Int. Journal of Math. Analysis, Vol. 6, 2012, no. 30, 1491 - 1499

Topological Transitivity of Uniform Limit Functions on G-spaces

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Abstract. We define and study the notion of topological transitivity of a continuous self-map on a metric $G-$space termed as topologically G-transitive map and obtain its characterization. Observing that the uniform limit of a sequence of topologically $G-$transitive maps need be topologically $G-$transitive, we obtain results giving sufficient conditions for the topological $G-$transitivity of the limit function.

Mathematics Subject Classification: 26A21, 26A03, 37D45, 54H20

Keywords: chain transitivity, metric $G-$spaces, topological transitivity, orbital convergence, uniform convergence

1. **Introduction**

Several real world problems are modeled by a discrete dynamical system $x_{n+1} = f(x_n), n = 0, 1, 2, ...$, where X is metric space and f is a continuous self map on X representing the dynamics of the system for example population growth models. In most situations, the dynamics of the system is unknown but the experimental data reflect chaotic symptoms, for example, unpredictability and very small differences in initial values result in very different behaviour. So to predict the state of the system in future time, one must approximate the dynamics of the system by a suitable function in accordance with the experimental data available. Naturally, prediction obtained will be an approximated function. Analysis of the study done in this direction shows the importance of the approximation and convergence of chaotic functions in the applied context.

It is well known that if a sequence of continuous functions converges uniformly then the limit function is continuous. Also, the limit function of a uniformly convergent sequence of Riemann integrable functions is Riemann integrable. However, differentiability of the limit function is not assured by the uniform convergence of a sequence of differentiable functions. An interesting problem in this direction is the investigation of which dynamical properties possessed by the functions in the sequence is inherited by the limit function [1, 2, 3, 11, 14, 15, 17]. For example, the limit function of a uniformly convergent sequence of topologically transitive functions need not be topologically transitive [2, 17]. Sufficient conditions have been obtained under which uniform limit function is topologically transitive [11, 14]. In [12], authors give a good survey of topological transitivity. In [17], author's study the Devaney's chaos of the uniform limit function. Some of the studies done in this direction have interesting applications in physics and engineering [10, 13, 16]. In [6, 9], different kinds of convergence of sequence of real valued functions have been studied.

In section 2, we give definitions and state known results required for remaining sections. In section 3, we define and study topological transitivity of functions on metric $G-$spaces and obtain a characterization of topological $G-$transitivity. In section 4, we define $G-$uniform convergence, give examples to justify that uniform limit of topological $G-$transitive functions need not be $G-$transitive and obtain sufficient conditions under which limit function is topologically $G-$transitive.

2. **Preliminaries**

Throughout (X, d) denotes a metric space with metric d, $\mathbb{R}$ denotes the set of real numbers, $\mathbb{Z}$ denotes the set of integers and $\mathbb{N}$ denotes the set of positive integers. By a G-space we mean a triple (X, G, θ), where X is a Hausdorff space, G is a topological group and $\theta : G \times X \to X$ is a continuous action of G on X [4]. Henceforth, $\theta(g, x)$ will be denoted by $g.x$. For $A \subset X, g \in X$ we have $g.A = \{g.a | a \in A\}$. For $x \in X$, the set $G(x) = \{g.x | g \in G\}$, is called the *G-orbit* of x in X. By trivial action of G on X we mean $g.x = x$ for all $g \in G, x \in X$. If X, Y are G-spaces, then a continuous map $h : X \to Y$ is called *equivariant* if $h(g.x) = g.h(x)$ for each g in G and each x in X. We call h *pseudoequivariant* if $h(G(x)) = G(h(x))$ for each x in X. An equivariant map is clearly pseudoequivariant but converse is not true [5]. In [7, 8] several interesting results using pseudoequivariant maps have been obtained. By a *metric G-space*, we mean a metric space (X, d) on which a topological group G acts.

Recall that a sequence $\{f_n\}_{n\in\mathbb{N}}$ of self-maps on a metric space (X, d) is said to converge uniformly to a self map f on X if for every $\varepsilon > 0$, there exists $n_0 \in \mathbb{N}$ (only depending on ε) such that $d(f_n(x), f(x)) < \varepsilon$ for all $n \geq n_0$ and for every $x \in X$. We write $f_n \xrightarrow{u} f$.

Definition 2.1. Let (X, d) be a metric space and $f : X \to X$ be a continuous map. Then f is called *topologically transitive* if for every pair U and V of non-empty open subsets of X, there is some $k \in \mathbb{N}$ such that $f^k(U) \cap V \neq \emptyset$.

For a continuous self-map f on a compact metric space X, let $tr(f)$ denote the set of all points x of X whose orbit $O(f,x) = \{f^n(x)|n \geq 0\}$ is dense in X. We recall the following result from [11]

Theorem 2.2. *Let (X,d) be a compact metric space and $f : X \to X$ be a continuous map. Then following are equivalent*

(i) *f is transitive;*
(ii) *f is onto and there is a point with dense orbit;*
(iii) *$tr(f)$ is a dense $G_\delta-$set in X.*

Using the equivalent conditions listed above, the following characterization about the transitivity of uniform limit of continuous self-maps on a compact metric space is obtained [11]:

Theorem 2.3. *Let $\{f_n\}_{n\in\mathbb{N}}$ be a sequence of transitive self-maps on a compact metric space (X,d) uniformly convergent to a map f. Then f is transitive if and only if there are $x_0 \in D = \cap\{tr(f_n) \mid n \in \mathbb{N}\}$ and $x_1 \in X$ such that*

$$\overline{\{f_n^{k_n}(x_0)|n \in \mathbb{N}\}} \cap \overline{O(f,x_1)} \neq \emptyset$$

for every sequence $\{k_n\}_{n\in\mathbb{N}}$ of non-negative integers.

Although transitivity is not inherited by uniform limits but another related dynamical property 'chain transitivity' is inherited by uniform limits. We recall the following [11]:

Definition 2.4. Let (X,d) be a compact metric space and $f : X \to X$ be a continuous map. For a real number $\delta > 0$, a sequence $\{x_n\}_{n\geq 0}$ in X is called a δ-*chain* if $d(f(x_n), x_{n+1}) < \delta$ for all n. The map f is called *chain-transitive* if for all $x, y \in X$ and every $\delta > 0$, there is a finite δ-chain $\{x_0, x_1, ..., x_n\}$ such that $x_0 = x$ and $x_n = y$.

Following is proved in [11].

Theorem 2.5. *Let $\{f_n\}_{n\in\mathbb{N}}$ be a sequence of chain transitive self-maps on a metric space (X,d) uniformly convergent to a map f. Then f is chain-transitive.*

The following strong form of convergence ensures the transitivity of a limit function on metric spaces.

Definition 2.6. [11] Let $\{f_n\}_{n\in\mathbb{N}}$ be a sequence of continuous self maps on a metric space (X,d). Then $\{f_n\}$ is called orbitally convergent to a map $f : X \to X$ if for every $\varepsilon > 0$, there exists $k \in \mathbb{N}$ such that $d(f_n^m(x), f^m(x)) < \varepsilon$ for every $x \in X$, $m \in \mathbb{N}$ and for all $n \geq k$.

Remark 2.7. Orbital convergence is stronger than uniform convergence.

Theorem 2.8. *Let (X,d) be a metric space and $\{f_n\}_{n\in\mathbb{N}}$ be a sequence of topologically transitive maps on X orbitally convergent to a map f. Then f is also topologically transitive.*

3. Transitivity on G-spaces

Definition 3.1. Let (X, d) be a metric $G-$space and $f : X \to X$ be a continuous map then f is called $G-$*transitive* if for every pair of non-empty open subsets U and V of X, there exists $n \in \mathbb{N}$ and $g \in G$ such that $g.f^n(U) \cap V \neq \emptyset$.

Remark 3.2. Under trivial action of G on X, notions of transitive and $G-$transitive coincide. Under non-trivial action of G on X, if f is transitive then it is $G-$transitive. But following example shows that every $G-$transitive map need not be transitive.

Example 3.3. Let $X = \{\pm\frac{1}{n}, \pm(1-\frac{1}{n}) | n \in \mathbb{N}\}$ under usual metric. Consider action of $\mathbb{Z}_2$, additive group of integers mod 2, on X given by $0.t = t$ and $1.t = -t$, $t \in X$. Then $f : X \to X$ defined by

$$f(x) = \begin{cases} x_+, & \textit{if } x \in \{\frac{1}{n}, 1-\frac{1}{n} \mid n \neq 1, n \in \mathbb{N}\}, \\ x_-, & \textit{if } x \in \{-\frac{1}{n}, -(1-\frac{1}{n}) \mid n \neq 1, n \in \mathbb{N}\}, \\ x, & \textit{if } x \in \{-1, 0, 1\}, \end{cases}$$

where x_+ (respectively x_-) denotes element of X immediate to right (respectively to left) of x. Then f is $\mathbb{Z}_2-$ transitive but not transitive.

Theorem 3.4. *Let X be a compact metric $G-$space with no isolated points and $f : X \to X$ be a pseudoequivariant onto map. Then following conditions are equivalent:*

(i) *There exists $x_0 \in X$ such that $\{g.f^n(x_0) \mid g \in G, n \geq 0\}$ is dense in X.*
(ii) *For every pair of non-empty open sets U and V in X, there exists $n \in \mathbb{N}$ and $g \in G$ such that $g.f^n(U) \cap V \neq \emptyset$.*

Proof. (i) implies (ii): We have closure of $\{g.f^n(x_0) \mid g \in G, n \geq 0\}$ equals X. Let U and V be non-empty open sets of X then there exists $g \in G, n \geq 0$ such that $g.f^n(x_0) \in U$ and $k \in G, m \geq 0$ such that $k.f^m(x_0) \in V$ respectively. We can assume $n < m$. Observe that

$$g.f^n(x_0) \in U \text{ implies } x_0 \in f^{-n}(g^{-1}.U)$$

and therefore using pseudoequivariancy of f we find $g' \in G$ such that

$$f^m(x_0) \in f^{m-n}(g^{-1}.U) = g'.f^{m-n}(U).$$

Since $k.f^m(x_0) \in V$ and we have $k'' = k.g' \in G$ satisfying

$$k.f^m(x_0) \in k.g'.f^{m-n}(U) = k''.f^{m-n}(U),$$

we get

$$k''.f^p(U) \cap V \neq \emptyset,$$

where $p = m - n > 0$.
Conversely suppose (ii) is satisfied. Since X is compact metric space, it is separable and hence second countable. Let $\{W_n | n \in \mathbb{N}\}$ be a countable basis

for X. Then for any $m \geq 0$, $f^{-m}(W_n)$ is a non-empty open set in X for every $n \in \mathbb{N}$. In view of (ii), for any non-empty open set V of X, there exists $k \in \mathbb{N}, g_n \in G$ such that

$$g_n.f^{k-m}(W_n) \cap V \neq \emptyset.$$

Therefore

$$\cup\{g_n.f^{-m}(W_n) \mid m \geq 0\}$$

is dense in X and hence

$$\cup\{[\cap\{X - g_n.f^{-m}(W_n) \mid m \geq 0\}] \mid n \in \mathbb{N}\}$$

is a countable union of nowhere dense sets in X. Applying Baire category theorem, we get $x \in X$ and

$$x \notin [\cup\{(\cap\{X - g_n.f^{-m}(W_n) \mid m \geq 0\}) \mid n \in \mathbb{N}\}].$$

Thus we obtain $m \geq 0$ such that for all $n \in \mathbb{N}$ we have

$$x \notin (X - g_n.f^{-m}(W_n)).$$

Now pseudoequivariancy of f gives $k_n \in G$ such that for some $m \geq 0$ and for all $n \in \mathbb{N}$

$$x \in g_n.f^{-m}(W_n) = f^{-m}(k_n.W_n).$$

But this gives $k_n^{-1}.f^m(x) \in W_n$ for all $n \in \mathbb{N}$ and hence

$$\{g.f^n(x) \mid g \in G, n \geq 0\}$$

is dense in X. This completes our proof. □

4. G-transitivity of uniform limit maps

Definition 4.1. Let (X, d) be a metric $G-$space. A sequence of functions $\{f_n\}$ from X to X is said to converge $G-$*uniformly* to a function $f : X \to X$ (written as $f_n \xrightarrow{G.u} f$) if for every $\epsilon > 0$ there exists $n_0 \in \mathbb{N}$ such that $d(g.f_n(x), k.f(x)) < \epsilon$ for all $g, k \in G$, for all $x \in X$ and for all $n \geq n_0$.

Example 4.2. Let $X = [-1, 1]$ and let $\mathbb{Z}_2$ act on X by $0.t = t$ and $1.t = -t$, $t \in X$. Consider $f_n : X \to X$ defined by $f_n(x) = \frac{x}{n}, n \in \mathbb{N}$. Then $f_n \xrightarrow{G.u} 0$.

Remark 4.3. If $f_n \xrightarrow{G.u} f$ then $f_n \xrightarrow{u} f$. The converse need not be true as observed in the following example.

Example 4.4. Let $X = [0, 1]$ and let $\mathbb{Z}_2$ act on X by $0.t = t$ and $1.t = 1 - t$, $t \in X$. Consider $f_n : X \to X$ defined by $f_n(x) = \frac{x}{n}, n \in \mathbb{N}$. Then $f_n \xrightarrow{u} 0$ but not $G-$uniformly.

We recall that a uniform limit of transitive maps need not be transitive. The following example shows that uniform limit of $G-$transitive maps also need not be $G-$transitive.

Example 4.5. Let $I = [0, 1]$. For any given $n \in \mathbb{N}$, $n \geq 3$, let $a_i = \frac{i}{n}$, $i \in \{0, 1, 2, ..., n\}$ and let $f_n : I \to I$ be the piecewise linear map satisfying $f_n(a_i) = a_i$, $i \in \{0, 1, 2, ..., n\}, f_n = (\frac{a_0+a_1}{2}) = a_2, f_n(\frac{a_{n-1}+a_n}{2}) = a_{n-2}$ and for $i \in \{1, 2, ..., n-2\}, f_n(\frac{2a_i+a_{i+1}}{3}) = a_{i+2}, f_n(\frac{a_i+2a_{i+1}}{3}) = a_{i-2}$. Then $\{f_n\}$ converges to identity function uniformly [17]. On the other hand if $\mathbb{Z}_2$ acts on I as in Example 4.4, then each f_n is topologically $\mathbb{Z}_2-$ transitive but identity function is not topologically $\mathbb{Z}_2-$ transitive.

Definition 4.6. Let $f : X \to X$ be a continuous map on a metric $G-$space (X, d) and δ be a positive real number. A sequence $\{x_n\}_{n\in\mathbb{N}}$ in X is said to be a $G - \delta$ *chain* if there exists $g_n \in G$ such that $d(g_n.f(x_n), x_{n+1}) < \delta$ for all n. The map f is called $G-$*chain transitive* if for each pair of points $x, y \in X$ and for every $\delta > 0$ there is a finite $G - \delta$ chain $x_0, ..., x_n$ such that $x_0 = x$ and $x_n = y$.

Theorem 4.7. *Let $\{f_n\}$ be a sequence of $G-$chain transitive maps on a metric $G-$space (X, d) converging $G-$uniformly to a map f. Then f is also $G-$chain transitive.*

Proof. Let $a, b \in X$ and $\varepsilon > 0$. Since $f_n \xrightarrow{G.u.} f$, take $m \in \mathbb{N}$ such that $d(g.f_m(x), k.f(x)) < \varepsilon/2$ for all $g, k \in G$. Since f_m is $G-$chain transitive, there exists $x_0, ..., x_n$ such that $x_0 = a$ and $x_n = b$ and $g_i \in G$ such that

$$d(g_i.f_m(x_i), x_{i+1}) < \varepsilon/2$$

for every $i < n$. Therefore

$$d(g_i.f(x_i), x_{i+1}) \leq d(g_i.f(x_i), g_i.f_m(x_i)) + d(g_i.f_m(x_i), x_{i+1}) < \varepsilon$$

for every $i < n$. This proves $x_0, ..., x_n$ is a $G-$chain from a to b with respect to f and hence f is $G-$chain transitive. □

Note 4.8. Under trivial action of G on X, the above result coincides with Theorem 2.5.

We can deduce the following corollary.

Corollary 4.9. *Let f be the $G-$uniform limit of a sequence of $G-$transitive self maps on a compact metric $G-$space then f is $G-$chain transitive.*

We obtain a sufficient condition under which limit function of a sequence of $G-$transitive maps is $G-$transitive.

Definition 4.10. Let $\{f_n\}$ be a sequence of continuous self-maps on a metric $G-$space (X, d). We say that $\{f_n\}$ is $G-$*orbitally convergent* to a map $f : X \to X$ if for every $\varepsilon > 0$, there exists $p \in \mathbb{N}$ such that $d(g.f_n^m(x), k.f^m(x)) < \varepsilon$ for all $x \in X$, for all $m \in \mathbb{N}$, for all $g, k \in G$ and for all $n \geq p$.

Theorem 4.11. *Let $\{f_n\}$ be a sequence of $G-$transitive maps on a metric $G-$space (X, d) converging $G-$orbitally to a map $f : X \to X$. Then f is also $G-$transitive.*

Proof. Let A, B be non-empty sets in X. Since $\{f_n\}$ converges $G-$orbitally to f, for every $\varepsilon > 0$ there exists $p \in \mathbb{N}$ such that $d(g.f_n^m(x), k.f^m(x)) < \varepsilon$ for each $x \in X$, for all $m \in \mathbb{N}$, for all $g, k \in G$ and for all $n \geq p$. Since B is open, there exists $\varepsilon > 0$ and $q \in X$ such that $B(q, \varepsilon) \subset B$, where $B(q, \varepsilon)$ denotes open ball centered at q and of radius of ε. As $\{f_n\}$ converges $G-$orbitally to f, corresponding to $\varepsilon/2$, there exists $p \in \mathbb{N}$ such that $d(g.f_n^m(x), k.f^m(x)) < \varepsilon/2$, for every $x \in X$, for all $m \in \mathbb{N}$, for all $g, k \in G$ and for all $n \geq p$. Choose an $n \geq p$. Then $G-$transitivity of f_n implies there exists $m \in \mathbb{N}$ and $g \in G$ such that

$$g.f_n^m(A) \cap B(q, \varepsilon/2) \neq \emptyset$$

and hence there exists $a \in A$ satisfying $d(g.f_n^m(a), q) < \varepsilon/2$. Thus

$$d(g.f^m(a), q) \leq d(g.f^m(a), g.f_n^m(a)) + d(g.f_n^m(a), q) < \varepsilon/2 + \varepsilon/2 = \varepsilon$$

which implies

$$g.f^m(a) \in B(q, \varepsilon) \subset B$$

and hence

$$g.f^m(A) \cap B \neq \emptyset.$$

This proves f is topologically $G-$transitive. □

Note 4.12. Under trivial action of G on X, above result coincides with Theorem 2.8.

For a continuous self map f on a compact metric $G-$space X, we define $tr_G(f)$ to be the set of all points $x \in X$ for which $O_G(f, x) = \{g.f^n(x) | g \in G, n \geq 0\}$ is dense in X, that is

$$tr_G(f) = \{x \in X \mid \overline{O_G(f, x)} = X\}.$$

Lemma 4.13. *Let (X, d) be a compact metric $G-$space and $f : X \to X$ be a continuous onto map. Then f is $G-$transitive if and only if $tr_G(f)$ is a dense $G_\delta-$set.*

Proof. Since X is a compact metric space, it is separable and hence has a countable dense set say $\{x_n\}$. Proof follows by noting that

$$tr_G(f) = \cap_{n=0}^{\infty}[\cap_{k=1}^{\infty}[\cup_{g \in G, m \in \mathbb{N}\cup\{0\}}[g.f^m(B(x_n, \frac{1}{k}))]]],$$

where $B(x, \varepsilon)$ denotes the open ball of radius ε centered at x, and hence $tr_G(f)$ is a dense $G_\delta-$set. □

Theorem 4.14. *Let (X, d) be a compact metric $G-$space with no isolated points and let $f_n : X \to X$ be a sequence of $G-$transitive onto maps converging $G-$uniformly to a continuous self-map f on X. Then f is $G-$transitive if and only if there exists $x_0 \in D = \cap\{tr_G(f_n) \mid n \in \mathbb{N}\}$ and $x_1 \in X$ such that*

$$\overline{\{g.f_n^{k_n}(x_0) | n \in \mathbb{N}, g \in G\}} \cap \overline{\{g.f^n(x_1) | n \in \mathbb{N}, g \in G\}} \neq \emptyset$$

for every sequence $\{k_n\}$ of non-negative integers.

Proof. Sincef_n is $G-$transitive therefore by previous Lemma $tr_G(f_n) \neq \emptyset$ and is a dense $G_\delta-$set for all $n \in \mathbb{N}$. This gives $D = \cap\{tr_G(f_n) \mid n \in \mathbb{N}\}$ is a countable intersection of open dense sets in the Baire space X. Also, observe that f is an onto function.

Now, if f is $G-$transitive then it is sufficient to take x_0 any point in D and x_1 a point for which $O_G(f, x_1)$ is dense in X.

Conversely, suppose that there are points x_0 and x_1 satisfying condition in the hypothesis. Then by Theorem 3.4 it is sufficient to show that $O_G(f, x_1)$ is dense in X. We take an open disc $B(p, \varepsilon)$ in X. Since $O_G(f_n, x_0)$ is dense in X for all $n \in \mathbb{N}$, there is some $k_n \in \mathbb{N}$ and $g_n \in G$ such that

$$g_n.f_n^{k_n}(x_0) \in B(p, \varepsilon/2).$$

Since

$$\overline{\{g.f_n^{k_n}(x_0) | n \in \mathbb{N}, g \in G\}} \cap \overline{O_G(f, x_1)} \neq \emptyset$$

there is some $g_m.f_m^{k_m}(x_0)$ and $g_s.f^s(x_1)$ such that

$$d(g_m.f_m^{k_m}(x_0), g_s.f^s(x_1))\varepsilon/2.$$

This gives

$$\begin{aligned} d(g_s.f^s(x_1), p) & \\ & \leq d(g_s.f^s(x_1), g_m.f_m^{k_m}(x_0)) + d(g_m.f_m^{k_m}(x_0), p) \\ & < \varepsilon/2 + \varepsilon/2 = \varepsilon, \end{aligned}$$

which implies $g_s.f^s(x_1) \in B(p, \varepsilon)$. Thus $\overline{O_G(f, x_1)} = X$ and hence f is $G-$transitive. □

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Received: January, 2012