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Generalized Closed Sets in Bigeneralized Topological Spaces

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Abstract

The purpose of the present paper is to introduce the concept of $\mu_{(m,n)}$-closed sets in bigeneralized topological spaces and study some of their properties. We introduce the notion of $g_{(m,n)}$-continuous functions on bigeneralized topological spaces and investigate some of their characterizations.

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1 Introduction

Generalized closed sets in a topological space were introduced by Levin [9] in order to extend many of the important properties of closed sets to a larger family. For instance, it was shown that compactness, normality and completeness in a uniform space are inherited by generalized closed subsets. The study of bitopological spaces was first initiated by Kelly [8] and thereafter a large number of papers have been done to generalize the topological concepts to bitopological setting. Fukutake [7] introduced generalized closed sets and pairwise generalized closure operator in bitopological spaces. He defined a set A of a bitopological space X to be $\tau_i\tau_j$-generalized closed sets if τ_j-$Cl(A) \subseteq U$ whenever $A \subseteq U$ and U is τ_i-open in X. Also, he defined a new closure operator and strongly pairwise $T_{\frac{1}{2}}$-space. Á. Császár [3] introduced the concepts of generalized neighborhood systems and generalized topological spaces. He also introduced the concepts of continuous functions and associated interior and closure operators on generalized neighborhood systems and generalized topological spaces. In particular, he investigated characterizations for the generalized continuous function by using a closure operator defined on generalized neighborhood systems. C. Boonpok [2] introduced the concept of bigeneralized topological spaces and studied (m, n)-closed sets and (m, n)-open sets in bigeneralized topological spaces.

In this paper, we introduce the notions of $\mu_{(m,n)}$-closed sets in bigeneralized topological spaces and study some of their properties. We introduce the notion of $g_{(m,n)}$-continuous functions on bigeneralized topological spaces and investigate some of their characterizations.

2 Preliminaries

We recall some basic definitions and notations. Let X be a set and denote $\exp X$ the power set of X. A subset μ of $\exp X$ is said to be a *generalized topology* (briefly GT) on X if $\emptyset \in \mu$ and an arbitrary union of elements of μ belongs to μ [3]. Let μ be a GT on X, the elements of μ are called *μ-open sets* and the complements of μ-open sets are called *μ-closed sets.* Let M_μ denote the union of all elements of μ. We say μ is *strong* [4] if $M_\mu = X$. If $A \subseteq X$, then $i_\mu(A)$ denotes the union of all μ-open sets contained in A and $c_\mu(A)$ is the intersections of all μ-closed sets containing A [5]. According to [6], for $A \subseteq X$ and $x \in X$, we have $x \in c_\mu(A)$ iff $x \in M \in \mu$ implies $M \cap A \neq \emptyset$. Let (X, μ) be a generalized topological space. A subset M of X is said to be *μ-semi-open* iff $M \subseteq c_\mu(i_\mu(M))$, *$\mu$-preopen* iff $M \subseteq i_\mu(c_\mu(M))$, *$\mu$-$\alpha$-open* iff $M \subseteq i_\mu(c_\mu(i_\mu(M)))$, finally *$\mu$-$\beta$-open* iff $M \subseteq c_\mu(i_\mu(c_\mu(M)))$ [5]. Let (X, μ) and (Y, μ') be generalized topological spaces. A map $f : (X, \mu) \to (Y, \mu')$ is said to be *(μ, μ')-continuous* iff $M' \in \mu'$ implies $f^{-1}(M') \in \mu$ [3].

Theorem 2.1. [3] *Let (X, μ) be a generalized topological space. Then*

(1) $c_\mu(A) = X - i_\mu(X - A)$;

(2) $i_\mu(A) = X - c_\mu(X - A)$.

Proposition 2.2. [10] *Let (X, μ) be a generalized topological space. For subsets A and B of X, the following properties holds:*

(1) $c_\mu(X - A) = X - i_\mu(A)$ and $i_\mu(X - A) = X - c_\mu(A)$;

(2) If $(X - A) \in \mu$, then $c_\mu(A) = A$ and if $A \in \mu$, then $i_\mu(A) = A$;

(3) If $A \subseteq B$, then $c_\mu(A) \subseteq c_\mu(B)$ and $i_\mu(A) \subseteq i_\mu(B)$;

(4) $A \subseteq c_\mu(A)$ and $i_\mu(A) \subseteq A$;

(5) $c_\mu(c_\mu(A)) = c_\mu(A)$ and $i_\mu(i_\mu(A)) = i_\mu(A)$.

Definition 2.3. [2] Let X be a nonempty set and let μ_1, μ_2 be generalized topologies on X. A triple (X, μ_1, μ_2) is said to be a *bigeneralized topological space* (briefly *BGTS*).

Let (X, μ_1, μ_2) be a bigeneralized topological space and A a subset of X. The closure of A and the interior of A with respect to μ_m are denoted by $c_{\mu_m}(A)$ and $i_{\mu_m}(A)$, respectively, for $m = 1, 2$.

Definition 2.4. [2] A subset A of a bigeneralized topological space (X, μ_1, μ_2) is called (m, n)*-closed* if $c_{\mu_m}(c_{\mu_n}(A)) = A$, where $m, n = 1, 2$ and $m \neq n$. The complement of (m, n)-closed set is called (m, n)*-open*.

Proposition 2.5. [2] *Let (X, μ_1, μ_2) be a bigeneralized topological space and A a subset of X. Then A is (m, n)-closed if and only if A is both μ-closed in (X, μ_m) and (X, μ_n).*

Proposition 2.6. [2] *Let (X, μ_1, μ_2) be a bigeneralized topological space. Then A is (m, n)-open if and only if $i_{\mu_m}(i_{\mu_n}(A)) = A$.*

3 Generalized closed sets

In this section, we introduce $\mu_{(m,n)}$-closed sets in bigeneralized topological space and study some of their properties.

Definition 3.1. A subset A of a bigeneralized topological space (X, μ_1, μ_2) is said to be (m, n) *generalized closed* (briefly $\mu_{(m,n)}$*-closed*) set if $c_{\mu_n}(A) \subseteq U$ whenever $A \subseteq U$ and U is μ_m-open set in X, where $m, n = 1, 2$ and $m \neq n$. The complement of $\mu_{(m,n)}$-closed set is said to be (m, n) *generalized open* (briefly $\mu_{(m,n)}$*-open*) set.

The family of all $\mu_{(m,n)}$-closed (resp. $\mu_{(m,n)}$-open) sets of (X, μ_1, μ_2) is denote by $\mu_{(m,n)}$-$C(X)$ (resp. $\mu_{(m,n)}$-$O(X)$), where $m, n = 1, 2$ and $m \neq n$.

A subset A of a bigeneralized topological space (X, μ_1, μ_2) is called *pairwise μ-closed* if A is $\mu_{(1,2)}$-closed and $\mu_{(2,1)}$-closed. The complement of pairwise μ-closed set is called *pairwise μ-open*.

Remark 1. *Every (m, n)-closed set is $\mu_{(m,n)}$-closed. The converse is not true as can be seen from the following example.*

Example 3.2. *Let $X = \{a, b, c\}$. Consider two generalized topologies $\mu_1 = \{\emptyset, \{a\}, \{a, b\}, X\}$ and $\mu_2 = \{\emptyset, \{c\}, \{b, c\}\}$ on X. Then $\{a\}$ is $\mu_{(1,2)}$-closed but is not $(1, 2)$-closed.*

Proposition 3.3. *Let (X, μ_1, μ_2) be a bigeneralized topological space and A a subset of X. If A is μ_n-closed, then A is $\mu_{(m,n)}$-closed, where $m, n = 1, 2$ and $m \neq n$.*

Remark 2. *The union of two $\mu_{(m,n)}$-closed sets is not a $\mu_{(m,n)}$-closed set in general as can be seen from the following example.*

Example 3.4. *Let $X = \{a, b, c, d\}$. Consider two generalized topologies $\mu_1 = \{\emptyset, \{a, b\}, \{b, c\}, \{a, b, c\}\}$ and $\mu_2 = \{\emptyset, \{a, b, d\}, \{b, c, d\}, X\}$ on X. Then $\{a\}$ and $\{c\}$ are $\mu_{(1,2)}$-closed but $\{a\} \cap \{c\} = \{a, c\}$ is not $\mu_{(1,2)}$-closed.*

Remark 3. *The intersection of two $\mu_{(m,n)}$-closed sets is not a $\mu_{(m,n)}$-closed set in general as can be seen from the following example.*

Example 3.5. *Let $X = \{a, b, c\}$. Consider two generalized topologies $\mu_1 = \{\emptyset, \{a\}, \{a, b\}, X\}$ and $\mu_2 = \{\emptyset, \{c\}, \{b, c\}\}$ on X. Then $\{a\}$ and $\{b\}$ are $\mu_{(1,2)}$-closed but $\{a\} \cup \{b\} = \emptyset$ is not $\mu_{(1,2)}$-closed.*

Remark 4. *The intersection of two $\mu_{(m,n)}$-open sets is not a $\mu_{(m,n)}$-open set in general as can be seen from the following example.*

Example 3.6. *Let $X = \{a, b, c, d\}$. Consider two generalized topologies $\mu_1 = \{\emptyset, \{a, b\}, \{b, c\}, \{a, b, c\}\}$ and $\mu_2 = \{\emptyset, \{a, b, d\}, \{b, c, d\}, X\}$ on X. Then $\{a, b, d\}$ and $\{b, c, d\}$ are $\mu_{(1,2)}$-open but $\{a, b, d\} \cap \{b, c, d\} = \{b, d\}$ is not $\mu_{(1,2)}$-open.*

Remark 5. *The union of two $\mu_{(m,n)}$-open sets is not a $\mu_{(m,n)}$-open set in general as can be seen from the following example.*

Example 3.7. *Let $X = \{a, b, c\}$. Consider two generalized topologies $\mu_1 = \{\emptyset, \{a\}, \{a, b\}, X\}$ and $\mu_2 = \{\emptyset, \{c\}, \{b, c\}\}$ on X. Then $\{a\}$ and $\{b, c\}$ are $\mu_{(1,2)}$-open but $\{a\} \cup \{b, c\} = X$ is not $\mu_{(1,2)}$-open.*

Proposition 3.8. *Let (X, μ_1, μ_2) be a bigeneralized topological space. If A is $\mu_{(m,n)}$-closed and F is (m, n)-closed, then $A \cap F$ is $\mu_{(m,n)}$-closed, where $m, n = 1, 2$ and $m \neq n$.*

Proof. Let $A \cap F \subseteq U$ and U is μ_m-open set. Then $A \subseteq U \cup (X - F)$ and so $c_{\mu_n}(A) \subseteq U \cup (X - F)$. Therefore, $c_{\mu_n}(A) \cap F \subseteq U$. Since F is (m, n)-closed, we obtain $c_{\mu_n}(A \cap F) \subseteq c_{\mu_n}(A) \cap c_{\mu_n}(F) \subseteq U$. Hence, $A \cap F$ is $\mu_{(m,n)}$-closed. $\square$

Remark 6. *$\mu_{(1,2)}$-$C(X)$ is generally not equal to $\mu_{(2,1)}$-$C(X)$ as can be seen from the following example.*

Example 3.9. *Let $X = \{a, b, c\}$. Consider two generalized topologies $\mu_1 = \{\emptyset, \{a\}, \{a, b\}, X\}$ and $\mu_2 = \{\emptyset, \{c\}, \{b, c\}\}$ on X. Then $\mu_{(1,2)}$-$C(X) = \{\{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, X\}$ and $\mu_{(2,1)}$-$C(X) = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, X\}$. Thus $\mu_{(1,2)}$-$C(X) \neq \mu_{(2,1)}$-$C(X)$.*

Proposition 3.10. *Let μ_1 and μ_2 be generalized topologies on X. If $\mu_1 \subseteq \mu_2$, then $\mu_{(2,1)}$-$C(X) \subseteq \mu_{(1,2)}$-$C(X)$.*

Proposition 3.11. *For each element x of a bigeneralized topological space $(X, \mu_1, \mu_2))$, $\{x\}$ is μ_m-closed or $X - \{x\}$ is $\mu_{(m,n)}$-closed, where $m, n = 1, 2$ and $m \neq n$.*

Proof. Let $x \in X$ and the singleton $\{x\}$ be not μ_m closed. Then $X - \{x\}$ is not μ_m-open, if $X \in \mu_m$, then X is only μ_m-open set which contains $X - \{x\}$, hence $X - \{x\}$ is $\mu_{(m,n)}$-closed and if $X \notin \mu_m$, then $X - \{x\}$ is $\mu_{(m,n)}$-closed. $\square$

Proposition 3.12. *Let A be a subset of a bigeneralized topological space $(X, \mu_1, \mu_2))$. If A is $\mu_{(m,n)}$-closed, then $c_{\mu_n}(A) - A$ contains no nonempty μ_m-closed set, where $m, n = 1, 2$ and $m \neq n$.*

Proof. Let A be a $\mu_{(m,n)}$-closed set and $F \neq \emptyset$ is μ_m-closed set such that $F \subseteq c_{\mu_n}(A) - A$. Since $A \in \mu_{(m,n)}$-$C(X)$, we have $c_{\mu_n}(A) \subseteq X - F$. Thus $F \subseteq c_{\mu_n}(A) \cap (X - c_{\mu_n}(A)) = \emptyset$, this is a contradiction. Then $c_{\mu_n}(A) - A$ contains no nonempty μ_m-closed set. $\square$

The converse of the above proposition is not true as seen from the following example.

Example 3.13. *Let $X = \{a, b, c\}$. Consider two generalized topologies $\mu_1 = \{\emptyset, \{a\}, \{a, c\}\}$ and $\mu_2 = \{\emptyset, \{b\}, X\}$ on X. If $A = \{a\}$, then $c_{\mu_2}(A) - A = \{c\}$ does not contain any nonempty μ_1-closed set. But A is not $\mu_{(1,2)}$-closed.*

Proposition 3.14. *Let μ_1 and μ_2 be generalized topologies on X. If A is a $\mu_{(m,n)}$-closed set, then $c_{\mu_m}(\{x\}) \cap A \neq \emptyset$ holds for each $x \in c_{\mu_n}(A)$, where $m, n = 1, 2$ and $m \neq n$.*

Proof. Let $x \in c_{\mu_n}(A)$. Suppose that $c_{\mu_m}(\{x\}) \cap A = \emptyset$. Then $A \subseteq X - c_{\mu_m}(\{x\})$. Since A is $\mu_{(m,n)}$-closed and $X - c_{\mu_m}(\{x\})$ is μ_m-open. Thus $c_{\mu_n}(A) \subseteq X - c_{\mu_m}(\{x\})$. Hence, $c_{\mu_n}(A) \cap c_{\mu_m}(\{x\}) = \emptyset$. This is a contradiction. $\square$

Proposition 3.15. *If A is a $\mu_{(m,n)}$-closed set of (X, μ_1, μ_2) such that $A \subseteq B \subseteq c_{\mu_n}(A)$, then B is $\mu_{(m,n)}$-closed set, where $m, n = 1, 2$ and $m \neq n$.*

Proof. Let A be a $\mu_{(m,n)}$-closed set and $A \subseteq B \subseteq c_{\mu_n}(A)$. Let $B \subseteq U$ and U is μ_m-open. Then $A \subseteq U$. Since A is $\mu_{(m,n)}$-closed, we have $c_{\mu_n}(A) \subseteq U$. Since $B \subseteq c_{\mu_n}(A)$, then $c_{\mu_n}(B) \subseteq c_{\mu_n}(A) \subseteq U$. Hence, B is $\mu_{(m,n)}$-closed. $\square$

Theorem 3.16. *For a bigeneralized topological space (X, μ_1, μ_2), μ_m-$O(X) \subseteq \mu_n$-$C(X)$ if and only if every subset of X is $\mu_{(m,n)}$-closed set, where $m, n = 1, 2$ and $m \neq n$.*

Proof. Suppose that μ_m-$O(X) \subseteq \mu_n$-$C(X)$. Let A be a subset of X such that $A \subseteq U$, where $U \in \mu_m$-$O(X)$. Then $c_{\mu_n}(A) \subseteq c_{\mu_n}(U) = U$ and hence A is $\mu_{(m,n)}$-closed.

Conversely, suppose that every subset of X is $\mu_{(m,n)}$-closed. Let $U \in \mu_m$-$O(X)$. Since U is $\mu_{(m,n)}$-closed, we have $c_{\mu_n}(U) \subseteq U$. Therefore, $U \in \mu_n$-$C(X)$ and hence μ_m-$O(X) \subseteq \mu_n$-$C(X)$. $\square$

Theorem 3.17. *A subset A of a bigenralized topological space (X, μ_1, μ_2) is $\mu_{(m,n)}$-open if and only if every subset F of X, $F \subseteq i_{\mu_n}(A)$ whenever F is μ_m-closed and $F \subseteq A$, where $m, n = 1, 2$ and $m \neq n$.*

Proof. Suppose that A is $\mu_{(m,n)}$-open. We shall show that $F \subseteq i_{\mu_n}(A)$ whenever F is μ_m-closed and $F \subseteq A$. Let $F \subseteq A$ and F is μ_m-closed. Then $X - A \subseteq X - F$ and $X - F$ is μ_m-open, we have $X - A$ is $\mu_{(m,n)}$-closed, then $c_{\mu_n}(X - A) \subseteq X - F$. Thus $X - i_{\mu_n}(A) \subseteq X - F$ and hence $F \subseteq i_{\mu_n}(A)$.

Conversely, suppose that $F \subseteq i_{\mu_n}(A)$ whenever F is μ_m-closed and $F \subseteq A$. Let $X - A \subseteq U$ and U is μ_m-open. Then $X - U \subseteq A$ and $X - U$ is μ_m-closed. By assumption, we have $X - U \subseteq i_{\mu_n}(A)$, then $X - i_{\mu_n}(A) \subseteq U$. Therefore, $c_{\mu_n}(X - A) \subseteq U$. Thus $X - A$ is $\mu_{(m,n)}$-closed. Hence, A is $\mu_{(m,n)}$-open. □

Theorem 3.18. *Let A and B be subsets of a bigenralized topological space (X, μ_1, μ_2) such that $i_{\mu_n}(A) \subseteq B \subseteq A$. If A is $\mu_{(m,n)}$-open, B is $\mu_{(m,n)}$-open, where $m, n = 1, 2$ and $m \neq n$.*

Proof. Suppose that $i_{\mu_n}(A) \subseteq B \subseteq A$. Let F be μ_m-closed such that $F \subseteq B$. Since A is $\mu_{(m,n)}$-open, $F \subseteq i_{\mu_n}(A)$. Since $i_{\mu_n}(A) \subseteq B$, we have $i_{\mu_n}(i_{\mu_n}(A)) \subseteq i_{\mu_n}(B)$. Consequently, $i_{\mu_n}(A) \subseteq i_{\mu_n}(B)$. Hence, $F \subseteq i_{\mu_n}(B)$. Therefore, B is $\mu_{(m,n)}$-open. □

Proposition 3.19. *If a subset A of a bigeneralized topological space (X, μ_1, μ_2) is $\mu_{(m,n)}$-closed, then $c_{\mu_n}(A) - A$ is $\mu_{(m,n)}$-open, where $m, n = 1, 2$ and $m \neq n$.*

Proof. Suppose that A is $\mu_{(m,n)}$-closed. We shall show that $c_{\mu_n}(A) - A$ is $\mu_{(m,n)}$-open. Let $X - (c_{\mu_n}(A) - A) \subseteq U$ and U is μ_m-open. Then $X - U \subseteq (c_{\mu_n}(A) - A)$ and $X - U$ is μ_m-closed. Thus, we have $c_{\mu_n}(A) - A$ does not contain nonempty μ_m-closed set by Proposition 3.12. Consequently, $X - U = \emptyset$, then $U = X$. Therefore, $c_{\mu_n}(X - (c_{\mu_n}(A) - A)) \subseteq U$ so we obtain $X - (c_{\mu_n}(A) - A)$ is $\mu_{(m,n)}$-closed. Hence, $c_{\mu_n}(A) - A$ is $\mu_{(m,n)}$-open. □

4 Generalized continuous functions

In this section, we introduce $g_{(m,n)}$-continuous functions on bigeneralized topological spaces and investigate some of their properties.

Definition 4.1. Let (X, μ_X^1, μ_X^2) and (Y, μ_Y^1, μ_Y^2) be generalized topological spaces. A function $f : (X, \mu_X^1, \mu_X^2) \to (Y, \mu_Y^1, \mu_Y^2)$ is said to be (m, n)-*generalized continuous* (briefly $g_{(m,n)}$-*continuous*) if $f^{-1}(F)$ is $\mu_{(m,n)}$-closed in X for every μ_n-closed F of Y, where $m, n = 1, 2$ and $m \neq n$.

A function $f : (X, \mu_X^1, \mu_X^2) \to (Y, \mu_Y^1, \mu_Y^2)$ is said to be *pairwise generalized continuous* (briefly *pairwise g-continuous*) if f is $g_{(1,2)}$-continuous and $g_{(2,1)}$-continuous.

Theorem 4.2. *For an injective function $f : (X, m^1_X, m^2_X) \to (Y, m^1_Y, m^2_Y)$, the following properties are equivalent:*

(1) f is $g_{(m,n)}$-continuous,

(2) For each $x \in X$ and for every μ_n-open set V containing $f(x)$, there exists a $\mu_{(m,n)}$-open set U containing x such that $f(U) \subseteq V$;

(3) $f(c_{\mu^n_X}(A)) \subseteq c_{\mu^n_Y}(f(A))$ for every subset A of X;

(4) $c_{\mu^n_X}(f^{-1}(B)) \subseteq f^{-1}(c_{\mu^n_Y}(B))$ for every subset B of Y.

Proof. (1) $\Rightarrow$ (2): Let $x \in X$ and V be a μ_n-open subset of Y containing $f(x)$. Then by (1), $f^{-1}(V)$ is $\mu_{(m,n)}$-open of X containing x. If $U = f^{-1}(V)$, then $f(U) \subseteq V$.

(2) $\Rightarrow$ (3): Let A be a subset of X and $f(x) \notin c_{\mu^n_Y}(f(A))$. Then, there exists a μ_n-open subset V of Y containing $f(x)$ such that $V \cap f(A) = \emptyset$. Then by (2), there exists a $\mu_{(m,n)}$-open set such that $f(x) \in f(U) \subseteq V$. Hence, $f(U) \cap f(A) = \emptyset$ implies $U \cap A = \emptyset$. Consequently, $x \notin c_{\mu^n_X}(A)$ and $f(x) \notin f(c_{\mu^n_X}(A))$.

(3) $\Rightarrow$ (4): Let B be a subset of Y. By (3), we obtain $f(c_{\mu^n_X}(f^{-1}(B))) \subseteq c_{\mu^n_Y}(f(f^{-1}(B)))$. Thus $c_{\mu^n_X}(f^{-1}(B)) \subseteq f^{-1}(c_{\mu^n_Y}(B))$.

(4) $\Rightarrow$ (1): Let F be a μ_n-closed subset of Y. Let U be a μ_m-open subset of X such that $f^{-1}(F) \subseteq U$. Since $c_{\mu^n_Y}(F) = F$ and by (4), $c_{\mu^n_X}(f^{-1}(F)) \subseteq U$. Hence, f is $g_{(m,n)}$-continuous. □

Definition 4.3. Let (X, μ^1_X, μ^2_X) and (Y, μ^1_Y, μ^2_Y) be generalized topological spaces. A function $f : (X, \mu^1_X, \mu^2_X) \to (Y, \mu^1_Y, \mu^2_Y)$ is said to be *g_m-continuous* if $f^{-1}(F)$ is μ_m-closed in X for every μ_m-closed F of Y, for $m = 1, 2$.

Definition 4.4. Let (X, μ^1_X, μ^2_X) and (Y, μ^1_Y, μ^2_Y) be generalized topological spaces. A function $f : (X, \mu^1_X, \mu^2_X) \to (Y, \mu^1_Y, \mu^2_Y)$ is said to be *g_m-closed* (resp. *g_m-open*) if $f(F)$ is μ_m-closed (resp. μ_m-open) of Y for every μ_m-closed (resp. μ_m-open) F of X, for $m = 1, 2$.

Proposition 4.5. *If $f : (X, \mu^1_X, \mu^2_X) \to (Y, \mu^1_Y, \mu^2_Y)$ is g_m-continuous and g_m-closed, then $f(A)$ is $\mu_{(m,n)}$-closed subset of Y for every $\mu_{(m,n)}$-closed subset A of X, where $m, n = 1, 2$ and $m \neq n$.*

Proof. Let U be a μ_m-open subset of Y such that $f(A) \subseteq U$. Then $A \subseteq f^{-1}(U)$ and $f^{-1}(U)$ is μ_m-open subset of X. Since A is $\mu_{(m,n)}$-closed, $c_{\mu^n_X}(A) \subseteq f^{-1}(U)$ and hence $f(c_{\mu^n_X}(A)) \subseteq U$. Therefore, we have $c_{\mu^n_Y}(f(A)) \subseteq c_{\mu^n_Y}(f(c_{\mu^n_X}(A))) = f(c_{\mu^n_X}(A)) \subseteq U$. Therefore, $f(A)$ is $\mu_{(m,n)}$-closed subset of Y. □

Lemma 4.6. *If $f : (X, \mu^1_X, \mu^2_X) \to (Y, \mu^1_Y, \mu^2_Y)$ is g_m-closed, then for each subset S of Y and each μ_m-open subset U of X containing $f^{-1}(S)$, there exists a μ_m-open subset V of Y such that $f^{-1}(V) \subseteq U$.*

Proposition 4.7. *If $f : (X, \mu_X^1, \mu_X^2) \to (Y, \mu_Y^1, \mu_Y^2)$ is injective, g_m-closed and $g_{(m,n)}$-continuous, then $f^{-1}(B)$ is $\mu_{(m,n)}$-closed subset of X for every $\mu_{(m,n)}$-closed subset B of Y, where $m, n = 1, 2$ and $m \neq n$.*

Proof. Let B be a $\mu_{(m,n)}$-closed subset of Y. Let U be a μ_m-open subset of X such that $f^{-}(B) \subseteq U$. Since f is g_m-closed and by Lemma 4.6, there exists a μ_m-open subset of Y such that $B \subseteq V$ and $f^{-1}(V) \subseteq U$. Since B is $\mu_{(m,n)}$-closed set and $B \subseteq V$, then $c_{\mu_Y^n}(B) \subseteq V$. Consequently, $f^{-1}(c_{\mu_Y^n}(B)) \subseteq f^{-1}(V) \subseteq U$. By Theorem 4.2, $c_{\mu_X^n}(f^{-1}(B)) \subseteq f^{-1}(c_{\mu_Y^n}(B)) \subseteq U$ and hence $f^{-1}(B)$ is $\mu_{(m,n)}$-closed subset of X. □

Definition 4.8. A bigeneralized topological space (X, μ_X^1, μ_X^2) is said to be $\mu_{(m,n)}$-$T_{\frac{1}{2}}$ *space* if, for every $\mu_{(m,n)}$-closed set is μ_n-closed, where $m, n = 1, 2$ and $m \neq n$.

Definition 4.9. A bigeneralized topological space (X, μ_X^1, μ_X^2) is said to be *pairwise μ-$T_{\frac{1}{2}}$ space* if it is both $\mu_{(1,2)}$-$T_{\frac{1}{2}}$ space and $\mu_{(2,1)}$-$T_{\frac{1}{2}}$ space.

Theorem 4.10. *A bigeneralized topological space is a $\mu_{(m,n)}$-$T_{\frac{1}{2}}$ space if and only if $\{x\}$ is μ_n-open or μ_m-closed for each $x \in X$, where $m, n = 1, 2$ and $m \neq n$.*

Proof. Suppose that $\{x\}$ is not μ_m-closed. Then $X - \{x\}$ is $\mu_{(m,n)}$-closed by Proposition 3.11. Since X is $\mu_{(1,2)}$-$T_{\frac{1}{2}}$ space, $X - \{x\}$ is μ_n-closed. Hence, $\{x\}$ is μ_n-open.

Conversely, let F be a $\mu_{(m,n)}$-closed set. By assumption, $\{x\}$ is μ_n-open or μ_m-closed for any $x \in c_{\mu_n}(F)$. Case (i) Suppose that $\{x\}$ is μ_n-open. Since $\{x\} \cap F \neq \emptyset$, we have $x \in F$. Case (ii) Suppose that $\{x\}$ is μ_m-closed. If $x \notin F$, then $\{x\} \subseteq c_{\mu_n}(F) - F$, which is a contradiction to Proposition 3.12. Therefore, $x \in F$. Thus is both cases, we conclude that F is μ_n-closed. Hence, (X, μ_X^1, μ_X^2) is a $\mu_{(m,n)}$-$T_{\frac{1}{2}}$ space. □

Definition 4.11. Let (X, μ_X^1, μ_X^2) and (Y, μ_Y^1, μ_Y^2) be generalized topological spaces. A function $f : (X, \mu_X^1, \mu_X^2) \to (Y, \mu_Y^1, \mu_Y^2)$ is said to be $g_{(m,n)}$-*irresolute* if $f^{-1}(F)$ is $\mu_{(m,n)}$-closed in X for every $\mu_{(m,n)}$-closed F of Y, where $m, n = 1, 2$ and $m \neq n$.

Proposition 4.12. *Let $f : (X, \mu_X^1, \mu_X^2) \to (Y, \mu_Y^1, \mu_Y^2)$ and $g : (Y, \mu_Y^1, \mu_Y^2) \to (Z, \mu_Z^1, \mu_Z^2)$ be functions, the following properties holds:*

(i) If f is $g_{(m,n)}$-irresolute and g is $g_{(m,n)}$-continuous, then $g \circ f$ is $g_{(m,n)}$-continuous;

(ii) If f and g are $g_{(m,n)}$-irresolute, then $g \circ f$ is $g_{(m,n)}$-irresolute;

(iii) Let (Y, μ_Y^1, μ_Y^2) be a $\mu_{(m,n)}$-$T_{\frac{1}{2}}$ space. If f and g are $g_{(m,n)}$-continuous, then $g \circ f$ is $g_{(m,n)}$-continuous.

Proof. (i) Let F be a μ_n-closed subset of Z. Since g is $g_{(m,n)}$-continuous, then $g^{-1}(F)$ is $\mu_{(m,n)}$-closed subset of Y. Since f is $g_{(m,n)}$-irresolute, then $(g \circ f)^{-1}(F) = f^{-1}(g^{-1}(F))$ is $\mu_{(m,n)}$-closed subset of X. Hence, $g \circ f$ is $g_{(m,n)}$-continuous.

(ii) Let F be a $\mu_{(m,n)}$-closed closed subset of Z. Since g is $g_{(m,n)}$-irresolute, then $g^{-1}(F)$ is $\mu_{(m,n)}$-closed subset of Y. Since f is $g_{(m,n)}$-irresolute, then $(g \circ f)^{-1}(F) = f^{-1}(g^{-1}(F))$ is $\mu_{(m,n)}$-closed subset of X. Hence, $g \circ f$ is $g_{(m,n)}$-irresolute.

(iii) Let F be a μ_n-closed closed subset of Z. Since g is $g_{(m,n)}$-continuous, then $g^{-1}(F)$ is $\mu_{(m,n)}$-closed subset of Y. Since (Y, μ_Y^1, μ_Y^2) is a $\mu_{(m,n)}$-$T_{\frac{1}{2}}$ space, then $g^{-1}(F)$ is μ_n-closed subset of Y. Since f is $g_{(m,n)}$-continuous, then $(g \circ f)^{-1}(F) = f^{-1}(g^{-1}(F))$ is $\mu_{(m,n)}$-closed subset of X. Hence, $g \circ f$ is $g_{(m,n)}$-continuous. □

Proposition 4.13. *Let (X, μ_X^1, μ_X^2) be a $\mu_{(m,n)}$-$T_{\frac{1}{2}}$ space. If $f : (X, \mu_X^1, \mu_X^2) \to (Y, \mu_Y^1, \mu_Y^2)$ is surjective, g_n-closed and $g_{(m,n)}$-irresolute, then (Y, μ_Y^1, μ_Y^2) is a $\mu_{(m,n)}$-$T_{\frac{1}{2}}$ space, where $m, n = 1, 2$ and $m \neq n$.*

Proof. Let F be a $\mu_{(m,n)}$-closed subset of Y. Since f is $g_{(m,n)}$-irresolute, we have $f^{-1}(F)$ is a $\mu_{(m,n)}$-closed subset of X. Since (X, μ_X^1, μ_X^2) is a $\mu_{(m,n)}$-$T_{\frac{1}{2}}$ space, $f^{-1}(F)$ is a μ_n-closed subset of X. It follows by assumption that F is a μ_n-closed subset of Y, Hence, (Y, μ_Y^1, μ_Y^2) is a $\mu_{(m,n)}$-$T_{\frac{1}{2}}$ space. □

Proposition 4.14. *Let (X, μ_X^1, μ_X^2) be a $\mu_{(m,n)}$-$T_{\frac{1}{2}}$ space. If $f : (X, \mu_X^1, \mu_X^2) \to (Y, \mu_Y^1, \mu_Y^2)$ is bijective, g_n-open and $g_{(m,n)}$-irresolute, then (Y, μ_Y^1, μ_Y^2) is a $\mu_{(m,n)}$-$T_{\frac{1}{2}}$ space, where $m, n = 1, 2$ and $m \neq n$.*

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