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On Contra α-$\mathcal{I}$-Continuous Functions

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Abstract

In this paper, α-$\mathcal{I}$-closed sets and α-$\mathcal{I}$-open sets are used to define and investigate a new class of functions called contra-α-$\mathcal{I}$-continuous functions on ideal topological spaces. Relationships between this new class and other classes of functions are established.

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1 Introduction

One of the important and basic topic in the theory of classical point set topology and several branches of mathematics, which have been researched by many authors, is continuity of functions. This concept has been extended to the setting of I-continuity of fuctions. Jankovic and Hamlett [7, 8] introduced the notion of I-open sets in topological spaces. Abd El-Monsef et al. [1] further investigated I-open sets and I-continuous functions. The notion of semi-I-open sets to obtain decomposition of continuity was introduced by Hatir and

Noiri [3, 4]. In addition to this, [3], [6] have introduced the notions of α-I-open sets, α-I-continuous functions and contra-α-continuous functions. The purpose of this paper is to give a new class of functions called contra-α-I-contiuous function in an ideal topological space. Some characterizations and several basic properties of this class of functions are obtained.

2 Preliminaries

Throughout this paper, $int(A)$ and $cl(A)$ denote the interior and closure of A, respectively. An ideal $\mathcal{I}$ on a topological space $(X,\ \tau)$ is a nonempty collection of subsets of X which satisfies (i) $A \in \mathcal{I}$ and $B \subset A$ implies $B \in \mathcal{I}$ and (ii) $A \in \mathcal{I}$ and $B \in \mathcal{I}$ implies $A \cup B \in \mathcal{I}$. Given a topological space $(X,\ \tau)$ with an ideal $\mathcal{I}$ on X and if $\mathcal{P}(X)$ is the set of all subsets of X, then the set operator $(.)^* : \mathcal{P}(X) \to \mathcal{P}(X)$, called the local function [15] of A with respect to τ and $\mathcal{I}$, is defined as follows: for $A \subset X$, $A^*(\tau, \mathcal{I}) = \{x \in X | U \cap A \notin \mathcal{I}$, for every open set U of X containing $x\}$. A Kuratowski closure operator $cl^*(.)$ for a topology $\tau^*(\tau,\ \mathcal{I})$ called the $*$-topology, finer than τ is defined by $cl^*(A) = A \cup A^*(\tau,\ \mathcal{I})$ when there is no chance of confusion. $A^*(\mathcal{I})$ is denoted by A^*. If $\mathcal{I}$ is an ideal on X, then $(X,\ \tau,\ \mathcal{I})$ is called an ideal topological space. By a space, we always mean a topological space $(X,\ \tau)$ with no separation properties assumed. A subset A of $(X,\ \tau)$ is said to be semiopen [9] (resp. α-open [13]) if $A \subset cl(int(A))$ (resp. $A \subset int(cl(int(A)))$. The complement of semiopen (resp. α-open) is called semiclosed (resp. α-closed).

A subset S of an ideal topological space $(X,\ \tau,\ \mathcal{I})$ is α-$\mathcal{I}$-open [3] (resp. β-$\mathcal{I}$-open [3]) is $S \subset int(cl^*(int(S)))$ (resp. $S \subset cl(int(cl^*(S))))$. The complement of a α-$\mathcal{I}$-open set is called α-$\mathcal{I}$-closed [3]. The intersection of all α-$\mathcal{I}$-closed sets containing S is called the α-$\mathcal{I}$-closure of S and is denoted by ${}_{\alpha\mathcal{I}}cl(S)$. The α-$\mathcal{I}$-interior of S is defined by the union of all α-$\mathcal{I}$-open sets contained in S and is denoted by ${}_{\alpha\{\mathcal{I}}Int(S)$. The family of all α-$\mathcal{I}$-open (resp. α-$\mathcal{I}$-closed) sets of $(X,\ \tau,\ \mathcal{I})$ is denoted by $\alpha\mathcal{I}O(X)$ (resp. $\alpha\mathcal{I}C(X)$). The family of all α-$\mathcal{I}$-open (resp. α-$\mathcal{I}$-closed) sets of $(X,\ \tau,\ \mathcal{I})$ containing a point $x \in X$ is denoted by $\alpha\mathcal{I}O(X,\ x)$ (resp. $\alpha\mathcal{I}C(X,\ x)$). A function $f : (X,\ \tau) \to (Y,\ \sigma)$ is said to be contra-continuous [2] (resp. contra-α-continuous [6]) if $f^{-1}(V)$ is closed (resp. α-closed) in X for every open set V of Y.

3 Contra-α-$\mathcal{I}$-continuous functions

We have introduced the following definition

Definition 3.1. A function $f : (X, \tau, \mathcal{I}) \rightarrow (Y, \sigma)$ is called *contra-α-$\mathcal{I}$-continuous* if $f^{-1}(V)$ is α-$\mathcal{I}$-open in $(X, \tau, \mathcal{I})$ for every closed set V of Y.

Proposition 3.1. *(i) Every contra-α-$\mathcal{I}$-continuous function is contra α-continuous.*
(ii) Every contra-continuous function is contra-α-$\mathcal{I}$-continuous.

Proof. Follows from Remark 3.4 in [10] ■

The converse of Proposition are need not be true as shown in the following examples.

Example 3.1 *Let* $X = \{a, b, c, d\}$, $\tau_1 = \{X, \phi, \{a\}, \{a, b\}, \{a, b, c\}\}$, $\tau_2 = \{X, \phi, \{a\}, \{b, c\}, \{a, b, c\}\}$ *and* $\mathcal{I} = \{\phi, \{a\}\}$. *Define* $f : (X, \tau, \tau, \mathcal{I}) \rightarrow (X, \tau_2)$ *by* $f(a) = d$, $f(b) = a$, $f(c) = c$ *and* $f(d) = b$ *is contra α-continuous but not contra-α-$\mathcal{I}$-continuous.*

Example 3.2 *Let* $X = \{a, b, c, d\}$, $\tau_1 = \{X, \phi, \{b\}, \{b, d\}, \{b, c, d\}\}$, $\tau_2 = \{X, \phi, \{a\}, \{d\}, \{a, d\}\}$ *and* $\mathcal{I} = \{\phi, \{c\}, \{d\} \{c, d\}\}$. *Clearly, the identity function* $f : (X, \tau_1, \mathcal{I}) \rightarrow (X, \tau_2)$ *is contra α-$\mathcal{I}$-continuous but not contra-continuous.*

Definition 3.2. Let A be a subset of a topological space (X, τ). The set $\bigcap\{U \in \tau | A \subset U\}$ is called the *kernal of A* [11] and is denoted by $ker(A)$.

Lemma 3.1. *[5] The following properties hold for subsets* A, B *of a space* X*:*
(1) $x \in ker(A)$ *if and only if* $A \cap F \neq \phi$ *for any closed set* F *of* X *containing* x*;*
(2) $A \subset ker(A)$ *and* $A = ker(A)$ *if* A *is open in* X*;*
(3) If $A \subset B$ *then* $ker(A) \subset ker(B)$.

Theorem 3.1. *The following are equivalent for a function* $f : (X, \tau, \mathcal{I}) \rightarrow (Y, \sigma)$*:*
(1) f *is contra-α-$\mathcal{I}$-continuous;*
(2) for every closed subset F *of* Y, $f^{-1}(F) \in \alpha\mathcal{I}O(X)$*;*
(3) for each $x \in X$ *and each closed set* F *of* Y *containing* $f(x)$*, there exists* $U \in \alpha\mathcal{I}O(X)$ *such that* $f(U) \subset F$*;*
(4) $f({}_{\alpha\mathcal{I}}cl(A))) \subset ker(f(A))$ *for every subset* A *of* X*;*
(5) ${}_{\alpha\mathcal{I}}cl(f^{-1}(B)) \subset f^{-1}(ker(B))$ *for every subset* B *of* Y.

Proof. The implications (1) ⇒ (2) and (2) ⇒ (3) are obivious.

(3) $\Rightarrow$ (2): Let F be any closed set of Y and $x \in f^{-1}(F)$. Then $f(x) \in F$ and there exists $U_x \in \alpha\mathcal{I}O(X,\ x)$ such thst $f(U_x) \subset F$. Therefore, we obtain $f^{-1}(F) = \bigcup\{U_x | x \in f^{-1}(F)\}$. Therefore $f^{-1}(F) \in \alpha\mathcal{I}O(X)$.

(2) $\Rightarrow$ (4): Let A be any subset of X. Suppose that $y \notin ker(f(A))$. Then by Lemma 3.1. there exists a closed set F of Y containing y such that $f(A) \cap F = \phi$. Thus, we have $A \cap f^{-1}(F) = \phi$ and ${}_{\alpha\mathcal{I}}cl(A) \cap f^{-1}(F) = \phi$. Therefore, we obtain $f({}_{\alpha\mathcal{I}}cl(A)) \cap F = \phi$ and $y \notin f({}_{\alpha\mathcal{I}}cl(A))$. This implies that $f({}_{\alpha\mathcal{I}}cl(A)) \subset ker(f(A))$.

(4) $\Rightarrow$ (5): Let B be any subset of Y. By (4) and Lemma 3.1., we have $f({}_{\alpha\mathcal{I}}cl(f^{-1}(B))) \subset ker(f(f^{-1}(B))) \subset ker(B)$ and ${}_{\alpha\mathcal{I}}cl(f^{-1}(B)) \subset f^{-1}(ker(B))$.

(5) $\Rightarrow$ (1): Let V be any open set of Y. Then by Lemma 3.1. we have ${}_{\alpha\mathcal{I}}cl(f^{-1}(V)) \subset f^{-1}(ker(V)) = f^{-1}(V)$ and ${}_{\alpha\mathcal{I}}cl(f^{-1}(V)) = f^{-1}(V)$. This shows that $f^{-1}(V)$ is α-$\mathcal{I}$-closed in $(X,\ \tau,\ \mathcal{I})$. ∎

Theorem 3.2. *If a function $f : (X,\ \tau,\ \mathcal{I}) \to (Y,\ \sigma)$ is contra-α-$\mathcal{I}$-continuous and Y is regular, then f is α-$\mathcal{I}$-continuous.*

Proof. Let x be an arbitrary point of X and V an open set of Y containing $f(x)$. Since Y is regular, there exists an open set W in Y containing $f(x)$ such that $cl(W) \subset V$. Since f is contra-α-$\mathcal{I}$-continuous, so by Theorem 3.1. there exists $U \in \alpha\mathcal{I}O(X,\ \tau)$ such that $f(U) \subset cl(W)$. Then $f(U) \subset cl(W) \subset V$. Hence, f is α-$\mathcal{I}$-continuous. ∎

Definition 3.3. A function $f : (X,\ \tau,\ \mathcal{I}) \to (Y,\ \sigma)$ is said to satisfy the α-$\mathcal{I}$-interiority condition if ${}_{\alpha\mathcal{I}}Int(f^{-1}(cl(V))) \subset f^{-1}(V)$ for each open set V of $(Y,\ \sigma)$.

Theorem 3.3. *If $f : (X,\ \tau,\ \mathcal{I}) \to (Y,\ \sigma)$ is a contra-α-$\mathcal{I}$-continuous function and satisfies the $\mathcal{I}$-interiority condition, then f is α-$\mathcal{I}$-continuous.*

Proof. Let V be any open set of Y. Since f is contra-α-$\mathcal{I}$-continuous and $cl(V)$ is closed, by Theorem 3.1. $f^{-1}(cl(V))$ is α-$\mathcal{I}$-open in X. By hypothesis of f, $f^{-1}(V) \subset f^{-1}(cl(V)) \subset_{\alpha\mathcal{I}} Int(f^{-1}(cl(V))) \subset_{\alpha\mathcal{I}} Int(f^{-1}(V)) \subset f^{-1}(V)$. Therefore, we obtain $f^{-1}(V) =_{\alpha\mathcal{I}} Int(f^{-1}(V))$ and consequently $f^{-1}(V) \in \beta\mathcal{I}O(X)$. This shows that f is a $\alpha\mathcal{I}$-continuous function. ∎

Theorem 3.4. *Let $(X,\ \tau,\ \mathcal{I})$ be any ideal topological space and let $f : (X,\ \tau,\ \mathcal{I}) \to (Y,\ \sigma)$ be a function and $g : X \to X \times Y$ be the graph function, given by $g(x) = (x, f(x))$ for every $x \in X$. Then f is contra-α-$\mathcal{I}$-continuous if and only if g is contra-α-$\mathcal{I}$-continuous.*

Proof. Let $x \in X$ and let W be a closed subset of $X \times Y$ containing $g(x)$. Then $W \cap (\{x\} \times Y)$ is closed in $\{x\} \times Y$ containing $g(x)$. Also $\{x\} \times Y$ is homeomorphic to Y. Hence $\{y \in Y | (x,\ y) \in W\}$ is a closed subset of Y. Since f is contra-α-$\mathcal{I}$-continuous, $\bigcup\{f^{-1}(y) \in Y | (x,\ y) \in W\}$ is a α-$\mathcal{I}$-open subset of $(X,\ \tau,\ \mathcal{I})$. Further, $x \in \bigcup\{f^{-1}(y) | (x,\ y) \in W\} \subset g^{-1}(W)$. Hence $g^{-1}(W)$ is α-$\mathcal{I}$-open. Then g is contra-α-$\mathcal{I}$-continuous. Conversely, let F be a closed subset of Y. Then $X \times F$ is a closed subset of $X \times Y$. Since g is contra-α-$\mathcal{I}$-continuous. $g^{-1}(X \times F)$ is a α-$\mathcal{I}$-open subset of X. Also, $g^{-1}(X \times F) = f^{-1}(F)$. Hence f is contra-α-$\mathcal{I}$-continuous. ∎

Definition 3.4. An ideal topological space $(X,\ \tau,\ \mathcal{I})$ is said to be α-$\mathcal{I}$-T_2 if for each distinct point $x,\ y \in X$, there exist $U,\ V \in \alpha\mathcal{I}O(X)$ containing x and y, respectively, such that $U \cap V = \phi$.

Theorem 3.5. *If $(X,\ \tau,\ \mathcal{I})$ is an ideal topological space and for each pair of distinct points x_1 and x_2 in X there exists a function f into a Urysohn space $(Y,\ \sigma)$ such that $f(x_1) \neq f(x_2)$ and f is contra-α-$\mathcal{I}$-continuous at x_1 and x_2, then the space $(X,\ \tau,\ \mathcal{I})$ is α-$\mathcal{I}$-T_2.*

Proof. Let x_1 and x_2 be any distinct points in X. Then by hypothesis there is a Urysohn space $(Y,\ \sigma)$ and a function $f : (X,\ \tau,\ \mathcal{I}) \to (Y,\ \sigma)$, which satisfies the conditions of this theorem. Let $y_i = f(x_i)$ for $i = 1,\ 2$. Then $y_1 \neq y_2$. Since (Y, σ) is Urysohn, there exists open neighbourhoods U_{y_1} and U_{y_2} of y_1 and y_2 respectively in Y such that $cl(U_{y_1}) \cap cl(U_{y_2}) = \phi$. Since f is contra-α-$\mathcal{I}$-continuous at x_i, there exists a α-$\mathcal{I}$-open neighbourhoods W_{x_i} of x_i in X such that $f(W_{x_i}) \subset cl(U_{y_i})$ for $i = 1,\ 2$. Hence we get $W_{x_1} \cap W_{x_2} = \phi$ because $cl(U_{y_1}) \cap cl(U_{y_2}) = \phi$. Then $(X,\ \tau,\ \mathcal{I})$ is a α-$\mathcal{I}$-T_2 space. ∎

Corollary 3.1. *If f is contra-α-$\mathcal{I}$-continuous injective function of an ideal topological space $(X,\ \tau,\ \mathcal{I})$ into a Urysohn space $(Y,\ \sigma)$, then $(X,\ \tau,\ \mathcal{I})$ is a α-$\mathcal{I}$-T_2 space.*

Proof. For each pair of distinct points x_1 and x_2 in X, f is contra-α-$\mathcal{I}$-continuous function of X into a Urysohn space $(Y,\ \sigma)$ such that $f(x_1) \neq f(x_2)$ because f is injective. Hence by Theorem 3.5., the space $(X,\ \tau,\ \mathcal{I})$ is a α-$\mathcal{I}$-T_2.
∎

Recall that a topological space $(X,\ \tau)$ is said to be *Ultra Hausdorff* [14] if every two distinct points of X can be separated by disjoint clopen sets.

Theorem 3.6. *If f is a contra-α-$\mathcal{I}$-continuous injective function of an ideal topological space $(X,\ \tau,\ \mathcal{I})$ into ultra Hausdorff space $(Y,\ \sigma)$, then $(X,\ \tau,\ \mathcal{I})$ is α-$\mathcal{I}$-T_2 space.*

Proof. Let x_1 and x_2 be any distinct points in X. Then since f is injective and Y is Ultra Hausdorff $f(x_1) \neq f(x_2)$ and there exists clopen sets V_1, V_2 such that $f(x_1) \in V_1$, $f(x_2) \in V_2$ and $V_1 \cap V_2 = \phi$. Then $x_i \in f^{-1}(V_i) \in \alpha\mathcal{I}O(X)$ for $i = 1,\ 2$ and $f^{-1}(V_1) \cap f^{-1}(V_2) = \phi$. Thus, $(X,\ \tau,\ \mathcal{I})$ is a α-$\mathcal{I}$-T_2 space. ■

Definition 3.5. The graph $G(f)$ of a function $f : (X,\ \tau,\ \mathcal{I}) \to (Y,\ \sigma)$ is said to be *contra-α-$\mathcal{I}$-closed* in $X \times Y$ if for each $(x,\ y) \in (X \times Y)\backslash G(f)$, there exists $U \in \alpha\mathcal{I}O(X)$ and a closed set V of Y containing y such that $(U \times V) \cap G(f) = \phi$.

Lemma 3.2. *The graph $f : (X,\ \tau,\ \mathcal{I}) \to (Y,\ \sigma)$ is contra-α-$\mathcal{I}$-closed in $X \times Y$ if and only if for each $(x,\ y) \in (X \times Y)\backslash G(f)$, there exists $U \in \alpha\mathcal{I}O(X,\ x)$ such that $f(U) \cap cl(W) = \phi$.*

Theorem 3.7. *If $f : (X,\ \tau,\ \mathcal{I}) \to (Y,\ \sigma)$ is a contra-α-$\mathcal{I}$-continuous function and Y is a Urysohn space, then $G(f)$ is contra-α-$\mathcal{I}$-closed in $X \times Y$.*

Proof. Let $(x,\ y) \in (X \times Y)\backslash G(f)$. Then $y \neq f(x)$ and there exist open set V, W of Y such that $f(x) \in V$, $y \in W$ and $cl(U) \cap cl(W) = \phi$. Since f is contra-α-$\mathcal{I}$-continuous, there exists $U \in \alpha\mathcal{I}O(X,\ x)$ such that $f(U) \subset cl(V)$. Therefore we obtain $f(U) \cap cl(W) = \phi$. This shows that $G(f)$ is contra-α-$\mathcal{I}$-closed. ■

Theorem 3.8. *If $f : (X,\ \tau,\ \mathcal{I}) \to (Y,\ \sigma)$ is a contra-α-$\mathcal{I}$-continuous function and $(Y,\ \sigma)$ is T_2, then $G(f)$ is contra-α-$\mathcal{I}$-closed.*

Proof. Let $(x,\ y) \in (X \times Y)\backslash G(f)$. Then $y \neq f(x)$ and there exist an open set V in Y such that $f(x) \in V$ and $y \notin V$. Since f is α-$\mathcal{I}$-continuous, there exists $U \in \alpha\mathcal{I}O(X,\ \tau)$ such that $f(U) \subset cl(V)$. Therefore, $f(U) \cap (Y - V) = \phi$ and $Y - V$ is a closed set of Y containing y. This shows that $G(f)$ is contra-α-$\mathcal{I}$-closed. ■

Definition 3.6. An ideal topological space $(X,\ \tau,\ \mathcal{I})$ is said to be *α-$\mathcal{I}$-connected* if X cannot be expressed as the union of two nonempty disjoint α-$\mathcal{I}$-open sets.

Theorem 3.9. *A contra-α-$\mathcal{I}$-continuous image of an α-$\mathcal{I}$-connected space is connected.*

Proof. The proof is clear. ■

Theorem 3.10. *Let $(X,\ \tau,\ \mathcal{I})$ be a α-$\mathcal{I}$-connected space and Y be a T_1-space. If $f : (X,\ \tau,\ \mathcal{I}) \to (Y,\ \sigma)$ is a contra-α-$\mathcal{I}$-continuous function then it is a constant function.*

Proof. Since Y is a T_1 space, $\Lambda = \{f^{-1}(\{y\}) : y \in Y\}$ is a disjoint α-$\mathcal{I}$-open partition of X. If $|\Lambda| \geq 2$, then X is the union of atleast two non-empty α-$\mathcal{I}$-open sets. Since $(X,\ \tau,\ \mathcal{I})$ is $\mathcal{I}$-connected, $|\Lambda| = 1$. Hence f is constant. ■

Definition 3.7. An ideal topological space $(X,\ \tau,\ \mathcal{I})$ is said to be *α-$\mathcal{I}$-normal* if each pair of nonempty disjoint closed sets can be separated by disjoint α-$\mathcal{I}$-open sets.

Definition 3.8. A topological space $(X,\ \tau)$ is said to be *ultra normal* [14] if each pair of nonempty disjoint closed sets can be separated by disjoint clopen sets.

Theorem 3.11. *If $f : (X,\ \tau,\ \mathcal{I}) \to (Y,\ \sigma)$ is a contra-α-$\mathcal{I}$-continuous closed injective function and Y is a ultra-normal space, then $(X,\ \tau,\ \mathcal{I})$ is an α-$\mathcal{I}$-normal space.*

Proof. Let F_1 and F_2 be a disjoint closed subsets of X. Since f is closed and injective, $f(F_1)$ and $f(F_2)$ are disjoint closed subsets of Y. Since Y is ultra normal, $f(F_1)$ and $f(F_2)$ are separated by disjoint clopen sets V_1 and V_2, of Y respectively. Hence $F_i \subset f^{-1}(V_i),\ f^{-1}(V_i) \in \alpha\mathcal{I}O(X)$ for $i = 1,\ 2$ and $f^{-1}(V_1) \cap f^{-1}(V_2) = \phi$. Thus, $(X,\ \tau,\ \mathcal{I})$ is a α-$\mathcal{I}$-normal. ■

Definition 3.9. A collection $\{G_\alpha : \alpha \in \wedge\}$ is called a β-$\mathcal{I}$-closed cover of a subset $\wedge$ of an ideal space $(X,\ \tau,\ \mathcal{I})$ if $A \subset \bigcup\{G_\alpha : X \backslash G_\alpha \in \alpha\mathcal{I}O(X),\ \alpha \in \wedge\}$.

Definition 3.10. An ideal topological space $(X,\ \tau,\ \mathcal{I})$ is said to be *α-$\mathcal{I}$-closed compact* if for every α-$\mathcal{I}$-closed cover $\{W_i : i \in \Delta\}$, there exists a finite subset Δ_o of Δ such that $X - \bigcup\{U_i : i \in \Delta_o\} \in \mathcal{I}$.

Lemma 3.3. *[12] For any function $f : (X,\ \tau,\ \mathcal{I}) \to (Y,\ \sigma)$. $f(\mathcal{I})$ is an ideal on Y.*

Theorem 3.12. *If a function $f : (X, \tau, \mathcal{I}) \to (Y, \sigma)$ is a contra-α-$\mathcal{I}$-continuous and the set A is α-$\mathcal{I}$-closed compact relative to X, then $f(A)$ is $f(\mathcal{I})$-compact in Y.*

Proof. Let $f : (X, \tau, \mathcal{I}) \to (Y, \sigma)$ be a contra-α-$\mathcal{I}$-continuous surjection and $\{V_i : i \in \Delta\}$ be an open cover of Y. Then $\{f^{-1}(V_i) : i \in \Delta\}$ is a α-$\mathcal{I}$-closed cover of X. From the assumption, there exists a finite subset Δ_o of Δ such that $X \setminus \bigcup\{f^{-1}(V_i) : i \in \Delta_o\} \in \mathcal{I}$. Therefore, $Y \setminus \bigcup\{V_i : i \in \Delta_o\} \in f(\mathcal{I})$ which shows that Y is $f(\mathcal{I})$-compact. ■

Theorem 3.13. *Let $f : (X, \tau, \mathcal{I}) \to (Y, \sigma)$ be a function and let $\{U_\alpha : \alpha \in \Delta\}$ be an α-$\mathcal{I}$-open cover of X. If the restriction function $f_{|}U_\alpha : (U_\alpha, \tau_{|U\alpha}, \mathcal{I}_{|U\alpha})$ is α-$\mathcal{I}$-continuous for each $\alpha \in \Delta$, then f is contra-α-$\mathcal{I}$-continuous.*

Proof. Obvious. ■

Definition 3.11. A function $f : (X, \tau, \mathcal{I}) \to (Y, \sigma, \mathcal{J})$ is said to be α-$\mathcal{I}$-irresolute if $f^{-1}(V)$ is α-$\mathcal{I}$-open in X for every α-$\mathcal{I}$-open set V of Y.

Theorem 3.14. *For the functions $f : (X, \tau, \mathcal{I}) \to (Y, \sigma, \mathcal{J})$ and $g : (Y, \sigma, \mathcal{J}) \to (Z, \eta)$ the following hold:*
(i) $g \circ f$ is α-$\mathcal{I}$-continuous, if f is contra-α-$\mathcal{I}$-continuous and g is contra continuous.
(ii) $g \circ f$ is contra-α-$\mathcal{I}$-continuous, if f is contra-α-$\mathcal{I}$-continuous and g is continuous.
(iii) $g \circ f$ is α-$\mathcal{I}$-continuous, if f is α-$\mathcal{I}$-irresolute and g is contra α-$\mathcal{J}$-continuous.

Remark 3.1. *The following examples shows that composition of any contra-α-$\mathcal{I}$-continuous functions need not be contra-α-$\mathcal{I}$-continuous function in general.*

Example 3.3 *Let $X = \{a, b, c, d\}$ with topology $\tau_1 = \{X, \phi, \{a\}, \{a, b\}, \{a, b, c\}\}$, $\tau_2 = \{X, \phi, \{a\}, \{b, c\}, \{a, b, c\}\}$, $\tau_3 = \{X, \phi, \{a\}, \{a, b, c\}\}$ and ideal $\mathcal{I}_\infty = \{\phi, \{a\}\}$, $\mathcal{I}_\in = \{\phi, \{b\}\}$. Define $f : (X, \tau_1, \mathcal{I}_\infty) \to (X, \tau_2, \mathcal{I}_\in)$ by by $f(a) = f(b) = d$, $f(c) = b$, $f(d) = c$ and $g : (X, \tau_2, \mathcal{I}_\in) \to (X, \tau_3)$ by $g(a) = d$, $g(b) = c$, $g(c) = b$ and $g(d) = a$. Then f and g are contra α-$\mathcal{I}$-continuous, because $F = \{b, c, d\}$ is closed in (X, τ_3) but $(g \circ f)^{-1}(F) = \{b, d\}$ which is not α-$\mathcal{I}$-open in (X, τ_1).*

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