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Transient Free Convection MHD Flow between Two Long Vertical Parallel Plates with Constant Temperature and Variable Mass Diffusion

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Abstract

The effect of a uniform transverse magnetic field on the unsteady transient free convection flow of an incompressible viscous electrically conducting fluid between two infinite vertical parallel plates with constant temperature and Variable mass diffusion is discussed. The Laplace transform method has been used to find the solutions for the velocity, temperature and concentration profiles. The velocity and skin-friction are studied for different parameters like Prandtl number, Schmidt number, magnetic parameter, buoyancy ratio parameter, and time.

Mathematics Subject Classification: 76R10, 80A20, 76W05

Keywords: Transient free convection, Heat transfer, Mass transfer, MHD, Mass diffusion

1 Introduction

Magnetohydrodynamic (MHD) flow between two parallel plates is a classical problem that has importance in many applications such as: MHD power generators and pumps etc. Singh et al.[1] have studied the transient free convection flow of a viscous incompressible fluid in a vertical parallel plate channel, when the walls are heated asymmetrically. Jha BK.[3] has studied the combined effect of natural convection and uniform transverse magnetic field on the unsteady couette flow. Narahari et al.[5] have studied the transient free convection flow between two infinite vertical parallel plates with constant heat flux at one boundary. Jha et al.[4] have presented the transient free convection flow in a vertical channel as a result of symmetric heating of the channel walls. Transient free convection flow of a viscous and incompressible fluid between two vertical walls as a result of asymmetric heating or cooling of the walls is studied by Singh and Paul[2]. Narahari [6] has studied the transient free convection flow of a viscous incompressible fluid between two infinite vertical parallel plates in the presence of constant temperature and mass diffusion.

2 Mathematical Analysis

In the present problem, we assume that the magnetic Reynolds number is so small that the induced magnetic field can be neglected in comparison to the applied one. A magnetic field (fixed relative to the plates) of uniform strength B_0 is assumed to be applied transversely to the plates. The $x^{'}$ - axis is considered along one of the vertical plates and $y^{'}$ - axis is taken normal to the plates. Initially, at time $t^{'} \leq 0$ the temperature of the fluid and the plates are same as $T_d^{'}$ and the concentration of the fluid is $C_d^{'}$. At $t^{'} > 0$, the temperature of the plate (at $y^{'} = 0$) is raised to $T_w^{'}$ and the concentration of the fluid near the plate (at $y^{'} = 0$) is raised linearly with time t, causing the flow of free convection currents. The governing equations under the usual Boussinesq's approximation are as follows:

$$\frac{\partial u^{'}}{\partial t^{'}} = g\beta\left(T^{'} - T_d^{'}\right) + g\beta^{*}\left(C^{'} - C_d^{'}\right) + \nu\frac{\partial^2 u^{'}}{\partial y^{'2}} - \frac{\sigma\beta_0^2 u^{'}}{\rho}, \tag{1}$$

$$\rho C_p \frac{\partial T^{'}}{\partial t^{'}} = k\frac{\partial^2 T^{'}}{\partial y^{'2}}, \tag{2}$$

$$\frac{\partial C^{'}}{\partial t^{'}} = D\frac{\partial^2 C^{'}}{\partial y^{'2}}. \tag{3}$$

The initial and boundary condition are as follows:

$$\left.\begin{array}{l} t^{'} \leq 0 : u^{'} = 0, T^{'} = T_d^{'}, C^{'} = C_d^{'} \ \text{ for } \ 0 \leq y^{'} \leq d, \\ t^{'} > 0 : u^{'} = 0, T^{'} = T_w^{'}, C^{'} = C_d^{'} + (C_w^{'} - C_d^{'})\frac{t^{'}\nu}{d^2} \ \text{ at } y^{'} = 0, \\ \qquad\quad u^{'} = 0, T^{'} = T_d^{'}, C^{'} = C_d^{'} \ \text{ at } \ y^{'} = d. \end{array}\right\} \tag{4}$$

Where $u^{'}$ is the velocity of the fluid, g-the acceleration due to gravity, β-volumetric coefficient of thermal expansion, $t^{'}$-time, d-the distance between two vertical plates, $T^{'}$-the temperature of the fluid, $T_d^{'}$-the temperature of the plate at $y^{'} = d$, β^{*}-volumetric coefficient of concentration expansion, $C^{'}$-species concentration in the fluid, $C_d^{'}$-species concentration at the plate $y^{'} = d$, ν-the kinematic viscosity, $y^{'}$-the coordinate axis normal to the plates, ρ-the density, C_p- the specific heat at constant pressure, k-the thermal conductivity of the fluid, D- the mass diffusion coefficient, $T_w^{'}$- temperature of the plate at $y^{'} = 0$, $C_w^{'}$-species concentration at the plate $y^{'} = 0$, B_0-the uniform magnetic field and σ is electrical conductivity.

Introducing the following non-dimensional quantities:

$$\left.\begin{array}{l} y = \frac{y^{'}}{d}, t = \frac{t^{'}\nu}{d^2}, u = \frac{u^{'}\nu}{d^2 g\beta(T_w^{'}-T_d^{'})} = \frac{u^{'}d}{\nu Gr}, Gr = \frac{g\beta(T_w^{'}-T_d^{'})d^3}{\nu^2}, \\ \theta = \frac{T^{'}-T_d^{'}}{T_w^{'}-T_d^{'}}, Pr = \frac{\mu C_p}{k}, C = \frac{C^{'}-C_d^{'}}{C_w^{'}-C_d^{'}}, Gm = \frac{g\beta^*(C_w^{'}-C_d^{'})d^3}{\nu^2}, \\ Sc = \frac{\nu}{D}, N = \frac{Gm}{Gr}, M = \frac{\sigma B_0^2 d^2}{\mu}, \mu = \rho\nu. \end{array}\right\} \tag{5}$$

Where u is the dimensionless velocity, y-dimensionless coordinate axis normal to the plates, t-dimensionless time, θ-the dimensionless temperature, C-the dimensionless concentration, Gr-thermal Grashof number, Gm-mass Grashof number, μ-the coefficient of viscosity, Pr-the Prandtl number, Sc-the Schmidt number, N-the buoyancy ratio parameter and M is magnetic parameter. Then the model is transformed in to the following non-dimensional form of equations:

$$\frac{\partial u}{\partial t} = \theta + NC + \frac{\partial^2 u}{\partial y^2} - Mu, \tag{6}$$

$$Pr\frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\partial y^2}, \tag{7}$$

$$Sc\frac{\partial C}{\partial t} = \frac{\partial^2 C}{\partial y^2}. \tag{8}$$

The initial and boundary conditions become:

$$\left.\begin{array}{l} t \le 0 : u = 0, \theta = 0, C = 0 \ for \ 0 \le y \le 1, \\ t > 0; u = 0, \theta = 1, C = t \ at \ y = 0, \\ \quad u = 0, \theta = 0, C = 0 \ at \ y = 1. \end{array}\right\} \tag{9}$$

The final solution of equations (6), (7) and (8) with boundary condition (9) is as under:

$$\begin{aligned} u(y,t) = {} & \tfrac{e^{Rt}}{2M}F_7(a,b,1,c_1,t) - \left(\tfrac{1}{2M} + \tfrac{N}{2MQ}\right)F_7(a,b,1,M,t) - \tfrac{N}{M}F_8(a,b,1,M,t) \\ & + \tfrac{Ne^{Qt}}{2MQ}F_7(a,b,1,c_2,t) - \tfrac{e^{Rt}}{2M}F_7(a,b,Pr,R,t) + \tfrac{1}{2M}F_7(a,b,Pr,0,t) \\ & - \tfrac{Ne^{Qt}}{2MQ}F_7(a,b,Sc,Q,t) + \tfrac{N}{M}F_9(a,b,Sc,t) + \tfrac{N}{2MQ}F_7(a,b,Sc,0,t), \end{aligned} \tag{10}$$

$$\theta(y,t) = \textstyle\sum_{n=0}^{\infty}[\frac{1}{2}\left\{F_1(a,Pr,0,t) - F_1(b,Pr,0,t)\right\}], \tag{11}$$

$$C(y,t) = \textstyle\sum_{n=0}^{\infty}[F_3(a,Sc,t) - F_3(b,Sc,t)]. \tag{12}$$

3 Skin-friction

The skin-friction has been studied for $Sc \neq 1$ and $Pr \neq 1$. Therefore using the expressions (10) the skin-friction τ_0 and τ_1 in non-dimensional form are given by:

$$\tau_0 = \frac{\tau_0^{'}\nu}{dg\beta(T_w^{'} - T_d^{'})} = \left(\frac{du}{dy}\right)_{y=0}$$
$$= -\frac{e^{Rt}}{2M}F_{10}(d_1, d_2, 1, c_1, t) + \left(\frac{1}{2M} + \frac{N}{2MQ}\right)F_{10}(d_1, d_2, 1, M, t)$$
$$+ \frac{N}{M}\sum_{n=0}^{\infty}\{F_5(d_1, 1, M, t) + F_5(d_2, 1, M, t)\} - \frac{Ne^{Qt}}{2MQ}F_{10}(d_1, d_2, 1, c_2, t)$$
$$+ \frac{e^{Rt}}{2M}F_{10}(d_1, d_2, Pr, R, t) - \frac{1}{2M}F_{10}(d_1, d_2, Pr, 0, t) + \frac{Ne^{Qt}}{2MQ}F_{10}(d_1, d_2, Sc, Q, t)$$
$$-\frac{N}{M}\sum_{n=0}^{\infty}\{F_6(d_1, Sc, t) + F_6(d_2, Sc, t)\} - \frac{N}{2MQ}F_{10}(d_1, d_2, Sc, 0, t). \quad (13)$$

$$\tau_1 = -\left(\frac{du}{dy}\right)_{y=1}$$
$$= \sum_{n=0}^{\infty}\left[\frac{e^{Rt}}{M}F_4(d_3, 1, c_1, t) - \left(\frac{1}{M} + \frac{N}{MQ}\right)F_4(d_3, 1, M, t) - \frac{2N}{M}F_5(d_3, 1, M, t)\right.$$
$$+ \frac{Ne^{Qt}}{MQ}F_4(d_3, 1, c_2, t) - \frac{e^{Rt}}{M}F_4(d_3, Pr, R, t) + \frac{1}{M}F_4(d_3, Pr, 0, t)$$
$$\left.-\frac{Ne^{Qt}}{MQ}F_4(d_3, Sc, Q, t) + \frac{2N}{M}F_6(d_3, Sc, t) + \frac{N}{MQ}F_4(d_3, Sc, 0, t)\right]. \quad (14)$$

Where $a = 2n + y, b = 2 + 2n - y, R = \frac{M}{Pr-1}$ with $Pr \neq 1, Q = \frac{M}{Sc-1}$ with $Sc \neq 1, c_1 = M + R, c_2 = M + Q, d_1 = 2n, d_2 = 2 + 2n$ and $d_3 = 1 + 2n$. Other symbols/expressions are defined in appendix.

4 Results and Discussions

The numerical values of the velocity and skin-friction are computed for different parameters like Prandtl number Pr, Schmidt number Sc, magnetic parameter M, buoyancy ratio parameter N and time t. When $N = 0$, there is no mass transfer and the buoyancy force is due to the thermal diffusion only. $N > 0$ means that mass buoyancy force acts in the same direction of thermal buoyancy force, while $N < 0$ means that mass buoyancy force acts in the opposite direction. The values of the main parameters considered are: the magnetic parameter $M = 1.0, 2.0, 3.0$; time $t = 0.2, 0.4, 0.6$; buoyancy ratio parameter $N = 0.2, 0.4, -0.2, -0.4$; Prandtl number $Pr = 0.71$(for air), 7 (for water) and 10(for Gasoline at 1 atm. Pressure at 20^0C) and Schmidt number $Sc = 0.22$ (for Hydrogen), 0.78 (for Ammonia) and 2.01 (for Ethyl Benzene). Graphs has been plotted for the velocity profiles to show the effects of different parameters. Fig -1 illustrates the effect of magnetic parameter M and buoyancy ratio parameter N on the velocity of fluid. It is observed that the velocity decreases with increase of magnetic parameter M. Fruther, the velocity increases in the presence of aiding flows ($N > 0$), but it decreases in the presence of opposing flows ($N < 0$). Fig -2 illustrates the effect of Prandtl number Pr and time t on the velocity of fluid. It is observed that the velocity increases with increase of time t, but, it decreases with increases of Prandtl number Pr. Fig -3 illustrates the effect of Schmidt number Sc on the velocity of fluid, which shows that, the velocity decreases with increases of Schmidt number Sc.

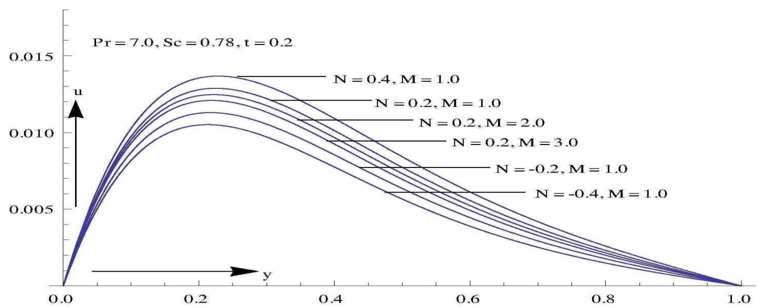

Figure 1: Velocity profiles

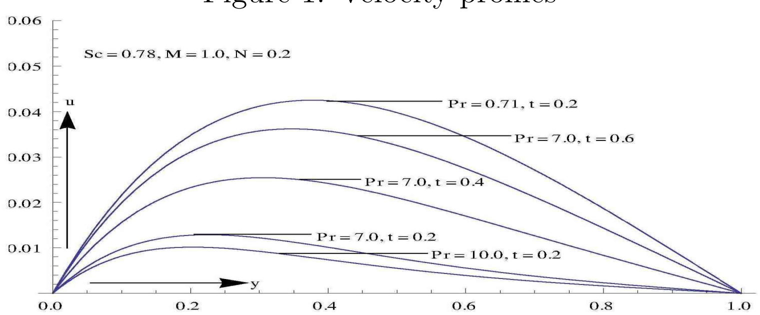

Figure 2: Velocity profiles

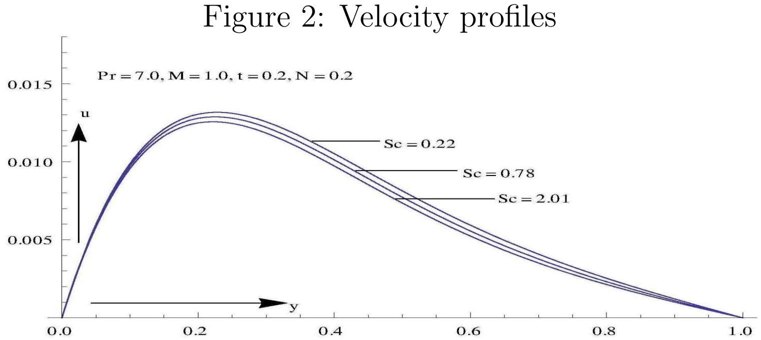

Figure 3: Velocity profiles

The numerical values of skin-friction τ_0 and τ_1 are presented in Table -1. It is clear that the values of skin-friction at the plate ($y = 0$) is greater than the values of skin-friction at the plate ($y = 1$). It is observed that the skin-friction decreases with increase of M, Pr and Sc, but it increases with increases of t. It is also observed that the skin-friction increases in the presence of aiding flows ($N > 0$), but it decreases in the presence of opposing flows ($N > 0$).

Table 1: Skin-friction and volume flux for $Sc \neq 1$, $Pr \neq 1$

Pr	Sc	M	N	t	τ_0	τ_1
7.0	0.78	1.0	0.2	0.2	0.142848	0.0107722
7.0	0.78	2.0	0.2	0.2	0.140409	0.0099557
7.0	0.78	3.0	0.2	0.2	0.138107	0.0092150
7.0	0.78	1.0	0.4	0.2	0.149815	0.0121329
7.0	0.78	1.0	-0.2	0.2	0.128916	0.0080508
7.0	0.78	1.0	-0.4	0.2	0.121950	0.0066902
7.0	0.78	1.0	0.2	0.4	0.205288	0.0412055
7.0	0.78	1.0	0.2	0.6	0.251099	0.0723188
0.71	0.78	1.0	0.2	0.2	0.264908	0.0954563
10	0.78	1.0	0.2	0.2	0.126266	0.0076185
7.0	0.22	1.0	0.2	0.2	0.1443090	0.0119295
7.0	2.01	1.0	0.2	0.2	0.141007	0.0098702

5 Coclusions

In the present paper, the general analytical solutions for the unsteady MHD transient free convection flow of an electrically conducting fluid between two infinite vertical parallel plates with constant temperature and variable mass diffusion has been determined. The solutions for the model have been determined by using Laplace transform method. The conclusions of the study are as follows:

- The velocity and skin-friction of the fluid increase in case of aiding flows ($N > 0$) and decrease with opposing flows ($N < 0$).

- The velocity and skin-friction of the fluid increase with increasing the value of time t but decrease with increasing the value of the Prandtl numberPr, Schmidt number Sc, magnetic parameter M.

Appendix:

$F_1(D_1, D_2, D_3, D_4) = e^{-a_3} erfc(a_1) + e^{a_3} erfc(a_2)$,

$F_2(D_1, D_2, D_3, D_4) = e^{-a_3} erfc(a_1) b_1 + e^{a_3} erfc(a_2) b_2$,

$F_3(D_1, D_2, D_3) = \left(D_3 + \frac{D_1^2 D_2}{2}\right) erfc\left(\frac{D_1\sqrt{D_2}}{2\sqrt{D_3}}\right) - \left(D_1\sqrt{\frac{D_2 D_3}{\pi}}\right) e^{-\frac{D_1^2 D_2}{4D_3}}$,

$F_4(D_1, D_2, D_3, D_4) = \frac{1}{\sqrt{\pi D_4}} e^{-a_3 - a_1^2} \sqrt{D_2} + \frac{1}{\sqrt{\pi D_4}} e^{a_3 - a_2^2} \sqrt{D_2}$
$+ e^{-a_3} \sqrt{D_2 D_3} erfc(a_1) - e^{a_3} \sqrt{D_2 D_3} erfc(a_2),$
$F_5(D_1, D_2, D_3, D_4) = \frac{1}{\sqrt{\pi D_4}} e^{-a_3 - a_1^2} b_1 \sqrt{D_2} + \frac{1}{\sqrt{\pi D_4}} e^{a_3 - a_2^2} b_2 \sqrt{D_2} + \frac{1}{4\sqrt{D_3}} e^{-a_3} erfc(a_1)$
$+ e^{-a_3} \sqrt{D_2 D_3} b_1 erfc(a_1) - \frac{1}{4\sqrt{D_3}} e^{a_3} erfc(a_2) - e^{a_3} \sqrt{D_2 D_3} b_2 erfc(a_2),$
$F_6(D_1, D_2, D_3) = \frac{2}{\sqrt{\pi}} e^{-\frac{D_1^2 D_2}{4D_3}} \sqrt{D_2 D_3} - D_1 D_2 erfc(\frac{D_1 \sqrt{D_2}}{2\sqrt{D_3}}),$
$F_7(a, b, D_1, D_2, D_3) = \sum_{n=0}^{\infty} \{F_1(a, D_1, D_2, D_3) - F_1(b, D_1, D_2, D_3)\},$
$F_8(a, b, D_1, D_2, D_3) = \sum_{n=0}^{\infty} \{F_2(a, D_1, D_2, D_3) - F_2(b, D_1, D_2, D_3)\},$
$F_9(a, b, D_1, D_2) = \sum_{n=0}^{\infty} \{F_3(a, D_1, D_2) - F_3(b, D_1, D_2)\}$
and $F_{10}(d_1, d_2, D_1, D_2, D_3) = \sum_{n=0}^{\infty} \{F_4(d_1, D_1, D_2, D_3) + F_4(d_2, D_1, D_2, D_3)\}.$
Here $a_1 = \left(\frac{D_1 \sqrt{D_2}}{2\sqrt{D_4}} - \sqrt{D_3 D_4}\right), a_2 = \left(\frac{D_1 \sqrt{D_2}}{2\sqrt{D_4}} + \sqrt{D_3 D_4}\right), a_3 = D_1 \sqrt{D_2 D_3},$
$b_1 = \left(\frac{D_4}{2} - \frac{D_1}{4\sqrt{D_3}}\right), b_2 = \left(\frac{D_4}{2} + \frac{D_1}{4\sqrt{D_3}}\right).$

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