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Inclusion Properties for Certain Subclasses of Analytic Functions Defined by a Generalized Multiplier Transformation

S. R. Swamy

Department of Computer Science and Engineering
R V College of Engineering, Mysore Road
Bangalore-560 059, India
mailtoswamy@rediffmail.com

Abstract

Using the principle of subordination, we obtain some inclusion properties of certain subclasses of analytic functions defined by a generalized multiplier transformation. Also inclusion properties of these classes involving the generalized integral operator are obtained.

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1. INTRODUCTION

Let A_p denote the class of functions of the form

(1.1) $$f(z) = z^p + \sum_{k=p+1}^{\infty} a_k z^k, (p \in N = \{1,2,3...\}).$$

which are analytic in the open unit disc $U = \{z \in C : |z| < 1\}$. If f and g are analytic in U, we say that f is subordinate to g, written $f \prec g$, if there exists a Schwarz function $w(z)$, which (by definition) is analytic in U with $w(0)=0$ and $|w(z)|<1$, for all $z\in U$, such that $f(z)=g(w(z)),\ z\in U$. Furthermore, if the function g is univalent in U, then we have the following equivalence (See [7],[14] and [15]):

$$f\prec g \Leftrightarrow f(0)=g(0) \text{ and } f(U)\subset g(U).$$

For $0\le\eta<p,\ p\in N$, we denote by $S_p^*(\eta), K_p(\eta)$ and C_p the subclasses of A_p consisting of all analytic functions which are, respectively, p-valent starlike of order η, p-valent convex of order η, and p-valent close-to-convex functions in U ([17],[19] and [22]).

Let Λ be the class of all functions ϕ which are analytic and univalent in U and for which $\phi(U)$ is convex with $\phi(0)=1$ and $\mathrm{Re}(\phi(z))>0, z\in U$.

Making use of the principle of subordination between analytic functions, we introduce the subclasses $S_p^*(\eta;\phi), K_p(\eta;\phi)$ and $C_p(\eta,\rho;\phi,\varphi)$ of the class A_p, $0\le\eta<p,\ 0\le\rho<p$ and $\phi,\varphi\in\Lambda$, which are defined by:

$$S_p^*(\eta;\phi)=\left\{f\in A_p:\frac{1}{p-\eta}\left(\frac{zf'(z)}{f(z)}-\eta\right)\prec\phi(z), z\in U\right\},$$

$$K_p(\eta;\phi)=\left\{f\in A_p:\frac{1}{p-\eta}\left(1+\frac{zf''(z)}{f'(z)}-\eta\right)\prec\phi(z), z\in U\right\},$$

and

$$C_p(\eta,\rho;\phi,\varphi)=\left\{f\in A_p:\frac{1}{p-\rho}\left(\frac{zf'(z)}{g(z)}-\rho\right)\prec\varphi(z), z\in U, where, g\in S_p^*(\eta;\phi)\right\}.$$

From these definitions, we can obtain some well known subclasses of A_p, by special choices of the functions ϕ and φ. For example, we have $S_p^*\left(\eta;\frac{1+z}{1-z}\right)=S_p^*(\eta),\ K_p\left(\eta;\frac{1+z}{1-z}\right)=K_p(\eta)$ and $C_p\left(0,0;\frac{1+z}{1-z},\frac{1+z}{1-z}\right)=C_p$.

Now we define the new generalized multiplier transformation $I^n_{p,\alpha,\beta}$ on A_p as below:

Definition 1.1. Let $p \in N$, $n \in N_0 = N \bigcup \{0\}$, $\beta \geq 0$ and α a real number with $\alpha + p\beta > 0$. Then for $f \in A_p$, we define the operator $I^n_{p,\alpha,\beta}$ by

$$I^0_{p,\alpha,\beta} f(z) = f(z),$$

$$I^1_{p,\alpha,\beta} f(z) = \frac{\alpha f(z) + \beta z f'(z)}{\alpha + p\beta},$$

$$\ldots,$$

$$I^n_{p,\alpha,\beta} f(z) = I_{p,\alpha,\beta}(I^{n-1}_{p,\alpha,\beta} f(z)).$$

Remark 1.2. We observe that $I^n_{p,\alpha,\beta} : A_p \to A_p$, is a linear operator and for $f(z)$ given by (1.1), we have

$$I^n_{p,\alpha,\beta} f(z) = z^p + \sum_{k=p+1}^{\infty} \left(\frac{\alpha + k\beta}{\alpha + p\beta} \right)^n a_k z^k. \tag{1.2}$$

It follows from (1.2) that

$$I^n_{p,\alpha,0} f(z) = f(z), \tag{1.3}$$

$$(\alpha + p\beta) I^{n+1}_{p,\alpha,\beta} f(z) = \alpha I^n_{p,\alpha,\beta} f(z) + \beta z (I^n_{p,\alpha,\beta} f(z))', \beta > 0, \tag{1.4}$$

and

$$I^{n_1}_{p,\alpha,\beta}(I^{n_2}_{p,\alpha,\beta} f(z) = I^{n_2}_{p,\alpha,\beta}(I^{n_1}_{p,\alpha,\beta} f(z), \text{ for all } n_1, n_2 \in N_0.$$

We note that

- $I^n_{1,\alpha,\beta} f(z) = I^n_{\alpha,\beta} f(z)$ (See [24]).
- $I^n_{p,\alpha,1} f(z) = I^n_p(\alpha) f(z), \alpha > -p$ (See [1], [21] and [23]).
- $I^n_{p,l+p-p\beta,\beta} f(z) = I^n_p(\beta, l) f(z), l > -p, \beta \geq 0$ (See Catas [8]).
- $I^n_{p,0,\beta} f(z) = D^n_p f(z)$ (See [4], [12] and [16]).

Remark 1.3. a) i) $I^n_p(\alpha) f(z)$ was considered in [1], [21] and [23] for $\alpha \geq 0$ and $I^n_p(\beta, l) f(z)$ was defined in [8] for $l \geq 0, \beta \geq 0$, ii) $I^n_p(l) f(z) = I^n_p(1, l) f(z), l > -p$. So our results in this paper are improvement of corresponding results proved earlier for $I^n_p(\alpha) f(z)$ or $I^n_p(\beta, l) f(z)$ to $\alpha > -p$ or $l > -p$, respectively.

b) i) $I_p^n(\beta,0)f(z) = D_p^n(\beta)f(z), \beta \geq 0,$ was mentioned in Aouf et.al. [3], ii) $D_1^n(\beta), \beta \geq 0,$ was introduced by Al-Oboudi [2],iii) $D_1^n(1)f(z) = D^n f(z)$ was defined by Salagean [20] and was considered for $n \geq 0$ in [5] , iv) $I_1^n(\alpha)f(z), \alpha \geq 0,$ was investigated in [9] and [10] and v) $I_1^n(1)$ was due to Uralegaddi and Somanatha [25].

Next, by using the operator $I_{p,\alpha,\beta}^n$, we introduce the following subclasses of analytic functions for $\phi, \varphi \in \Lambda, p \in N, n \in N_0, \beta \geq 0, \alpha$ a real number with $\alpha + p\beta > 0,\ 0 \leq \eta < p$ and $0 \leq \rho < p$.

$$S_{p,\alpha,\beta}^n(\eta;\phi) = \left\{ f \in A_p : I_{p,\alpha,\beta}^n f(z) \in S_p^n(\eta;\phi) \right\},$$

$$K_{p,\alpha,\beta}^n(\eta;\phi) = \left\{ f \in A_p : I_{p,\alpha,\beta}^n f(z) \in K_p(\eta;\phi) \right\},$$

and

$$C_{p,\alpha,\beta}^n(\eta,\rho;\phi,\varphi) = \left\{ f \in A_p : I_{p,\alpha,\beta}^n f(z) \in C_p(\eta,\rho;\phi,\varphi) \right\}.$$

We also note that

(1.5) $$f(z) \in K_{p,\alpha,\beta}^n(\eta;\phi) \Leftrightarrow \frac{zf'(z)}{p} \in S_{p,\alpha,\beta}^n(\eta;\phi).$$

In particular, we set

(1.6) $$S_{p,\alpha,\beta}^n\left(\eta;\frac{1+\mathrm{A}z}{1+\mathrm{B}z}\right) = S_{p,\alpha,\beta}^n(\eta;\mathrm{A},\mathrm{B}), -1 \leq \mathrm{B} < \mathrm{A} \leq 1,$$

and

(1.7) $$K_{p,\alpha,\beta}^n\left(\eta;\frac{1+\mathrm{A}z}{1+\mathrm{B}z}\right) = K_{p,\alpha,\beta}^n(\eta;\mathrm{A},\mathrm{B}), -1 \leq \mathrm{B} < \mathrm{A} \leq 1.$$

In section 2, some preliminary results are mentioned. In section 3, we show that $S_{p,\alpha,\beta}^{n+1}(\eta;\phi) \subset S_{p,\alpha,\beta}^n(\eta;\phi)$, $K_{p,\alpha,\beta}^{n+1}(\eta;\phi) \subset K_{p,\alpha,\beta}^n(\eta;\phi)$ and $C_{p,\alpha,\beta}^{n+1}(\eta,\rho;\phi,\varphi) \subset C_{p,\alpha,\beta}^n(\eta,\rho;\phi,\varphi)$. In section 4, we study inclusion properties of classes $S_{p,\alpha,\beta}^n(\eta;\phi), K_{p,\alpha,\beta}^n(\eta;\phi)$ and $C_{p,\alpha,\beta}^n(\eta,\rho;\phi,\varphi)$, involving generalized Libera integral operator.

2. PRELIMINARY LEMMAS

The following lemmas will be required in our investigation.

Lemma 2.1 ([11]). Let ϕ be convex, univalent in U with $\phi(0)=1$ and $\mathrm{Re}(\kappa\phi(z)+\gamma)>0$, $\kappa,\gamma\in C$. If $p(z)$ is analytic in U with $p(0)=1$, then

$$p(z)+\frac{zp'(z)}{\kappa p(z)+\gamma}\prec\phi(z),(z\in U) \text{ implies } p(z)\prec\phi(z),(z\in U).$$

Lemma 2.2 ([15]). Let ϕ be convex, univalent in U and w be analytic in U with $\mathrm{Re}(w(z))\geq 0$. If $p(z)$ is analytic in U with $p(0)=\phi(0)$, then

$$p(z)+w(z)zp'(z)\prec\phi(z),(z\in U) \text{ implies } p(z)\prec\phi(z),(z\in U).$$

3. INCLUSION PROPERTIES INVOLVING THE OPERATOR $I^n_{p,\alpha,\beta}$.

Unless otherwise mentioned we shall assume that $\beta\geq 0,\alpha$ a real number with $\alpha+p\beta>0$, $p\in N$, $n\in N_0$, $0\leq\eta<p$ and $0\leq\rho<p$, throughout this paper.

Theorem 3.1. Let $f\in A_p$ and let $\phi\in\Lambda$ with $\mathrm{Re}((p-\eta)\phi(z)+\eta+(\alpha/\beta))>0$. Then

$$S^{n+1}_{p,\alpha,\beta}(\eta;\phi)\subset S^{n}_{p,\alpha,\beta}(\eta;\phi).$$

Proof. Let $f(z)\in S^{n+1}_{p,\alpha,\beta}(\eta;\phi)$ and set

$$p(z)=\frac{1}{p-\eta}\left(\frac{z(I^n_{p,\alpha,\beta}f(z))'}{I^n_{p,\alpha,\beta}f(z)}-\eta\right), \tag{3.1}$$

Where $p(z)$ is analytic in U with $p(0)=1$. Using (1.4) in (3.1), we get

$$\left(\frac{\alpha+p\beta}{\beta}\right)\frac{I^{n+1}_{p,\alpha,\beta}f(z)}{I^n_{p,\alpha,\beta}f(z)}=(p-\eta)p(z)+\eta+(\alpha/\beta). \tag{3.2}$$

Differentiating (3.2) logarithmically with respect to z, we obtain

$$\frac{1}{p-\eta}\left(\frac{z(I^{n+1}_{p,\alpha,\beta}f(z))'}{I^{n+1}_{p,\alpha,\beta}f(z)}-\eta\right)=p(z)+\frac{zp'(z)}{(p-\eta)p(z)+\eta+(\alpha/\beta)}, z\in U. \tag{3.3}$$

Applying Lemma 2.1 to (3.3), it follows that $p\prec\phi$, i.e. $f\in S^n_{p,\alpha,\beta}(\eta;\phi)$.

Theorem 3.2. Let $f\in A_p$ and let $\phi\in\Lambda$ with $\mathrm{Re}((p-\eta)\phi(z)+\eta+(\alpha/\beta))>0$. Then $K^{n+1}_{p,\alpha,\beta}(\eta;\phi)\subset K^{n}_{p,\alpha,\beta}(\eta;\phi)$.

Proof. Applying (1.5) and Theorem 3.1, we conclude that

$$f \in K^{n+1}_{p,\alpha,\beta}(\eta;\phi) \Rightarrow \frac{zf'}{p} \in S^{n+1}_{p,\alpha,\beta}(\eta;\phi) \subset S^{n}_{p,\alpha,\beta}(\eta;\phi)$$

$$\Leftrightarrow \frac{zf'}{p} \in S^{n}_{p,\alpha,\beta}(\eta;\phi)$$

$$\Leftrightarrow f \in K^{n}_{p,\alpha,\beta}(\eta;\phi).$$

Taking $\phi(z) = \frac{1+\mathrm{A}z}{1+\mathrm{B}z}, -1 \le \mathrm{B} < \mathrm{A} \le 1; z \in U$, in Theorem 3.1 and Theorem 3.2, we have the following corollary.

Corollary3.3. Let $f \in A_p$ .Then $S^{n+1}_{p,\alpha,\beta}(\eta;\mathrm{A},\mathrm{B}) \subset S^{n}_{p,\alpha,\beta}(\eta;\mathrm{A},\mathrm{B})$ and $K^{n+1}_{p,\alpha,\beta}(\eta,\mathrm{A},\mathrm{B}) \subset K^{n}_{p,\alpha,\beta}(\eta;\mathrm{A},\mathrm{B})$.

By using Lemma 2.2, we obtain the following inclusion relation for the class $C^{n}_{p,\alpha,\beta}(\eta,\rho;\phi,\varphi)$.

Theorem 3.4. Let $f \in A_p$ and let $\phi,\varphi \in \Lambda$ with $\mathrm{Re}((p-\eta)\phi(z)+\eta+(\alpha/\beta)) > 0$. Then $C^{n+1}_{p,\alpha,\beta}(\eta,\rho;\phi,\varphi) \subset C^{n}_{p,\alpha,\beta}(\eta,\rho;\phi,\varphi)$.

Proof. Let $f \in C^{n+1}_{p,\alpha,\beta}(\eta,\rho;\phi,\varphi)$, then by definition there exists a function $g \in S^{n+1}_{p,\alpha,\beta}(\eta;\phi)$ such that

$$\frac{1}{p-\rho}\left(\frac{z(I^{n+1}_{p,\alpha,\beta}f(z))'}{I^{n+1}_{p,\alpha,\beta}g(z)} - \rho\right) \prec \varphi(z), z \in U.$$

Now, let $p(z) = \frac{1}{p-\rho}\left(\frac{z(I^{n+1}_{p,\alpha,\beta}f(z))'}{I^{n}_{p,\alpha,\beta}g(z)} - \rho\right)$, where $p(z)$ is analytic in U with $p(0)=1$.

Using (1.4), we have

$$\left(\frac{\alpha+p\beta}{\beta}\right)I^{n+1}_{p,\alpha,\beta}f(z) = (\alpha/\beta)I^{n}_{p,\alpha,\beta}f(z) + [(p-\rho)p(z)+\rho]I^{n}_{p,\alpha,\beta}g(z). \tag{3.4}$$

Differentiating (3.4) with respect to z and multiplying by z, we get

(3.5) $$\left(\frac{\alpha+p\beta}{\beta}\right)z(I^{n+1}_{p,\alpha,\beta}f(z))' = \left(\frac{\alpha}{\beta}\right)z(I^{n}_{p,\alpha,\beta}f(z))' + [(p-\rho)p(z)+\rho]z(I^{n}_{p,\alpha,\beta}g(z))' + \\ + (p-\rho)zp'(z)(I^{n}_{p,\alpha,\beta}g(z).$$

Since $g \in S^{n+1}_{p,\alpha,\beta}(\eta;\phi)$, then by Theorem 3.1, we have $g \in S^{n}_{p,\alpha,\beta}(\eta;\phi)$. Let

$$h(z)=\frac{1}{p-\eta}\left(\frac{z(I^{n}_{p,\alpha,\beta}g(z))'}{I^{n}_{p,\alpha,\beta}g(z)}-\eta\right).$$

Applying (1.4) again, we get

(3.6) $$\left(\frac{\alpha+p\beta}{\beta}\right)\frac{I^{n+1}_{p,\alpha,\beta}g(z)}{I^{n}_{p,\alpha,\beta}g(z)} = (p-\eta)h(z)+\eta+(\alpha/\beta).$$

From (3.5) and (3.6), we have

$$\frac{1}{p-\rho}\left(\frac{z(I^{n+1}_{p,\alpha,\beta}f(z))'}{I^{n+1}_{p,\alpha,\beta}g(z)}-\rho\right) = p(z)+\frac{zp'(z)}{(p-\eta)h(z)+\eta+(\alpha/\beta)}, z\in U.$$

Since $0 \le \eta < p$ and $h(z) \prec \phi(z)$ in U, then $\mathrm{Re}((p-\eta)h(z)+\eta+(\alpha/\beta)) > 0$. So by taking $w(z)=1/[(p-\eta)h(z)+\eta+(\alpha/\beta)]$ and applying Lemma 2.2, we can show that $p \prec \varphi$, so that $f \in C^{n}_{p,\alpha,\beta}(\eta,\rho;\phi,\varphi)$, which proves Theorem 3.4.

4. INCLUSION PROPERTIES INVOLVING THE INTEGRAL OPERATOR F_c

In this section we consider the generalized Libera integral operator $F_{p,c}$ (See [6], [13] and [18]), defined by

(4.1) $$F_{p,c}(f)(z)=\frac{c+p}{z^c}\int_0^z t^{c-1}f(t)dt, (c>-p; f\in A_p).$$

Theorem 4.1. Let $c>-p$ and let $\phi\in\Lambda$ with $\mathrm{Re}((p-\eta)\phi(z)+\eta+c)>0$. If $f\in S^{n}_{p,\alpha,\beta}(\eta;\phi)$, then $F_{p,c}(f)\in S^{n}_{p,\alpha,\beta}(\eta;\phi)$.

Proof. Let $f\in S^{n}_{p,\alpha,\beta}(\eta;\varphi)$ and set

(4.2) $$p(z)=\frac{1}{p-\eta}\left(\frac{z(I^{n}_{p,\alpha,\beta}F_{p,c}(f)(z))'}{I^{n}_{p,\alpha,\beta}F_{p,c}(f)(z)}-\eta\right),$$

Where p is analytic in U with $p(0)=1$. From (4.1), we have

(4.3) $$z(I^{n}_{p,\alpha,\beta}F_{p,c}(f)(z))'=(c+1)I^{n}_{p,\alpha,\beta}f(z)-cI^{n}_{p,\alpha,\beta}F_{p,c}(f)(z).$$

Using (4.3) in (4.2), we get

(4.4) $$(c+p)\frac{I^{n}_{p,\alpha,\beta}f(z)}{I^{n}_{p,\alpha,\beta}F_{p,c}(f)(z)}=(p-\eta)p(z)+\eta+c.$$

Differentiating (4.4) logarithmically with respect to z, we obtain

(4.5) $$\frac{1}{p-\eta}\left(\frac{z(I^{n}_{p,\alpha,\beta}f(z))'}{I^{n}_{p,\alpha,\beta}f(z)}-\eta\right)=p(z)+\frac{zp'(z)}{(p-\eta)p(z)+\eta+c}.$$

Applying Lemma 2.1 to (4.5), we conclude that $F_{p,c}(f)(z)\in S^{n}_{p,\alpha,\beta}(\eta;\phi)$.

Similarly applying (1.5) and Theorem 4.1, we have the following result.

Theorem 4.2. Let $c>-p$ and let $\phi\in\Lambda$ with $\mathrm{Re}((p-\eta)\phi(z)+\eta+c)>0$. If $f\in K^{n}_{p,\alpha,,\beta}(\eta;\phi)$, then $F_{p,c}(f)\in K^{n}_{p,\alpha,\beta}(\eta;\phi)$.

From Theorem 4.1 and Theorem 4.2, we have the following corollary:

Corollary 4.3. Let $f\in A_p$ and $c>-p$. If $f\in S^{n}_{p,\alpha,\beta}(\eta;\mathrm{A},\mathrm{B})$ (or $K^{n}_{p,\alpha,\beta}(\eta;\mathrm{A},\mathrm{B})$), then $F_{p,c}(f)\in S^{n}_{p,\alpha,\beta}(\eta;\mathrm{A},\mathrm{B})$ (or $K^{n}_{p,\alpha,\beta}(\eta;\mathrm{A},\mathrm{B})$).

Theorem 4.4. Let $c>-p$ and let $\phi,\varphi\in\Lambda$ with $\mathrm{Re}((p-\eta)\phi(z)+\eta+c)>0$. If $f\in C^{n}_{p,\alpha,,\beta}(\eta,\rho;\phi,\varphi)$, then $F_{p,c}(f)\in C^{n}_{p,\alpha,\beta}(\eta,\rho;\phi,\varphi)$.

Proof**.** Let $f\in C^{n}_{p,\alpha,\beta}(\eta,\rho;\phi,\varphi)$. Then there exists a function $g\in S^{n}_{p,\alpha,\beta}(\eta;\phi)$ such that

$$\frac{1}{p-\rho}\left(\frac{z(I^{n}_{p,\alpha,\beta}f(z))'}{I^{n}_{p,\alpha,\beta}g(z)}-\rho\right)\prec\varphi(z),z\in U.$$

We set

$$p(z)=\frac{1}{p-\rho}\left(\frac{z(I^{n}_{p,\alpha,\beta}F_{p,c}(f)(z))'}{I^{n}_{p,\alpha,\beta}F_{p,c}(g)(z)}-\rho\right)$$

where p is analytic in U with $p(0)=1$. Since $g\in S^{n}_{p,\alpha,\beta}(\eta;\phi)$, we have from Theorem 4.1, that $F_{p,c}(g)\in S^{n}_{p,\alpha,\beta}(\eta;\phi)$. Using (4.3) we obtain

$$[(p-\rho)p(z)+\rho]I^{n}_{p,\alpha,\beta}F_{p,c}(g)(z)+cI^{n}_{p,\alpha,\beta}F_{p,c}(f)(z)=(c+p)I^{n}_{p,\alpha,\beta}f(z).$$

Then by simple calculations, we get

$$(c+p)\frac{z(I^{n}_{p,\alpha,\beta}f(z))'}{I^{n}_{p,\alpha,\beta}F_{p,c}(g)(z)}=[(p-\rho)p(z)+\rho][(p-\eta)h(z)+\eta+c]+(p-\rho)zp'(z),$$

where, $h(z)=\frac{1}{p-\eta}\left(\frac{z(I^{n}_{p,\alpha,\beta}F_{p,c}(g)(z))'}{I^{n}_{p,\alpha,\beta}F_{p,c}(g)(z)}-\eta\right)$. Hence, we have

$$\frac{1}{p-\rho}\left(\frac{z(I^{n}_{p,\alpha,\beta}f(z))'}{I^{n}_{p,\alpha,\beta}g(z)}-\rho\right)=p(z)+\frac{zp'(z)}{(p-\eta)h(z)+\eta+c}.$$

The remaining part of the proof is similar to that of Theorem 3.4 and so we omit it.

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