in X.

**Theorem 4.5.** If A ⊆ Y ⊆ X and suppose that A is rgb closed set in X then A is rgb-closed set relative to Y.

**Proof.** Given that A ⊆ Y ⊆ X and A is rgb-closed set in X. To prove that A is rgb-closed set relative to Y. Let us assume that A ⊆ Y ∩ U, where U is regular open in X. Since A is rgb-closed set, A ⊆ U implies bcl (A) ⊆ U. It follows that Y ∩ bcl (A) ⊆ Y ∩ U. That is A is rgb-closed set relative to Y.

**Theorem 4.6.** If A is both regular open and rgb-closed set in X, then A is regular closed set.
**On regular generalized b-closed set**

**Proof.** Since A is regular open and rgb closed in X, bcl (A) ⊆ U. But A ⊆ bcl (A). Therefore A = bcl (A). Hence A is regular closed set.

**Theorem 4.7.** For \( x ∈ X \), then the set \( X - \{x\} \) is a rgb-closed set or regular open.

**Proof.** Suppose that \( X - \{x\} \) is not regular open, then X is the only regular open set containing \( X - \{x\} \). (i.e.) bcl(\( X - \{x\} \)) ⊆ X. Then \( X - \{x\} \) is rgb-closed in X.

**5. Regular generalized b- open sets and regular generalized b- neighbourhoods**

In this section we introduce regular generalized b-open sets (briefly rgb-open) and regular generalized b-neighbourhoods (briefly rgb-nbhd) in topological spaces by using the notions of rgb-open sets and study some of their properties.

**Definition 5.1:** A subset A of a topological space \((X, τ)\), is called regular generalized b- open set (briefly rgb-open set) if \( Ac \) is rgb-closed in X. We denote the family of all rgb-open sets in X by rgb-O(X).

**Theorem 5.2:** If A and B are rgb-open sets in a space X. Then \( A ∩ B \) is also rgb-open set in X.

**Proof.** If A and B are rgb-open sets in a space X. Then \( Ac \) and \( Bc \) are rgb-closed sets in a space X. By theorem 3.6 \( Ac ∪ Bc \) is also rgb-closed set in X. (i.e.) \( Ac ∪ Bc = (A ∩ B)c \) is a rgb-closed set in X. Therefore \( A ∩ B \) rgb-open set in X.

**Remark 5.3** The union of two rgb-open sets in X is generally not a rgb-open set in X.

**Example 5.4:** Let \( X = \{a, b, c\} \) with \( τ = \{X, φ, \{a\}, \{b\}, \{a,b\}\} \). If \( A = \{b\}, B = \{c\} \) are rgb-open sets in X, then \( A ∪ B = \{b, c\} \) is not rgb open set in X.

**Remark 5.5.** If A and B are rgb-open sets in X. Then \( A ∩ B \) is not rgb-open set in X.

**Example 5.6:** Let \( X = \{a, b, c\} \) with \( τ = \{X, φ, \{a\}, \{a,b\}, \{a,c\}\} \). If \( A = \{a, b\}, B = \{b, c\} \), then A and B are rgb-open sets in X, But \( A ∩ B = \{b\} \) is not rgb open set in X.

**Theorem 5.7:** If int \( (B) ⊆ B ⊆ A \) and if A is rgb-open in X, then B is rgb-open in X.
Proof. Suppose that \( \text{int}(B) \subseteq B \subseteq A \) and \( A \) is rgb-open in \( X \) then \( A^c \subseteq B^c \subseteq C \) (\( A^c \)). Since \( A^c \) is rgb-closed in \( X \), by theorem B is rgb-open in \( X \).

**Definition 5.8:** Let \( x \) be a point in a topological space \( X \) and let \( x \in X \). A subset \( N \) of \( X \) is said to be a rgb-nbhd of \( x \) iff there exists a rgb-open set \( G \) such that \( x \in G \subseteq N \).

**Definition 5.9:** A subset \( N \) of Space \( X \) is called a rgb-nbhd of \( A \subseteq X \) iff there exists a rgb-open set \( G \) such that \( A \subseteq G \subseteq N \).

**Theorem 5.10:** Every nbhd \( N \) of \( x \in X \) is a rgb-nbhd of \( X \).

Proof. Let \( N \) be a nbhd of point \( x \in X \). To prove that \( N \) is a rgb-nbhd of \( x \). By definition of nbhd, there exists an open set \( G \) such that \( x \in G \subseteq N \). Hence \( N \) is a rgb-nbhd of \( x \).

**Remark 5.11.** In general, a rgb-nbhd of \( x \in X \) need not be a nbhd of \( x \) in \( X \) as seen from the following example.

**Example 5.12:** Let \( X = \{a, b, c\} \) with topology \( \tau = \{X, \emptyset, \{a\}, \{b\}, \{b,c\}\} \). Then rgb-\( O(X) = \{X, \emptyset, \{a\}, \{a,b\}, \{a,c\}\} \). The set \( \{a,c\} \) is rgb-nbhd of point \( c \), since the rgb-open sets \( \{c\} \) is such that \( c \in \{c\} \subseteq \{a,c\} \). However, the set \( \{a,c\} \) is not a nbhd of the point \( c \), since no open set \( G \) exists such that \( c \in G \subseteq \{a,c\} \).

**Remark 5.13** The rgb-nbhd \( N \) of \( x \in X \) need not be a rgb-open in \( X \).

**Example 5.14:** Let \( X = \{a, b, c\} \) with topology \( \tau = \{X, \emptyset, \{a\}, \{a,b\}, \{a,c\}\} \). Then rgb-\( O(X) = \{X, \emptyset, \{a\}, \{a,b\}, \{a,c\}\} \). The set \( \{b,c\} \) is rgb-open sets \( \{c\} \), but it is a rgb-nbhd if \( \{c\} \). Since \( \{c\} \) is rgb-open set such that \( c \in \{c\} \subseteq \{b,c\} \).

**Theorem 5.15.** If a subset \( N \) of a space \( X \) is rgb-open, then \( N \) is rgb-nbhd of each of its points.

Proof. Suppose \( N \) is rgb-open. Let \( x \in N \). We claim that \( N \) is rgb-nbhd of \( x \). For \( N \) is a rgb-open set such that \( x \in N \subseteq N \). Since \( x \) is an arbitrary point of \( N \), it follows that \( N \) is a rgb-nbhd of each of its points.

**Remark 5.16:** In general, a rgb-nbhd of \( x \in X \) need not be a nbhd of \( x \) in \( X \) as seen from the following example.

**Example 5.17:** Let \( X = \{a, b, c\} \) with topology \( \tau = \{X, \emptyset, \{a\}, \{b\}, \{a,b\}\} \). Then rgb-\( O(X) = \{X, \emptyset, \{a\}, \{b\}, \{a,b\}, \{a,c\}\} \). The set \( \{b, c\} \) is rgb-nbhd of point \( b \), since the rgb-open sets \( \{b\} \) is such that \( b \in \{b\} \subseteq \{b, c\} \). Also the set \( \{b, c\} \) is rgb-nbhd of point \( \{c\} \). Since the rgb-open set \( \{c\} \) is such that \( c \in \{c\} \subseteq \{b,c\} \), \( \{b, c\} \) is rgb-nbhd of each of its points. However, the set \( \{b, c\} \) is not a rgb-open set in \( X \).
Theorem 5.18. Let $X$ be a topological space. If $F$ is rgb-closed subset of $X$ and $x \in F^c$. Prove that there exists a rgb-nbhd $N$ of $x$ such that $N \cap F = \emptyset$

Proof: Let $F$ be rgb-closed subset of $X$ and $x \in F^c$. Then $F^c$ is rgb-open set of $X$. So by theorem 6.2 $F^c$ contains a rgb-nbhd of each of its points. Hence there exists a rgb-nbhd $N$ of $x$ such that $N \subset F^c$. (i.e.) $N \cap F = \emptyset$

Definition 5.19. Let $x$ be a point in a topological space $X$. The set of all rgb-nbhd of $x$ is called the rgb-nbhd system at $x$, and is denoted by rgb-N(x).

Theorem 5.20. Let a rgb-nbhd $N$ of $X$ be a topological space and each $x \in X$. Let $rgb-N(X, \tau)$ be the collection of all rgb-nbhd of $x$. Then we have the following results.

(i) $\forall x \in X$, rgb-N(x) $\neq \emptyset$.
(ii) $N \in rgb-N(x) \Rightarrow x \in N$.
(iii) $N \in rgb-N(x)$, $M \supset N$ $\Rightarrow M \in rgb-N(x)$.
(iv) $N \in rgb-N(x)$ $\Rightarrow$ there exists $M \in rgb-N(x)$ such that $M \subset N$ and $M \in rgb-N(y)$ for every $y \in M$.

Proof: (i) Since $X$ is rgb-open set, it is a rgb-nbhd of every $x \in X$. Hence there exists at least one rgb-nbhd (namely-X) for each $x \in X$. Therefore $rgb-N(x) \neq \emptyset$ for every $x \in X$.
(ii) If $N \in rgb-N(x)$, then $N$ is rgb-nbhd of $x$. By definition of rgb-nbhd, $x \in N$.
(iii) Let $N \in rgb-N(x)$ and $M \supset N$. Then there is a rgb-open set $G$ such that $x \in G \subset N$. Since $N \subset M$, $x \in G \subset M$ and so $M$ is rgb-nbhd of $x$. Hence $M \in rgb-N(x)$.
(iv) Let $N \in rgb-N(x)$, $M \in rgb-N(x)$. Then by definition of rgb-nbhd, there exists rgb-open sets $G_1$ and $G_2$ such that $x \in G_1 \subset N$ and $x \in G_2 \subset M$. Hence $x \in G_1 \cap G_2 \subset N \cap M$ -------- (1). Since $G_1 \cap G_2$ is a rgb-open set,(being the intersection of two rgb-open sets), it follows from (1) that $N \cap M$ is a rgb-nbhd of $x$. Hence $N \cap M \in rgb-N(x)$.
(v) Let $N \in rgb-N(x)$, Then there is a rgb-open set $M$ such that $x \in M \subset N$. Since $M$ is rgb-open set, it is rgb-nbhd of each of its points. Therefore $M \in rgb-N(y)$ for every $y \in M$.

6. Conclusion

The classes of regular generalized b-closed set is defined using regular open set form a topology that lies between the class of the class of b-closed set and rg-closed set. The rgb-closed set can be used to derive a new decomposition of continuity, closed map and open map, homeomorphism, closure and interior and new separation axioms. This idea can be extended to bitopological and fuzzy topological spaces.
REFERENCES


Received: October, 2012