Optimal Pricing and Ordering Policy

for Three Parameter Weibull Deterioration

under Trade Credit

C. K. Tripathy* and L. M. Pradhan**

*Department of Statistics
Sambalpur University, Jyoti Vihar
Sambalpur -768019, India
c.tripathy@yahoo.com

**Department of Mathematics
Silicon Institute of Technology, Sason
Sambalpur-768200. India
lalitshivbaba@gmail.com

Abstract

This paper deals with development of an optimal pricing and ordering policy for items with three parameter Weibull deterioration where the supplier offers a delay in payments. Here the demand is considered to decrease with time. Optimal selling price and the ordering quantity have been derived in the end which maximizes the retailer’s profit. Optimal cycle time, selling price, ordering quantity and total profit per unit time for this model have been derived and the results are illustrated with the help of numerical examples and a sensitivity analysis at the end.

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1. Introduction

Generally in an EOQ model we assume that the retailer must be paid for the items as soon as the items were received. In practice the supplier hopes to stimulate his
products and so he will offer the retailer a delay period, namely the trade credit period. Before the end of the trade credit period the retailer can sell the goods and accumulate revenue and earn interest on the other hand a higher interest is charged if the payment is not settled by the end of the trade credit period. Therefore it is required for the retailer to delay the payment up to the last moment of the permissible period allowed by the supplier. Much work has been done in this respect for inventory with permissible delay in payments. Some of the prominent work are discussed below.

Goyal [7] explored the concept of trade credit. He computed interest earned on the sales revenue on unit purchase price. Aggarwal and Jaggi [8] considered the inventory model with an exponential deterioration rate under the permissible delay in payments. Jamal, Sarker, and Wang [1] then further generalized the model to allow for shortages. Huang and Chung [10] discussed the replenishment and payment policies to minimize the annual total average cost under cash discount and payment delay from the retailer’s point of view. Huang [9] extended one level trade credit into two level trade credits. He assumed that not only the supplier offers the retailer trade credit but also the retailer offers the trade credit to his customer. Shah and Raykundaliya [5] derived a retailer’s pricing and ordering strategy for weibull distribution deterioration under trade credit in declining market. Teng [3] also investigated EOQ model under conditions of permissible delay in payments. Abad and Jaggi[6] has made a joint approach for setting unit price and the length of the credit period for a seller when end demand is price sensitive. Chang [2] developed an EOQ model with deteriorating items under inflation when supplier credits linked to order quantity. Chung and Liao [4] has made a lot-sizing decision under trade credit depending on the ordering quantity.

In this paper we have assumed demand as a decreasing function of time and sale price. Deterioration is allowed on inventory which follows three parameter Weibull distributions. Here we have assumed that the supplier offers trade credit to the retailer for settlement of the account against purchases. The two cases considered here are interest earned on the generated sales during the permissible delay period and interest charged on unsold items in the stock after the permissible delay period. Section 2 contains basic assumptions and notations. Section 3 contains mathematical model. Numerical examples and sensitivity table have been given in section 4 and 5 respectively.

2. Basic assumptions and notations

The following are the assumptions applied in the development of the model:

a) We have made this model for a single item only.
b) Shortages are allowed and lead-time is zero.
c) An infinite planning horizon is assumed.
d) The demand of the product decreases with the increase of time and sale price.
e) The deterioration follows three parameter Weibull distribution. There is no replacement or repair for a deteriorated item.
f) The retailer gets revenue by selling the product and then deposited it in an interest earning account during the allowable credit period. At the end of this period, the retailer settles the account for all the units sold keeping the difference for day to day expenses, and starts paying the interest charges on the unsold items in the inventory.

The notations that are employed here:

i. $C$: The unit purchase cost.

ii. $P$: The unit sale price with $P > C$.

iii. $A$: The ordering cost per order.

iv. $\theta$: $\theta = \alpha \beta (t - \gamma)^{\beta - 1}$, The Three parameter Weibull distribution deterioration rate (unit/unit time). Where $0 \leq \alpha < 1$ is called the scale parameter, $\beta > 0$ is the shape parameter and $\gamma$ is the location parameter, $-\infty < \gamma < \infty$.

v. $h$: The inventory holding cost per unit per annum excluding interest charges.

vi. $Q$: The order quantity.

vii. $R(t, P) = a (1 - b t)^{P - \eta}$: Where $a > 0$ is fixed demand, $0 < b < 1$ is rate of change of demand and $\eta > 1$ is makeup parameter.

viii. $T$: Optimal cycle time.

ix. $M$: The permissible credit period.

x. $I_c$: The interest charged per monetary unit in stock per annum by the supplier.

xi. $I_e$: The interest earned per monetary unit per year, where $I_e < I_c$.

xii. $I(t)$: The inventory level at any instant of time $t$, $0 \leq t \leq T$.

xiii. $Z(P, T)$: The total profit over the total time period considered.

xiv. $SR$: Sales revenue.

xv. $PC$: Purchase cost.

xvi. $IHC$: Inventory holding cost excluding interest charges.

xvii. $OC$: Ordering cost.

### 3. Mathematical Model

The inventory level of the product at time $t$ over the period $[0, T]$ can be represented by the differential equation

$$
\frac{dI(t)}{dt} + \theta(t) I(t) = -R(t, P) \quad 0 \leq t \leq T
$$

(1)
Where \( \theta(t) = \alpha \beta (t - \gamma)^{\beta-1} \) is the three parameter Weibull distribution deterioration. Subject to the boundary conditions \( I(0) = Q \) and \( I(T) = 0 \).

The solution of equation (1) is

\[
I(t)e^{\alpha(t-\gamma)^\beta} = -aP^{-\eta} \int_0^t (1-b t)e^{\alpha(t-\gamma)^\beta} dt + Q e^{\alpha(t-\gamma)^\beta} \quad (2)
\]

Again using \( I(T) = 0 \) in equation (2) we get

\[
Q = e^{-\alpha(t-\gamma)^\beta} aP^{-\eta} \int_0^T (1-b t)e^{\alpha(t-\gamma)^\beta} dt \quad (3)
\]

Using equation (3) in equation (2) we get

\[
I(t)e^{\alpha(t-\gamma)^\beta} = aP^{-\eta} \left\{ \int_0^T (1-b t)e^{\alpha(t-\gamma)^\beta} dt - \int_0^t (1-b t)e^{\alpha(t-\gamma)^\beta} dt \right\} \quad (4)
\]

Integrating the above using the Taylor’s series expansion of the exponential function and neglecting the terms containing degree greater or equal to two of \( \alpha \) and \( b \) and also neglecting the factors involving \( \alpha b \) we get from equation(3),

\[
Q = aP^{-\eta} \left[ T + \alpha(t-\gamma)^{\beta+1} - \frac{b T^2}{2} - \alpha(-\gamma)^{\beta+1} - \alpha(-\gamma)^\beta T \right] \quad (5)
\]

And from equation (4)

\[
I(t) = aP^{-\eta} \left[ T - t + \alpha(T-\gamma)^{\beta+1} - \frac{b T^2}{2} - \frac{b T^2}{2} - \alpha T(t-\gamma)^\beta + \alpha \gamma T(t-\gamma)^\beta \right] \quad (6)
\]

Let us now calculate the different cost involved in this model.

Purchase cost, \( PC = \frac{CQ}{T} \) \quad (7)

Sales Revenue, \( SR = \frac{PQ}{T} \) \quad (8)

Ordering cost, \( OC = \frac{A}{T} \) \quad (9)

Inventory holding cost,

\[
IHC = \frac{h}{T} \int_0^T I(t) dt
\]
Optimal pricing and ordering policy

\[
= h a P^{-\eta} \left[ \frac{T}{2} + \frac{\alpha (T - \gamma)^{\beta+1}}{\beta + 1} - \frac{2\alpha (T - \gamma)^{\beta+2}}{(\beta + 1)(\beta + 2)T} - \frac{bT^2}{3} + \frac{\alpha (-\gamma)^{\beta+1}}{\beta + 1} + \frac{2\alpha (-\gamma)^{\beta+2}}{(\beta + 1)(\beta + 2)T} \right]
\]

We now come across the two cases: Case-1 \( M \leq T \), case-2 \( M > T \).

**Case-1**: \( M \leq T \)

Under the assumption (b) above, the retailer sells \( R(M) M \) units by the end of the permissible trade credit \( M \) and has \( CR(M) M \) to pay the supplier. The supplier charges an interest rate \( I_\epsilon \) from time \( M \) onwards for the unsold items in the stock. Hence, the interest charged, \( IC_1 \) per time unit is

\[
IC_1 = \frac{Cl_c}{T} \int_{M}^{T} I(t) dt = Cl_c aP^{-\eta} \left[ \frac{T}{2} + \frac{\alpha (T - \gamma)^{\beta+1}}{\beta + 1} - \frac{2\alpha (T - \gamma)^{\beta+2}}{(\beta + 1)(\beta + 2)T} - \frac{bM^2}{2(\beta + 1)(\beta + 2)T} + \frac{2\alpha M (T - \gamma)^{\beta+1}}{(\beta + 1)T} \right] \tag{11}
\]

During \([0, M]\) the retailer sells the product and deposits the revenue into an interest earning account at the rate \( I_\epsilon \) per monetary unit per year.

We get the interest earned, \( IE_1 \) per time unit

\[
IE_1 = \frac{Pl_e}{T} \int_{0}^{M} R(t,P) t dt = \frac{al_e P^{-\eta}}{T} \left[ \frac{M^2}{2} - \frac{bM^3}{3} \right] \tag{12}
\]

Hence, the retailers profit per time unit is

\[
Z_1(T, P) = SR - PC - OC + IE_1 - IHC - IC_1 = \frac{PQ}{T} - \frac{CQ}{T} - \frac{A}{T} + IE_1 - IHC - IC_1
\]

\[
= (aP^{\theta} - CaP^{-\eta}) \left[ 1 + \frac{\alpha (T - \gamma)^{\beta+1}}{(\beta + 1)T} - \frac{bT^2}{2} - \frac{\alpha (-\gamma)^{\beta+1}}{(\beta + 1)T} + \frac{2\alpha (-\gamma)^{\beta+2}}{(\beta + 1)(\beta + 2)T} - \frac{bT^2}{3} \right]
\]

\[
- \frac{A}{T} + \frac{al_e P^{\eta}}{T} \left[ \frac{M^2}{2} - \frac{bM^3}{3} \right] - h aP^{-\eta} \left[ \frac{T}{2} + \frac{\alpha (T - \gamma)^{\beta+1}}{(\beta + 1)} - \frac{2\alpha (T - \gamma)^{\beta+2}}{(\beta + 1)(\beta + 2)T} - \frac{bT^2}{3} \right] \tag{13}
\]

\[
- \frac{\alpha (-\gamma)^{\beta+1}}{\beta + 1} + \frac{2\alpha (-\gamma)^{\beta+2}}{(\beta + 1)(\beta + 2)T} - Cl_c aP^{-\eta} \left[ \frac{T}{2} + \frac{\alpha (T - \gamma)^{\beta+1}}{(\beta + 1)} - \frac{2\alpha (T - \gamma)^{\beta+2}}{(\beta + 1)(\beta + 2)T} - \frac{bT^2}{3} \right]
\]
\[-M + \frac{M^2}{2T} - \frac{\alpha M (T - \gamma)^{\beta+1}}{(\beta+1)T} + \frac{2\alpha (M - \gamma)^{\beta+2}}{(\beta+1)(\beta+2)T} + \frac{bMT}{2} - \frac{bM^3}{6T} + \frac{\alpha (M - \gamma)^{\beta+1}}{(\beta+1)T} \]

(13)

**Case-2: \( M > T \)**

Here, the retailer sells \( R(T)T \) units in all by the end of the cycle time and has \( CR(T)T \) to pay the supplier in full by the end of the credit period \( M \). Hence, interest charges

\[
IC_2 = 0
\]

(14)

And the interest earned per time unit is

\[
IE_2 = \frac{Pf}{T} \left[ \int_0^T R(t, P) dt + R(T, P)T(M - T) \right]
\]

\[
= a I_c P^{1-\eta} \left[ M - MbT - \frac{T}{2} - \frac{2bT^2}{3} \right]
\]

(15)

The total cost; \( Z_2(T, P) \) of an inventory system per time unit is

\[
Z_2(T, P) = SR - PC - OC + IE_2 - IHC - IC_2
\]

\[
= (aP^{1-\eta} - CaP^{-\eta}) \left[ 1 + \frac{\alpha (T - \gamma)^{\beta+1}}{(\beta+1)T} - \frac{T}{2} - \frac{\alpha(-\gamma)^{\beta+1}}{(\beta+1)T} \right] \frac{A}{T} - \frac{bT^2}{2} + \frac{bT}{3} - \frac{2bT^2}{3} \]

\[
+ a I_c P^{1-\eta} \left[ \frac{T}{2} + \frac{2bT^2}{3} - \frac{aP^{-\eta}}{P} \right] \left[ \frac{\alpha (T - \gamma)^{\beta+1}}{\beta+1} - \frac{2\alpha (T - \gamma)^{\beta+2}}{(\beta+1)(\beta+2)T} \right]
\]

(16)

The necessary conditions for \( Z_i(T, P) \) to be optimum is

\[
\frac{\partial Z_1(T, P)}{\partial P} = 0
\]

(17)

And

\[
\frac{\partial Z_1(T, P)}{\partial T} = 0
\]

(18)

Then the solution obtained from (17) and (18), \( (T, P) = (T_1, P_1) \) (say) maximizes the profit \( Z_i(T, P) \) for which

\[
\frac{\partial^2 Z_1}{\partial P^2} < 0, \quad \frac{\partial^2 Z_1}{\partial T^2} < 0 \quad \text{and} \quad \left( \frac{\partial^2 Z_1}{\partial P^2} \right) \left( \frac{\partial^2 Z_1}{\partial T^2} \right) - \left( \frac{\partial^2 Z_1}{\partial T \partial P} \right) > 0
\]

(19)

Similarly the necessary conditions for \( Z_2(T, P) \) to be optimum is
Optimal pricing and ordering policy

\[
\frac{\partial Z_2(T,P)}{\partial P} = 0 \tag{20}
\]

And

\[
\frac{\partial Z_2(T,P)}{\partial T} = 0 \tag{21}
\]

Then the solution obtained from (20) and (21), \((T, P) = (T_2, P_2)\) (say) maximizes the profit \(Z_2(T, P)\) for which

\[
\frac{\partial^2 Z_2}{\partial P^2} < 0, \quad \frac{\partial^2 Z_2}{\partial T^2} < 0 \quad \text{and} \quad \left( \frac{\partial^2 Z_2}{\partial P^2} \right) - \left( \frac{\partial^2 Z_2}{\partial T \partial P} \right) > 0
\]

\(\tag{22}
\]

The above equations from (17) to (22) can be solved by the help of mathematica -5.1 software.

4. Numerical Examples

**Example 1:** \(M \leq T\)

Here we have taken

\[
[a, b, \eta, h, C, A, I, \alpha, \beta, \gamma, M] = [300000, 0.2, 2, 1, 20, 300, 0.12, 0.04, 0.2, 4, 1, 0.08]
\]

in their proper units. Using these data in equation (17), (18) and (19) we get

\[
P_1 = 41.5358, \quad T_1 = 0.530608, \quad \frac{\partial^2 Z_1}{\partial P^2} = -3.43648, \quad \frac{\partial^2 Z_1}{\partial T^2} = -2842.1
\]

and

\[
\left( \frac{\partial^2 Z_1}{\partial P^2} \right) - \left( \frac{\partial^2 Z_1}{\partial T \partial P} \right) = 9753.94 > 0, \quad \text{which maximizes the profit as}
\]

\[Z_1(T, P) = 2398.97 \quad \text{and} \quad Q = 75.7155\]

**Example 2:** \(M > T\)

Now we are taking

\[
[a, b, \eta, h, C, A, I, \alpha, \beta, \gamma, M] = [100000, 0.2, 2, 1, 20, 50, 1.54, 0.2, 2, 1, 0.16]
\]

in their proper units. Using these data in equation (20), (21) and (22) we get

\[
P_2 = 36.2306, \quad T_2 = 0.131579, \quad \frac{\partial^2 Z_1}{\partial P^2} = -2.32101, \quad \frac{\partial^2 Z_1}{\partial T^2} = -42379.5
\]

and

\[
\left( \frac{\partial^2 Z_1}{\partial P^2} \right) - \left( \frac{\partial^2 Z_1}{\partial T \partial P} \right) = 98283.6357 > 0, \quad \text{which maximizes the profit as}
\]

\[Z_2(T, P) = 1143.35 \quad \text{and} \quad Q = 9.63976\]
Using the data of example 1 the sensitivity analysis is carried out by changing values of $M, \alpha, \beta, \gamma, b$ from -50%, -25%, 25%, 50%. The variation in cycle time, selling price, purchase units and total profit per time unit are exhibited in the table given below.

5. Sensitivity Analysis

On the basis of the data given in example-1 above we have studied the sensitivity analysis changing one parameter at a time and keeping the rest fixed.

<table>
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<tr>
<th>Parameter</th>
<th>%change</th>
<th>$T$</th>
<th>$P$</th>
<th>$Q$</th>
<th>$Z_1$</th>
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<td>$M$</td>
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From the above table it can be observed that increasing the delay period decreases the cycle time and selling price but increases the order quantity and the total profit. Increasing in scale parameter decreases the cycle time, selling price, order quantity and total profit. When the shape parameter increases it shows variable changes in
each factor. Increasing in the location parameter decreases the Cycle time, order quantity and selling price but it shows variable changes towards the total profit. Again when the demand rate increases all factors decrease simultaneously.

6. Conclusions

Here in this model we have derived an optimal ordering and pricing policy for a retailer when the inventory are subject to Weibull three parameter deterioration with declining demand under trade credit policy. It is observed that the retailer should take the advantage of permissible delay payments more frequently and should replenish smaller order.

References


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