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Strongly g^*-Closed Sets in Topological Spaces

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Abstract

In this paper , the authors introduce and investigate the concept of strongly generalized g^* -closed sets(briefly strongly g^*-closed set) and investigate the relation between the associated topology.

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1 Introduction

Levine(1960) introduced the notion of generalized closed (briefly g-closed) sets in topological spaces and showed that compactness, countably compactness, para compactness and normality etc are all g-closed hereditary.Andrijevic(1986), Arya and Nour(1990), Bhattacharya and Lahiri(1987),Dontchev(1995,1996), Ganambal(1997), Levine(1960,1963),Maki(1993,1994,1996),Mashhour et.al(1982), Njastad(1965), Palaniappan(1993),Velicko(1968) and Veerakumar(2000) introduced and investigated semi-preopen sets, generalized semiopen sets, semi-generalized open sets, generalized semi-preopen sets, δ -generalized closed sets, θ -generalized closed sets, pre regular closed sets, generalized open sets, semi open sets,α-closed sets, regular generalized closed sets, H-closed sets and g^* -closed sets which are some of the weak and stronger form of open sets and complements of these sets are called the same type g-closed sets respectively.

Veerakumar (2000) introduced and investigated between closed sets and g^* -closed sets. The aim of this paper is to introduce and study stronger form of generalized g^*-closed sets in a topological space. Also we investigate topological properties of strongly g^*-closed sets. Throughout this paper (X, τ) represent non empty topological spaces on which no separation axioms are assumed, unlesses otherwise mentioned. For a subset A of (X, τ), cl(A) and int(A) represent the closure of A with respect to τ and the interior of A with respect to τ respectively.

2 Preliminaries

Before entering into our work, we recall the following definitions which are due to Levine.

Definition 2.1. *[13]: A subset A of a topological space (X, τ) is called a pre-open set if $A \subseteq int(cl(A))$ and pre-closed set if $cl(int(A)) \subseteq A$.*

Definition 2.2. *[8] A subset A of a topological space (X, τ) is called a semi-open set if $A \subseteq cl(int(A))$ and semi closed set if $int(cl(A)) \subseteq A$.*

Definition 2.3. *[14] A subset A of a topological space (X, τ) is called an α-open set if $A \subseteq int(cl(int(A)))$ and an α -closed set if $cl(int(cl(A))) \subseteq A$.*

Definition 2.4. *[1] A subset A of a topological space (X, τ) is called a semi pre-open set (β-open set) if $A \subseteq cl(int(cl(A)))$ and semi-preclosed set if $int(cl(int(A))) \subseteq A$.*

Definition 2.5. *[16] A subset A of a topological space (X, τ) is called a δ-closed set if $A = cl_\delta(A)$ where $cl_\delta(A) = \{x \in X : int(cl(U)) \cap A \neq \phi, U \in \tau$ and $x \in U\}$.*

Definition 2.6. *[16] A subset A of a topological space (X, τ) is called a θ-closed set if $A = cl_\theta(A)$ where $cl_\theta(A) = \{x \in X : (cl(U)) \cap A \neq \phi, U \in \tau$ and $x \in U\}$.*

Definition 2.7. *[9] A subset A of a topological space (X, τ) is called a g-closed if $cl(A) \subseteq U$, whenever $A \subseteq U$ and U is open in (X, τ).*

Definition 2.8. *[3] A subset A of a topological space (X, τ) is called a semi-generalized closed set (briefly sg-closed) if $scl(A) \subseteq U$, whenever $A \subseteq U$, U is semi open in (X, τ).*

Definition 2.9. *[2] A subset A of a topological space (X, τ) is called a generalized semi-closed set (briefly gs-closed) if $scl(A) \subseteq U$ whenever $A \subseteq U$, U is open in (X, τ).*

Definition 2.10. *[11] A subset A of a topological space (X, τ) is called a generalized α -closed (briefly $g\alpha$ -closed) if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is α-open in (X, τ).*

Definition 2.11. *[10] A subset A of a topological space $(X, \tau)$is called an α generalized closed set (briefly αg-closed) if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X, τ).*

Definition 2.12. *[14] A subset A of a topological space (X, τ) is called a generalized semi pre-closed set (briefly gsp-closed) if $spcl(A) \subseteq U$whenever $A \subseteq U$ and U is open in (X, τ).*

Definition 2.13. *[154] A subset A of a topological space (X, τ) is called a regular generalized closed set (briefly r-g-closed) if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular open in(X, τ).*

Definition 2.14. *[12] A subset A of a topological space (X, τ) is called a generalized pre closed set (briefly gp-closed) if $pcl(A) \subseteq U$, whenever $A \subseteq U$ and U is open in (X, τ).*

Definition 2.15. *[7] A subset A of a topological space (X, τ) is called a generalized pre regular closed set (briefly gpr-closed) if $pcl(A) \subseteq U$, whenever $A \subseteq U$ and U is regular open in (X, τ).*

Definition 2.16. *[6] A subset A of a topological space (X, τ) is called a $\theta-$ generalized closed set (briefly θg-closed) if $cl_\theta \subseteq U$ whenever $A \subseteq U$ and U is open in (X, τ).*

Definition 2.17. *[5] A subset A of a topological space (X, τ) is called a δ generalized closed set (briefly δg closed) if $cl_\delta(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X, τ).*

Definition 2.18. *[17] A subset A of a topological space (X, τ) is called a g^* -closed set if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is g-open in (X, τ).*

3 Strongly g^*-closed sets

In this section we have introduce the concept of strongly g^*-closed sets in topological space and we investigate the group of structure of the set of all strongly g^*-closed sets.
Definition 3.1. *Let (X, τ) be a topological space and A be its subset , then A is strongly g^*-closed set if $cl(int(A)) \subseteq U$ whenever $A \subseteq U$ and U is g-open.*

Theorem 3.1. *Every closed set is strongly g^*-closed set.*

Proof. The proof is immediate from the definition of closed set. □

Example 3.1. *The converse of the above theorem need not be true from the following example.*
Let X={a,b,c}.$\tau = \{\Phi, X, \{a\}, \{a, c\}\}$. Let A={a,b}. A is a strongly g^* closed set but not a closed set of (X, τ).

Theorem 3.2. *If a subset A of a topological space X is g^*-closed then it is strongly g^*-closed in X but not conversely.*

Proof. Suppose A is g^*-closed in X. Let G be an open set containing A in X. Then G contains cl(A). Now $G \supseteq cl(A) \supseteq cl(int(A))$. Thus A is strongly g^*-closed in X. □

Example 3.2. *The converse of the above theorem need not be true as seen from the following example.*
Let X={a,b,c} with topology $\tau = \{X, \phi, \{a\}, \{a, b\}\}$. In this topological space the subset $\{b\}$ is strongly g^*-closed but not g^*-closed set.

Theorem 3.3. *If A is a subset of a topological space X is open and strongly g^*-closed then it is closed.*

Proof. Suppose a subset A of X is both open and strongly g^*-closed. Now $A \supseteq cl(int(A)) \supseteq cl(A)$. Therefore $A \supseteq cl(A)$. Since $cl(A) \supseteq A$. We have $A \subseteq cl(A)$. Thus A is closed in X. □

Corollary 3.1. *If A is both open and strongly g^*-closed in X then it is both regular open and regular closed in X.*

Proof. As A is open $A = int(A) = int(cl(A))$, since A is closed. Thus A is regular open . Again A is open in X, cl(int(A))=cl(A). As A is closed cl(int(A))=A. Thus A is regular closed. □

Corollary 3.2. *If A is both open and strongly g^*-closed then it is rg-closed.*

Theorem 3.4. *If a subset A of a topological space X is both strongly g^*-closed and semi open then it is g^*-closed.*

Proof. Suppose A is both strongly g^*-closed and semi open in X, Let G be an open set containing A. As A is strongly g^*-closed , $G \supseteq cl(int(A))$. Now $G \supseteq cl(A)$. since A is semi open . Thus A is g^*-closed in X. □

Corollary 3.3. *If A subset A of a topological space X is both strongly g^*-closed and open then it is g^*-closed set.*

Proof. As every open set is semiopen by the above theorem the proof follows. □

Theorem 3.5. *A set A is strongly g^*-closed iff $cl(int(A)) - A$ contains no non empty closed set.*

Proof. **Necessary :** Suppose that F is non empty closed subset of cl(int(A)). Now $F \subseteq cl(int(A)) - A$ implies $F \subseteq cl(int(A)) \cap A^c$, since $cl(int(A)) - A = cl(int(A)) \cap A^c$. Thus $F \subseteq cl(int(A))$. Now $F \subseteq A^c$ implies $A \subseteq F^c$. Here F^c is g-open and A is stronglyg^*-closed, we have $cl(int(A)) \subseteq F^c$. Thus $F \subseteq (cl(int(A)))^c$. Hence $F \subseteq (cl(int(A))) \cap (cl(int(A)))^c = \phi$. Therefore $F = \phi \Rightarrow cl(int(A)) - A$ contains no non empty closed sets.
Sufficient: Let $A \subseteq G$, G is g-open. suppose that cl(int(A)) is not contained in G then $(cl(int(A)))^c$ is a non empty closed set of $cl(int(A)) - A$ which is a contradiction. Therefore $cl(int(A)) \subseteq G$ and hence A is strongly g^* -closed. □

Corollary 3.4. *A strongly g^*-closed set A is regular closed iff $cl(int(A)) - A$ is closed and $cl(int(A)) \supseteq A$.*

Proof. Assume A that A is regular closed. Since $cl(int(A)) = A, cl(int(A)) - A = \phi$ is regular closed and hence closed.
conversely assume that$cl(int(A))-A$is closed.By the above theorem $cl(int(A))-A$ contains no nonempty closed set.Therefore $cl(int(A)) - A = \Phi$.Thus A is regular closed. □

Theorem 3.6. *Suppose that $B \subseteq A \subseteq X$, B is strongly g^*-closed set relative to A and that both open and strongly g^*closed subset of X then B is strongly g^*closed set relative to X.*

Proof. Let $B \subseteq G$ and G be an open set in X. But given that $B \subseteq A \subseteq X$, therefore $B \subseteq A$ and $B \subseteq G$. This implies $B \subseteq A \cap G$. Since B is strongly g^*-closed relative to A, $cl(int(B)) \subseteq A \cap G.(ie) A \cap cl(int(B)) \subseteq A \cap G$. This implies $A \cap (cl(int(B))) \subseteq G$. Thus $(A \cap (cl(int(B)))) \cup (cl(int(B)))^c \subseteq G \cup (cl(int(B)))^c$ implies $A \cup (cl(int(B)))^c \subseteq G \cup (cl(int(B)))^c$. since A is strongly g^*closed in X, we have $(cl(int(A))) \subseteq G \cup (cl(int(B)))^c$. Also $B \subseteq A \Rightarrow cl(int(B)) \subseteq cl(int(A))$. Thus $cl(int(B)) \subseteq cl(int(A)) \subseteq G \cup (cl(int(B)))^c$. Therefore B is strongly g^* closed set relative to X. □

Corollary 3.5. Corrollary 3.14: *Let A be strongly g^*closed and suppose that F is closed then $A \cap F$ is strongly g^*closed set.*

Proof. To show that$A \cap F$ is stronglyg^*-closed, we have to show $cl(int(A \cap F)) \subseteq G$ whenever $A \cap F \subseteq G$ and G is g-open. $A \cap F$ is closed in A and so strongly g^*closed in B. By the above theorem $A \cap F$ is strongly g^*closed in X. Since $A \cap F \subseteq A \subseteq X$. □

Theorem 3.7. Theorem 3.15: *If A is strongly g^*closed and $A \subseteq B \subseteq cl(int(A))$ then B is strongly g^*closed .*

Proof. Given that $B \subseteq cl(int(A))$ then $cl(int(B)) \subseteq cl(int(A)), cl(int(B)) - B \subseteq cl(int(A)) - A$. Since $A \subseteq B$. As A is strongly g^* closed by the above theorem $cl(int(A)) - A$ contains no non empty closed set, $cl(int(B)) - B$ contains no empty closed set. Again by theorem 3.13, B is strongly g^*-closed set. □

Theorem 3.8. Theorem 3.16: *Let $A \subseteq Y \subseteq X$ and suppose that A is strongly g^*closed in X then A is strongly g^* closed relative to Y.*

Proof. Given that $A \subseteq Y \subseteq X$ and A is strongly g^*closed in X. To show that A is strongly g^*-closed relative to Y, let $A \subseteq Y \cap G$, where G is g-open in X. Since A is strongly g^*-closed in X,$A \subseteq G$ implies $cl(int(A)) \subseteq G$. (ie) $Y \cap cl(int(A)) \subseteq Y \cap G$, where $Y \cap cl(int(A))$ is closure of interior of A in Y. Thus A is strongly g^*closed relative to Y. □

Theorem 3.9. *If a subset A of a topological space X is gsp-closed then it is strongly g^*-closed but not conversely.*

Proof. Suppose that A is gsp- closed set in X, let G be open set containing A. Then $G \supseteq spcl(A), A \cup G \supseteq A \cup (int(cl(int(A))))$ which implies $G \supseteq int(cl(int(A)))$ as G is open. (ie)$G \supseteq cl(int(A)) - A$ is strongly g^*closed set in X. □

Example 3.3. Example 3.18: *The converse of the above theorem need not be true from the following example.*
Let X= {a,b,c} with topology $\tau = \{\Phi, X, \{a\}, \{b, c\}\}\}$ and B={b}. B is not strongly g^*closed . since {b} is a g-open set of (X, τ) such that $B \subseteq \{b\}$ but $cl(B) = cl(\{b\}) = \{b, c\} \subseteq \{b\}$. However B is a gsp-closed set of (X, τ).

Theorem 3.10. Theorem 3.19: *Every $\delta-$ closed set is a strongly g^*closed set.*

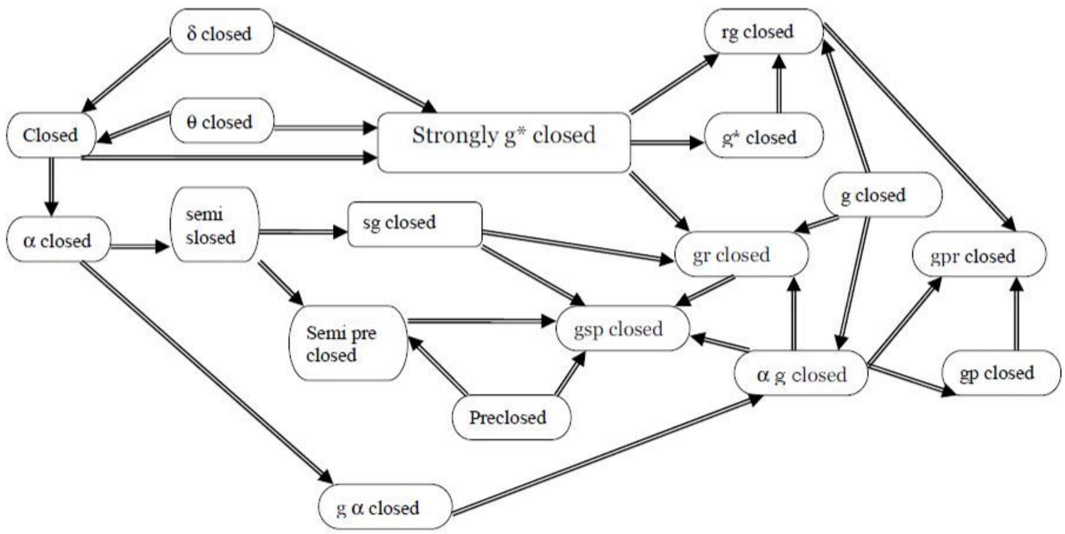
Proof. The Proof of the theorem is immediate from the definition. □

Example 3.4. *The converse of the above theorem need not be true from the following example.*
Let $X = \{a, b, c\}, \tau = \{\Phi, X, \{a, b\}\}D = \{a, c\}$. D is not a δ -closed set and also not even closed set. Hence D is strongly g^* closed set.

Theorem 3.11. *Every θ -closed set is a strongly g^*closed set.*

Proof. The Proof of the theorem is immediate from the definition. □

Example 3.5. *The converse of the above theorem need not be true from the following example.*
Let $X = \{a, b, c\}, \tau = \{\Phi, X, \{a\}, \{a, b\}\{a, c\}\}$ and E ={c}. Clearly E is closed and hence strongly g^*-closed. E is not $\theta-$ closed set of (X, τ).



Theorem 3.12. *Every strongly g^*-closed set in an α g-closed set and hence gs-closed, gsp-closed, gp-closed, gpr closed set and rg closed set but not conversely.*

Proof. Let A be a strongly g^*-closed set of (X, τ). By above theorem, A is g-closed. By implications (2.4) in Maki et.al(1993) A is α g-closed. From the investigations of Dontchev(1996) and Ganambal (1997), we know that every g-closed set is gs-closed, gsp-closed, gp-closed, gpr-closed and rg-closed. By above theorem every strongly g^*closed set is gs-closed, gsp closed and rg-closed. □

Example 3.6. *The converse of the above theorem need not be true from the following example.*
Let $X = \{a, b, c\}$, $\tau = \{\Phi, X, \{a, b\}\}D = \{b\}$. D is not a α g closed, gs closed, gp closed, gpr closed and regular-closed but not strongly g^*closed.

Remark 3.1. *The following are the implications of strongly g^*-closed set.*

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