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A Common Fixed Point Theorem for Fuzzy Weakly Contractive Mappings in Quasi-Metric Spaces

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Abstract

A common fixed point theorem for fuzzy weakly contractive mappings in the settings of quasi-metric spaces is proved.

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1 Introduction

The concept of weakly contractive point-to-point mappings is introduced by Alber and Guerr-Delabriere [1] in the settings of Hilbert spaces. Rhoades [12] showed that most results of [1] are still true for any Banach space. Also Bae [4] obtain fixed point theorems of multi-valued weakly contractive mapping.

Zhang and Song [14] proved a common fixed point theorem for a pair of generalized φ-weak contractions in complete metric space. Heilpern [6] first introduced the concept of fuzzy mappings and proved a fixed point theorem for fuzzy mappings. Since then many fixed point theorems for fuzzy mappings have been obtained by many authors. Azam and Beg [3] have introduced the concept of fuzzy weakly contractive mappings and proved a very interesting common fixed point theorem for two fuzzy weakly contractive mappings. Bose and Roychowdhury [7] considered such fuzzy mappings and its two generalized versions, and proved some fixed point theorems.

In this paper, we consider a generalized contractive type condition involving fuzzy mappings in Smyth-complete quasi-metric spaces and establish a common fixed point theorem which extends many known theorems.

2 Preliminaries

Throughout this paper we shall use the notations as in [5]

A quasi-metric on a nonempty set X is a nonnegative real valued function d on $X \times X$ such that, for all $x, y, z \in X$:

(a) $d(x,y) = d(y,x) = 0 \Leftrightarrow x = y$, and (b) $d(x,y) \le d(x,z) + d(z,y)$.

A pear (X,d) is called a quasi-metric space, if d is a quasi-metric on X. Each quasi-metric d on X induces a topology $\tau(d)$ which has a base the family of all d-balls $B(x,\varepsilon)$ where $B(x,\varepsilon) = \{y \in X : d(x,y) < \varepsilon\}$.

If d is a quasi-metric on X, then the function d^{-1} defined on $X \times X$ by $d^{-1}(x,y) = d(y,x)$ is also a quasi-metric on X. By $d \wedge d^{-1}$ we denote $\min\{d, d^{-1}\}$ and also we denote d^s the metric on X by $d^s(x,y) = \max\{d(x,y), d(y,x)\}$, for all $x, y \in X$.

A sequence $(x_n)_{n\in N}$ in a quasi-metric space (X,d) is called left K-Cauchy [15] if for each $\varepsilon > 0$ there is a $k \in N$ such that $d(x_n, x_m) < \varepsilon$ for all $n, m \in N$ with $m \ge n \ge k$.

A quasi metric space (X,d) is said to be Smyth-complete [16] if each left K-Cauchy sequence in (X,d) converges in the metric space (X,d^s).

Let (X,d) be a quasi-metric space and let $\mathscr{K}_0^S(X)$ be the collection of all nonempty compact subset of the metric space (X,d^s). Then the Hansdorff distance H_d on $\mathscr{K}_0^S(X)$ is defined by

$$H_d(A,B) = \max\{\sup d(a,B) : a \in A, \sup d(A,b) : b \in B\}$$

whenever $A, B \in \mathscr{K}_0^S(X)$.

A fuzzy set on X is an element of I^X where $I = [0,1]$. If A is a fuzzy

set and $x \in X$, then the function value $A(x)$ is called the grade of membership of x in A. The collection of all fuzzy sets in X is denoted by $\mathscr{F}(X)$.
Let $A \in \mathscr{F}(X)$ and $\alpha \in [0,1]$. The $\alpha -$level set of A, denoted by A_α, is defined by

$A_\alpha = \{x : A(x) \geq \alpha\}$ if $\alpha \in (0,1]$, $A_0 = \overline{\{x : A(x) \geq 0\}}$, where $\overline{B}$ denotes the closure of the set B.

Definition 2.1 [5] Let (X,d) be a quasi-metric space. The family $\mathscr{W}(X)$ of all fuzzy sets on (X,d) is defined by:

$$\mathscr{W}(X) = \{A \in I^X : A_\alpha \text{ is } d^s\text{-compact for each } \alpha \in [0,1] \text{ and } \sup\{A(x) : x \in X\} = 1.$$

Definition 2.2 [6] Let $A, B \in \mathscr{W}(X)$. Then A is said to be more accurate than B, denoted by $A \subset B$, if and only if $A(x) \leq B(x)$ for each $x \in X$.

Definition 2.3 [6] Let (X,d) be a quasi-metric space and let $A, B \in \mathscr{W}(X)$ and $\alpha \in [0,1]$. Then we define ,

$$p_\alpha(A,B) = \inf\{d(x,y) : x \in A_\alpha, y \in B_\alpha\} = d(A_\alpha, B_\alpha)$$
$$D_\alpha(A,B) = H_d(A_\alpha, B_\alpha)$$
$$p(A,B) = \sup\{p_\alpha(A,B) : \alpha \in [0,1]\}$$
$$D(A,B) = \sup\{D_\alpha(A,B) : \alpha \in [0,1]\}.$$

For $x \in X$ we write $p_\alpha(x,A)$ instead of $p_\alpha(\{x\},A)$.
We note that p_α is a non-decreasing function of α and D is a metric on $\mathscr{W}(X)$.

Definition 2.4 [5] A fuzzy mapping on a quasi-metric space (X,d) is a function F defined on X, which satisfies the following two conditions:

(1) $F(x) \in \mathscr{W}(X)$ for all $x \in X$

(2) If $a, z \in X$ such that $(F(z))(a) = 1$ and $p(a, F(a)) = 0$, then $(F(a))(a) = 1$

Definition 2.5 A fuzzy mapping $T : X \to \mathscr{W}(X)$ on a quasi-metric space (X,d) is said to be weakly contractive if

$$D(Tx,Ty) \leq d(x,y) - \varphi(d(x,y)) \quad \text{for each } x, y \in X,$$

where $\varphi : [0,\infty) \to [0,\infty)$ is a continuous and non-decreasing function such that φ is positive on $[0,\infty)$, $\varphi(0) = 0$ and $\lim\limits_{t\to\infty} \varphi(t) = \infty$.

Lemma 2.6 [5] Let (X,d) be a quasi-metric space. Then, for each $A \in \mathscr{W}(X)$ there exists $p \in X$ such that $A(p)=1$.

Lemma 2.7 [5] Let (X,d) be a quasi-metric space and let $A,B \in \mathscr{W}(X)$ and $x \in A_1$. There exists $y \in B_1$ such that $d(x,y) \le D_1(A,B)$.

Lemma 2.8 [5] Let (X,d) be a quasi-metric space and let $A,B \in \mathscr{W}(X)$.
Then $p(A,B) = p_1(A,B)$

Lemma 2.9 [5] Let (X,d) be a quasi-metric space and let $A \in \mathscr{W}(X)$ and $y \in A_1$.Then $p(x,A) \le d(x,y)$ for each $x \in X$.

Lemma 2.10 [6] Let $x \in X$, $A \in \mathscr{W}(X)$ and $\{x\}$ be a fuzzy set with membership function equal to a characteristic function of the set $\{x\}$, then $\{x\} \subset A$ if and only if $p_\alpha(x,A)=0$ for each $\alpha \in [0,1]$.

Lemma 2.11 [6] Let $x,y \in X$ and $A \in \mathscr{W}(X)$.
Then $p_\alpha(x,A) \le d(x,y) + p_\alpha(y,A)$

Lemma 2.12 [6] If $\{x_0\} \subset A$ then $p_\alpha(x_0,B) \le D_\alpha(A,B)$ for each $B \in \mathscr{W}(X)$

Lemma 2.13 [9] Let A and B be nonempty compact subsets of a metric space (X,d). If $a \in A$ then there exists a $b \in B$ such that $d(a,b) \le H(A,B)$.

Lemma 2.14 [5] Let (X,d) be a quasi-metric space and let $A \in \mathscr{W}(X)$. If $p(x,A)=0$, then there is $y \in cl_{\tau(d^{-1})}\{x\}$ such that $A(y)=1$.

Definition 2.15 [5] We say that a fuzzy mapping F on a quasi-metric space (X,d) has a fixed point if there exists $a \in X$ such that $(F(a))(a)=1$.

Definition 2.16 Two fuzzy mappings $T_1,T_2 : X \to \mathscr{W}(X)$ on a quasi-metric space (X,d) are called generalized $\varphi-$weak contractive if exists a continuous map $\varphi:[0,\infty)\to[0,\infty)$ with $\varphi(0)=0$ and $\varphi(t)>0$ for all $t>0$ such that for all $x,y \in X$

$$D(T_1x,T_2y) \le M(x,y) - \varphi(M(x,y))$$

where

$$M(x,y) = \max\{d(x,y), p(x,T_1(x)), p(y,T_2(y)), [p(x,T_2(y)) + p(y,T_1(x))]/2\}.$$

3 Main results.

The function $\varphi:[0,\infty)\to[0,\infty)$ is said to be lower semi-continuous (l.s.c.) in $x\in[0,\infty)$, if for any sequence $\{x_n\}$ with $\lim_{n\to\infty} x_n = x$ and $\lim_{n\to\infty}\varphi(x_n)=r$, then $\varphi(x)\le r$.

Theorem 3.1 Let (X,d) be a Smyth-complete quasi-metric space and $T_1,T_2:X\to\mathscr{W}(X)$ be two fuzzy generalized $\varphi-$weak contractive mappings satisfying the following condition:

$$\psi(D(T_1x,T_2y))\le\psi(M(x,y))-\varphi(M(x,y)) \qquad (3.1)$$

for each $x,y\in X$, and $\varphi:[0,\infty)\to[0,\infty)$ is l.s.c. function with $\varphi(0)=0$ and $\varphi(t)>0$ for all $t>0$, $\psi:[0,\infty)\to[0,\infty)$ is monotonically increasing continuous function with $\psi(0)=0$ and $\psi(t)>0$ for all $t>0$. Then T_1 and T_2 have common fixed point.

Proof. Let x_0 be an arbitrary point in X. By lemma 2.6, there exists $x_1\in X$ such that $(T_1(x_0))(x_1)=1$. By lemmas 2.6 and 2.7, there exists $x_2\in X$ such that $(T_2(x_1))(x_2)=1$ and $d(x_1,x_2)\le D_1(T_1(x_0),T_2(x_1))$.
Then we obtain

$$d(x_1,x_2)\le D_1(T_1(x_0),T_2(x_1))\le D(T_1(x_0),T_2(x_1))$$

Continuing this process, we construct a sequence $\{x_n\}_{n\in N}$ such that having chosen $x_n\in X$, we obtain $x_{n+1}\in X$ such that $\{x_{2n+1}\}\subset T_1(x_{2n})$, $\{x_{2n+2}\}\subset T_2(x_{2n+1})$ and

$$\begin{aligned} d(x_{2n},x_{2n+1})&\le D_1(T_1(x_{2n-1}),T_2(x_{2n}))\le D(T_1(x_{2n-1}),T_2(x_{2n}))\\ d(x_{2n+1},x_{2n+2})&\le D_1(T_1(x_{2n}),T_2(x_{2n+1}))\le D(T_1(x_{2n}),T_2(x_{2n+1})) \end{aligned} \qquad (3.2)$$

Hence, by the given hypothesis and as ψ is monotonically increasing and we have,

$$\begin{aligned} \psi(d(x_{2n},x_{2n+1}))&\le\psi(M(x_{2n-1},x_{2n}))-\varphi(M(x_{2n-1},x_{2n}))\le\psi(M(x_{2n-1},x_{2n}))\\ \psi(d(x_{2n+1},x_{2n+2}))&\le\psi(M(x_{2n},x_{2n+1}))-\varphi(M(x_{2n},x_{2n+1}))\le\psi(M(x_{2n},x_{2n+1})) \end{aligned} \qquad (3.3)$$

Where

$$\begin{aligned} M(x_{2n-1},x_{2n}) &= \max\{d(x_{2n-1},x_{2n}),\, p(x_{2n-1},T_1(x_{2n-1})),\, p(x_{2n},T_2(x_{2n})), \\ &[p(x_{2n-1},T_2(x_{2n})) + p(x_{2n},T_1(x_{2n-1}))]/2\} \\ &\le \max\{d(x_{2n-1},x_{2n}), d(x_{2n-1},x_{2n}), d(x_{2n},x_{2n+1}), \\ &[d(x_{2n-1},x_{2n+1}) + 0]/2\} \\ &= \max\{d(x_{2n-1},x_{2n}), d(x_{2n},x_{2n+1})\} \end{aligned} \tag{3.4}$$

So by (3.3)

$$M(x_{2n-1},x_{2n}) \le d(x_{2n-1},x_{2n}) \tag{3.5}$$

Similarly

$$M(x_{2n},x_{2n+1}) \le d(x_{2n},x_{2n+1}) \tag{3.6}$$

As ψ is monotonically increasing by (3.3), (3.4), (3.5) and (3.6) for $n \ge 0$ we have,

$$\begin{aligned} d(x_{2n},x_{2n+1}) \le M(x_{2n-1},x_{2n}) \le d(x_{2n-1},x_{2n}) \\ d(x_{2n+1},x_{2n+2}) \le M(x_{2n},x_{2n+1}) \le d(x_{2n},x_{2n+1}) \end{aligned} \tag{3.7}$$

which shows that the sequence of positive real numbers $\{d(x_n,x_{n+1})\}$ is monotone non-increasing and bounded below. So there exits $l \ge 0$ such that

$$\lim_{n\to\infty} d(x_n,x_{n+1}) = \lim_{n\to\infty} M(x_n,x_{n+1}) = l \tag{3.8}$$

Again by (3.3) and (3.7) for $n \ge 0$ we have,

$$\psi(d(x_{n+1},x_{n+2})) \le \psi(M(x_n,x_{n+1})) - \varphi(M(x_n,x_{n+1})) . \tag{3.9}$$

Since ψ is continuous and φ is lower semi-continuous, taking $n \to \infty$ we have

$$\psi(l) \le \psi(l) - \varphi(l) \quad (\varphi(l) \le \liminf_{n\to\infty} \varphi(M(x_{n-1},x_n)))$$

which is contradiction.

Therefore $\lim_{n\to\infty} d(x_n,x_{n+1}) = \lim_{n\to\infty} M(x_n,x_{n+1}) = 0$.

Next we show that $\{x_n\}$ left K-Cauchy sequence.

Let

$$C_n = \sup\{d(x_i,x_j) : i, j \ge n\}$$

Obviously $\{C_n\}$ is decreasing. So there exists $c \geq 0$ such that $\lim_{x\to\infty} C_n = c$.
For every $k \in \Gamma$, there exists $n(k),\ m(k) \in \Gamma$ such that $n(k) > m(k) \geq k$ and

$$C_k - \frac{1}{k} \leq d(x_{m(k)}, x_{n(k)}) \leq C_k \tag{3.10}$$

So

$$\lim_{k\to\infty} d(x_{m(k)}, x_{n(k)}) = c .$$

By (3.3) and for every $k \in \Gamma$ we have

$$\psi(d(x_{m(k)+1}, x_{n(k)+1})) \leq \psi(M(x_{m(k)}, x_{n(k)})) - \varphi(M(x_{m(k)}, x_{n(k)}) \tag{3.11}$$

where

$$\begin{aligned} M(x_{m(k)}, x_{n(k)}) = \max\{&d(x_{m(k)}, x_{n(k)}), p(x_{m(k)}, T_1(x_{m(k)})), p(x_{n(k)}, T_2(x_{n(k)})), \\ &[p(x_{m(k)}, T_2(x_{n(k)})) + p(x_{n(k)}, T_1(x_{m(k)}))]/2\} \\ \leq \max\{&d(x_{m(k)}, x_{n(k)}), d(x_{m(k)}, x_{m(k)+1}), d(x_{n(k)}, x_{n(k)+1}), \\ &[d(x_{m(k)}, x_{n(k)+1}) + d(x_{n(k)}, x_{m(k)+1})]/2\} \end{aligned} \tag{3.12}$$

As $k \to \infty$ in inequality (3.11) we have $\lim_{k\to\infty} M(x_{m(k)}, x_{n(k)}) = c$. Since ψ is continuous and φ is lower semi-continuous and (3.11) holds, we have $c \leq c - \varphi(c)$. Hence $\varphi(c) = 0$ and so $c = 0$.
Therefore, $\{x_n\}$ is a left K-Cauchy sequence in the Smyth-complete quasi-metric space (X, d) and so there exists a $z \in X$ such that $\lim_{n\to\infty} x_n = z$ ($\lim_{n\to\infty} d^s(z, x_n) = 0$ and $\lim_{n\to\infty} d^s(x_n, z) = 0$)

By Lemmas 2.11 and 2.12 we have:

$$p(z, T_2(z)) \leq d(z, x_{2n+1}) + D(T_1(x_{2n}), T_2(z)) \tag{3.13}$$

On the other hand

$$\begin{aligned} M(x_{2n-1}, z) \leq \max\{&d(x_{2n-1}, z), p(x_{2n-1}, T_1(x_{2n-1})), p(z, T_2(z)), \\ &[p(x_{2n-1}, T_2(z)) + p(z, T_1(x_{2n-1}))]/2\} \\ \leq \max\{&d(x_{2n-1}, z), d(x_{2n-1}, x_{2n}), p(z, T_2(z)), \\ &[d(x_{2n-1}, T_2(z)) + d(z, x_{2n})]/2\} \end{aligned} \tag{3.14}$$

Taking $n \to \infty$ in (3.14) we have

$$\lim_{n\to\infty} M(x_{2n-1}, z) = p(z, T_2(z)).$$

Also, taking $n \to \infty$ in (3.13) we have

$$p(z, T_2(z)) \le \lim_{n\to\infty} D(T_1(x_{2n}), T_2(z)) \qquad (3.15)$$

So, by (3.14), (3.15) and properties of ψ and φ we have

$$\begin{aligned}\psi(p(z,T_2(z))) &\le \lim_{n\to\infty}\psi(D(T_1(x_{2n}),T_2(z)))\\ &\le \lim_{n\to\infty}\psi(M(x_{2n},z)) - \lim_{n\to\infty}\varphi(M(x_{2n},z))\\ &= \psi(p(z,T_2(z))) - \varphi(p(z,T_2(z)))\end{aligned}$$

So $\varphi(p(z,T_2(z))) = 0$ and $p(z,T_2(z)) = 0$. Similarly, we have $p(z, F_1(z) = 0$. So by lemma 2.14 there exists $z^* \in cl_{\tau(d^{-1})}\{z\}$ such that $(T_2(z))(z^*) = 1$. Since $z^* \in cl_{\tau(d^{-1})}\{z\}$ we have $d(z, z^*) = 0$.

We will prove now that z^* is fixed point of F_2.

By $d(x_n, z^*) \le d(x_n, z) + d(z, z^*)$ we have $d(x_n, z^*) \to 0$ and $x_n \to z^*$ as $n \to \infty$.

Also by Lemmas 2.11 and 2.12 we have

$p(z^*, T_2(z^*)) \le d(z^*, x_{2n+1}) + p(x_{2n+1}, T_2(z^*)) \le d(z^*, x_{2n+1}) + D(T_1(x_{2n}), T_2(z^*))$ and as $n \to \infty$,

$$p(z^*, T_2(z^*)) \le +D(T_1(z), T_2(z^*)) \quad (3.16).$$

As ψ is monotonically increasing and inequality (3.1) we have

$$\psi(p(z^*, T_2(z^*))) \le \psi(D(T_1(z), T_2(z^*))) \le \psi(M(z, z^*)) - \varphi(M(z, z^*)) \qquad (3.17)$$

Using the inequality (3.1), we have

$$\psi(D(T_1(x_{2n}), T_2(z)) \le \psi(M(x_{2n}, z^*) - \varphi(M(x_{2n}, z^*))$$

where

$$\begin{aligned}M(x_{2n}, z^*) = \max\{&d(x_{2n}, z^*), p(x_{2n}, T_1(x_{2n})), p(z^*, T_2(z^*)),\\ &[p(x_{2n}, T_2(z^*)) + p(z^*, T_1(x_{2n}))]/2\}\end{aligned}$$

By Lemmas 2.9 and 2.12 we have

$p(x_{2n},T_1(x_{2n}))\le d(x_{2n},x_{2n+1})$; $p(x_{2n},T_2(z^*))\le d(x_{2n},z^*)+p(z^*,T_2(z^*))$;

$p(z^*,T_1(x_{2n}))\le d(z^*,x_{2n+1})$.

So

$$M(x_{2n},z^*)\le \max\{d(x_{2n},z^*),d(x_{2n},x_{2n+1}),p(z^*,T_2(z^*)),\\ [d(x_{2n},z^*)+d(z^*,x_{2n+1})]/2\} \tag{3.18}$$

As $n\to\infty$ in inequality (3.18) we have

$$M(z,z^*)=p(z^*,T_2(z*))$$

By inequality (3.17) we have

$$\psi(p(z^*,T_2(z^*)))\le\psi(M(z,z^*))-\varphi(M(z,z^*))\le\psi(p(z^*,T_2(z^*)))-\varphi(p(z^*,T_2(z^*)))$$

and so $p(z^*,T_2(z*))=0$.

Also, by Definition 2.15 we see that $T_2(z^*)(z^*)=1$ and so z^* is fixed point of T_2.

Similarly, by Lemma 2.14 there exists $z_1^*\in cl_{\tau(d^{-1})}\{z\}$ such that $T_1(z)(z_1^*)=1$ and z_1^* is fixed point of T_1.

By the triangle inequality $d(z^*,z_1^*)\le d(z^*,x_n)+d(x_n,z_1^*)$ and $d(z_1^*,z^*)\le d(z_1^*,x_n)+d(x_n,z^*)$.

So as $n\to\infty$, $d(z^*,z_1^*)=d(z_1^*,z^*)=0$ we have $z^*=z_1^*$ and z^* is common fixed point of T_1 and T_2.

Corollary 3.2 Let (X,d) be a Smyth-complete quasi-metric space and $T:X\to\mathscr{W}(X)$ be a fuzzy generalized $\varphi-$ weak contractive mappings satisfying the following condition:

$$\psi(D(Tx,Ty))\le\psi(M(x,y))-\varphi(M(x,y))$$

for each $x,y\in X$, and $\varphi:[0,\infty)\to[0,\infty)$ is l.s.c. function with $\varphi(0)=0$ and $\varphi(t)>0$ for all $t>0$, $\psi:[0,\infty)\to[0,\infty)$ is monotonically increasing continuous function with $\psi(0)=0$ and $\psi(t)>0$ for all $t>0$. Then T have a fixed point.

Proof. The proof of this corollary is similar to the Theorem 3.1, if we take $T_1=T_2$.

Corollary 3.3 Let (X,d) be a Smyth-complete quasi-metric space and

$T_1, T_2 : X \to \mathscr{W}(X)$ be a fuzzy generalized $\varphi-$ weak contractive mappings satisfying the following condition:

$$D(T_1x, T_2y) \le M(x, y) - \varphi(M(x, y))$$

for each $x, y \in X$ and $\varphi : [0, \infty) \to [0, \infty)$ is l.s.c function with $\varphi(0) = 0$ and $\varphi(t) > 0$ for all $t > 0$. Then T_1 and T_2 have a common fixed point.

Proof. The proof of this corollary is similar to the Theorem 3.1, if we take $\psi(t) = t$, for all $t \in [0, \infty)$.

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