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Effect of Rotation on Thermoelastic Waves in a Non-Homogeneous Infinite Cylinder

A. M. Abd-Alla[a] and S. R. Mahmoud[b1]

[a]Department of Mathematics, Faculty of science, Taif University, Saudi Arabia.

[b]Department of Mathematics, Faculty of Education, King Abdulaziz University, Saudi Arabia.

srhassan@kau.edu.sa

Abstract

In this paper, the problem involving a half-space or an infinite space with a cylindrical or spherical cavity have been subjected to certain boundary conditions. The material is elastic and has an inhomogeneity in the direction perpendicular to the boundary surface for the half-space problem with a cylindrical or spherical cavity. The system of fundamental equations have been solved by using an implicit finite-difference scheme. A numerical method is used to calculate the temperature, displacement and the components of stresses with time and through the radial of the body. Numerical results have been given and illustrated graphically for each case considered. The results indicate that the effect of rotation and inhomogeneity is very pronounced. Comparison has been made with the results in the absence of rotation.

Mathematics subject classification: 74B05

Keywords: Rotation, Stresses, Thermoelastic, Non-homogeneous, Isotropic Material.

1- Introduction

The dynamical problem of thermoelasticity has received much attention in the literature during the past decade. In recent years many thermoelastics problems concerning anisotropic materials have been solved by several workers. Mahmoud [1-2] studied the wave propagation in cylindrical poroelastic dry bones and effect of the non-homogeneity on wave propagation on orthotropic elastic media. Abd-Alla and Mahmoud [3] solved magneto-thermoelastic problem in rotating non-homogeneous orthotropic hollow cylindrical under the hyperbolic heat conduction model. Influences of rotation, magnetic field, initial stress and gravity on Rayleigh waves in a homogeneous orthotropic elastic half-space is investigated by Abd-Alla et al. [4,5]. Mahmoud et al [6]. studied effect of the rotation on the radial vibrations in a non-homogeneous orthotropic hollow cylinder. Abd-Alla and Abo-Dahab [7] studied time-harmonic sources in a generalized magneto-thermo-viscoelastic continuum with and without energy dissipation. Information about the behavior of inhomogeneous materials is important in understanding the response to a dynamic

[1] Permanent address: Mathematics Department, Faculty of Science, University of Sohag, Sohag 82524, Egypt (samsam73@yahoo.com)

input of composite materials and materials with local impurities. For this reason, this matter has attracted the attention of many researchers such as Abd-El-Salam, et al. [8] studied a numerical solution of magneto-thermoelastic problem in non-homogeneous isotropic cylinder by the finite-difference method. The effects of thermo elastic deformation due to sudden appearance of heat sources are an important of structural aspect studies in various contexts. El-Naggar et al. [9] studied the thermal stresses in a rotating non-homogeneous orthotropic hollow cylinder. Abd-Alla et al. [10] investigated the thermal stresses in a non-homogeneous orthotropic elastic multilayered cylinder. Abd-Alla et al. [11] studied the transient thermal stresses in rotating non-homogeneous cylindrically orthotropic composite tubes. Abd-Alla [12] investigated the wave propagation in hyperelstic media.

In this paper, we choose here a problem which is simple but physically realistic enough to lead to useful information and better understanding for the early times response of inhomogeneous material .The problem involves a half-space, or an infinite space with a cylindrical or spherical cavity subjected to certain boundary conditions. The material is elastic, and has an inhemogenity in the direction perpendicular to the boundary for the half-space problem and inhemogenity in the radii direction for the problem with a cylindrical or spherical cavity. In order to keep the formulation general, the equations of thermoelasticity are used in this analysis. These equations are solved using a numerical method which uses relation from the characteristic theory of finite difference scheme. Numerical examples of the temperature, radial displacement., and the stress are illustrated for isotropic material. The effects of inhomogeneity and the magnetic field are very pronounced. The numerical results due to this procedure, for this problem through the thickness of the cylinder, closely agree with the numerical solution by using the method of characteristics.

2. Formulation of the problem

The problems, we study involve a half-space, or an infinite body containing aspherical or cylindrical cavity of inner radius a and outer radius b. The bodies are made of inhomogeneous elastic materials. The inhemogenity is assumed to occur in the direction perpendicular to the boundary for the half-space problem and in the radial direction for the spherical and cylindrical problems. The bodies are initially at rest the half-space and the infinite bodies with spherical and cylindrical cavities are recarred respectively to Cartesian, spherical and cylindrical coordinate systems in which denotes the distance perpendicular to the surface for the half-space problem and the radial distance for the other two. A common formulation is used for three problems and the surface of the half-space is chosen to coincide with the plane r = b. [In the analysis} we consider a case for which the inhomogeneity occurs in Lame's constants λ and μ and all the other coefficients appearing in the equations are constant.

The symmetry conditions of the problem imply that all of the field variables depend on the distance rand time t' only. To keep the formulation general we use the thermoelasticity theory, i.e., we take the thermal effects caused by the deformation of the material into account. The heat conduction equation in the absence of heat sources is

$$\frac{\partial^2 T\prime}{\partial r^2}+\frac{P}{r}\frac{\partial T\prime}{\partial r}=\frac{1}{k1}\frac{\partial T\prime}{\partial t\prime}. \tag{1}$$

The equation of motion is

$$\frac{\partial \sigma_{rr}}{\partial r}+\frac{P(\sigma_{rr}-\sigma_{\theta\theta})}{r}+\rho\Omega^2 u=\rho\frac{\partial^2 u}{\partial t^{/2}}. \tag{2}$$

The nonzero components of the stress tensor are given in terms of the nonzero component of the displacement vector $\mathrm{u}(\mathrm{r},\mathrm{t}.')$,

$$\sigma_{rr}=(\lambda+2\mu)\frac{\partial u}{\partial r}+P\lambda\frac{u}{r}-\gamma T', \tag{3.a}$$

$$\sigma_{\theta\theta}=\lambda\frac{\partial u}{\partial r}+P(\lambda+2\mu)\frac{u}{r}-P(P-1)\mu\frac{u}{r}-\gamma T', \tag{3.b}$$

where σ_{rr} and $\sigma_{\theta\theta}$ are the stress components, uis the displacement vector $t'$is the time, ρ is the mass density, λ and μ are Lame's constants, $T'$is the absolute temperature and k_1 is the thermal diffusivity of the material. In these equations p takes the values 0.1 and 2 for the half-space, cylindrical and spherical problems respectively. We characterize the elastic constants λ , μ and density ρ of non-homogeneous material by

$$\lambda=\lambda_0 r^{2m},\ \ \mu=\mu_0 r^{2m} \quad and \quad \rho=\rho_0 r^{2m}, \tag{4}$$

where λ_0, μ_0 and ρ_0 are constants (they are the values λ , μ and ρ in homogeneous matter), and m is a rational number. Substituting from equations (4) into equations (2), we have

$$\frac{\partial^2 u}{\partial r^2}+\frac{\alpha_1}{r}\frac{\partial u}{\partial r}-\alpha_2\frac{u}{r^2}-\frac{\gamma_0}{\lambda_0+2\mu_0}\left[\frac{\partial T\prime}{\partial r}+\frac{2m}{r}T'\right]+\frac{\rho_0\Omega^2}{\lambda_0+2\mu_0}u=\frac{\rho_0}{\lambda_0+2\mu_0}\frac{\partial^2 u}{\partial t^2}. \tag{5}$$

It is convenient to introduce the following non-dimensionalization scheme,

$$b(U,R)=(u,r), \qquad t'=\frac{b}{v}t, \qquad T'=T_0 T, \tag{6}$$

where T_0 is a reference temperature and v is the dimension of velocity. Substituting from equations (6) into equation (1) and (5) we have :

$$\frac{\partial^2 T}{\partial R^2}+\frac{P}{R}\frac{\partial T}{\partial R}=\alpha\frac{\partial T}{\partial t}, \tag{7}$$

$$\frac{\partial^2 U}{\partial R^2}+\frac{\alpha_1}{R}\frac{\partial U}{\partial R}-\alpha_2\frac{U}{R^2}-\alpha_3\left[\frac{\partial T}{\partial R}+\frac{2m}{R}T\right]+\alpha_5 u=\alpha_4\frac{\partial^2 U}{\partial t^2}. \tag{8}$$

In terms of these non-dimensional variables, the stress components induced by the temperature $T$are related to displacement component U by:

$$\sigma_{RR}=\left(bR\right)^{2m}[\lambda_0+2\mu_0)\frac{\partial U}{\partial R}+P\lambda_0\frac{U}{R}-\gamma_0 T_0 T\Big], \tag{9.a}$$

$$\sigma_{\theta\theta}=(bR)^{2m}[\lambda_0\frac{\partial U}{\partial R}+P(\lambda_0+2\mu_0)\frac{U}{R}-P(P-1)\mu_0\frac{U}{R}-\gamma_0 T_0 T], \tag{9.b}$$

where $\alpha = \frac{vb}{k_1}$, $\alpha_1 = 2m + \rho$, $\alpha_2 = \frac{P((1-2m)\lambda_0 + P(3-P)\mu_0)}{\lambda_0 + 2\mu_0}$,
$\alpha_3 = \frac{\gamma_0 T_0}{\lambda_0 + 2\mu_0}$, $\alpha_4 = \frac{\rho_0 v^2}{\lambda_0 + 2\mu_0}$, $\gamma_0 = (3\lambda_0 + 2\mu_0)\alpha_1$,
$\alpha_5 = \frac{\rho_0 \Omega^2}{\lambda_0 + 2\mu_0}$,
where α_t is the coefficient of linear thermal expansion.

3. Boundary conditions

The above equations are solved subject to the initial condition
at $t = 0$, $T = 0$,
(10.a)
at $t = 0$, $U = \frac{\partial U}{\partial t} = 0.$ (10.b)

Problem I (P = 0)
For the half-space problem, the boundary conditions can be written as follow:
at $R = \frac{a}{b}$, $\frac{\partial U}{\partial R} = 0$, $T = 1$, (11.a)
at $R = 1$, $U = 0$, $\frac{\partial U}{\partial R} = 0$, (11.b)

Problem II (P=l).
(i) Case A
For the cylindrical problem, the boundary conditions can be expressed as:
at $R = \frac{a}{b}$, $U = 0$, $T = 1$, (12.a)
at $R = 1$, $\frac{\partial U}{\partial R} = 0$, $\frac{\partial T}{\partial R} = 0.$ (12.b)

(ii) Case B.
For the cylindrical problem, the boundary conditions can be expressed as:
at $R = \frac{a}{b}$, $U = 0$, $T = 1$, (13.a)
at $R = 1$, $U = \frac{\partial \sigma_{RR}}{\partial R} = 0$, $\frac{\partial T}{\partial R} = 0.$ (13.b)

Problem III (P=2) .
For the spherical problem, the boundary conditions can be expressed as:
at $R = \frac{a}{b}$, $U = 0$, $T = 1$, (14.a)
at $R = 1$, $\frac{\partial U}{\partial R} = 0$, $\frac{\partial T}{\partial R} = 0$. (14.b)

4. Numerical scheme

A finite difference scheme which is a modification of Mac Comack's scheme is described by Watchlman et al. [18], where it is used to obtain solutions to problem of thermal stress emanating from cylindrical cavity in a bounded medium. This scheme is a forward backward predictor corrector scheme. We take the finite difference grids with spatial intervals $h$in the direction R and $k$as the time step, and use the subscripts i and n to denote the ithdiscrete points in the Rdirection and the $n\ th$discrete time. A mesh is defined by:

$R_i = a_0 + ih, \qquad i = 0,1,2 \dots \dots \dots . j-1 \ t^n = kn, \qquad n = 0,1,2,3.$

, k being the time step, where $a_0 = \frac{a}{b}$. The functions T(R,t) and U(R,t) may be at any nodal location

$$T(R_i, t^n) = T_i^n, \tag{15.a}$$
$$U(R, t^n) = U_i^n. \tag{15.b}$$

The heat conduction equation (7) in difference form as follows:

$$T_i^{n+1} = T_i^n + \frac{\rho}{a}(T_{i+1}^n - 2T_i^n + T_{i-1}^n + \rho\left(\frac{h}{\frac{a}{b}+ih}\right)(T_{i+1}^n - T_i^n)). \tag{16}$$

Also, the equation of motion (8) may be expressed in the difference as follows:

$$U_i^{n+1} = (2+\alpha_5)U_i^n + U_i^{n-1} + \frac{\rho}{\alpha_4}[\, U_{i+1}^n - 2U_i^n + U_{i-1}^n + \alpha_1\left(\frac{h}{\frac{a}{b}+ih}\right)(U_{i+1}^n - U_i^n)$$
$$-\alpha_2\left(\frac{h}{\frac{a}{b}+ih}\right)^2 U_i^n - \alpha_3 h(T_{i+1}^n - T_i^n + \left(\frac{2mh}{\frac{a}{b}+ih}\right)\ T_i^n]\text{`}, \tag{17}$$

where $\rho = \frac{k}{h^2}, \qquad \rho_1 = \left(\frac{k}{h}\right)^2.$

5- Numertcal results and discussion

Let us consider the distribution of the transient temperature, displacement, and thermal stresses in an infinite elastic body. The copper material was chosen for purposes of numerical evolutions. The constants of the problem were taken as:

$\lambda_0 = 1.387 \times 10^{12} dyne/cm^2, \quad \mu_0 = 0.448 \times 10^{12} dyne/cm^2, \quad \rho_0 = 8.93 g/cm^3, \quad k_1 = 1.14\ cm^2/s, \quad \alpha_t = 1.67 \times 10^{-8}/^0\,C.$

Results are presented for cylinder with $a = 0.1$, $b = 1$and $T_0 = 1$. The numerical results are illustrated in terms of the non-dimensional displacement U, the stresses σ_{rr} , $\sigma_{\theta\theta}$, the temperature (T) and they are shown in figures 1-22. Figures 1, 2, 3 and 4 to the half- space problem. Figures (5-15) to the cylindrical problem. Figures (16)-(22) to the spherical problem. T_0 illustrate the non-dimensional radial distance R is used. We study the non-homogeneous case by taking $m = 0.5$ and the homogeneous case was studied by taking $m = 0$. We represented the numerical results graphically. Figures 1, 5 and 16 show the temperature variation for various nondimensional times t. It is noticed that the temperature decreases with increasing R in all three modes and satisfied the boundary conditions for three problems, the half-space, the cylindrical and the spherical respectively. Figure 2 shows the radial displacement U for the half-space: problem. From the figure, it decreases with R and it increases with t. Figures 3 and 4 show the radial stress and tangential stress for the half-space problem. It is noticed that they increase with increasing R and they decrease with increasing t. Figures 6, 7, 12, 13, 17 and 18 show the influence of the non-homogeneity of the material constants on the radial-displacement, also, they show the difference among the two cases namely non-homogeneous and homogeneous bodies for cylindrical problem and .spherical problem, respectively. From these figure the radial displacement decreases and then increases with increasing R and it decreases with increasing t for the non-homogeneous case. But for the homogeneous case it increases and then decreases with increasing $R$and it increases with increasing $t$and satisfies the boundary conditions. Figures 8, 9, 10 and 11 illustrate (the radial stress and tangential stress along the radial direction at various $t$for the cylindrical problem. It is noticed that they increase with increasing R for the non-homogeneous case; also they decrease with increasing R for the homogeneous case and satisfies the boundary conditions. Figures 14 and 15 show the radial stress σ_{rr} along the radial R at various t for non-homogeneous and homogeneous case. It is clear from these figures that the values of radial stress increases with increasing t for non-homogeneous cases and a dispersion curves with the radial$R$. But for the homogeneous case the radial stress increases with increasing$R$. Figures 19, 20, 21 and 22 illustrate the radial stress and tangential stress for different values of t. In all these cases, the stress is of a compressive character and decreases with increasing$t$, then a sudden jump occurs in the stress value at a certain point. The values of the jump discontinuity is the difference between the two maximum absolute values of the compressive stress. The results are specific. For the example considered, but other examples may have different trends because of the dependence of the results on the mechanical and thermal constants of the material.

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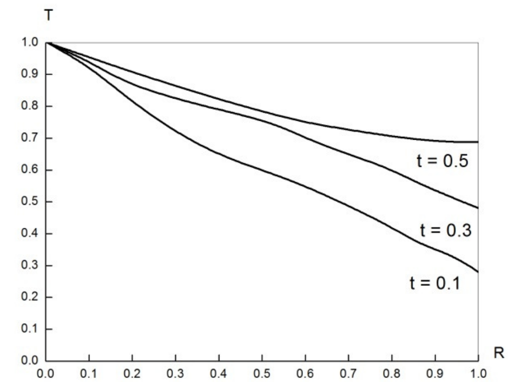

Fig.1 Show temperature distribution for problem I.

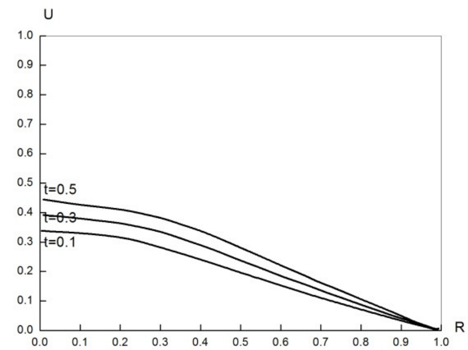

Fig.2 Show radial displacement distribution ($m = 0.0$) for problem I

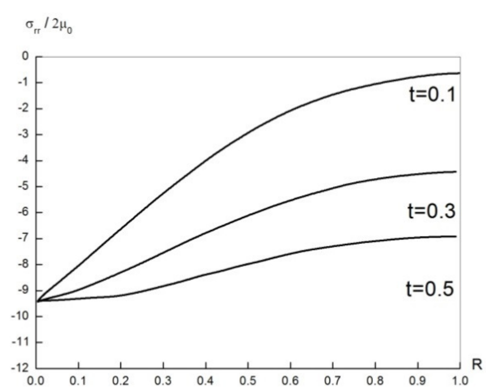

Fig.3 Show radial stress distribution ($m = 0.0$) for problem I

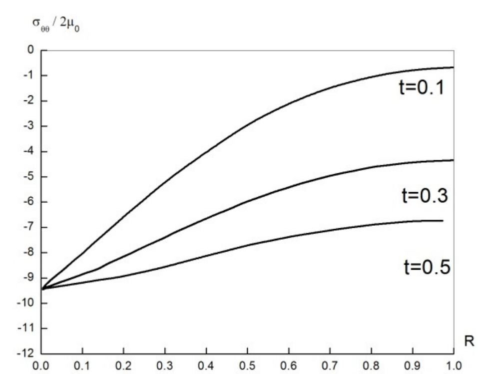

Fig.4 Show tangential stress distribution ($m = 0.0$) for problem I

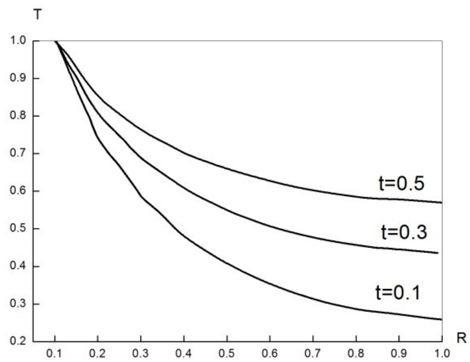

Fig.5 Show temperature distribution for problem II

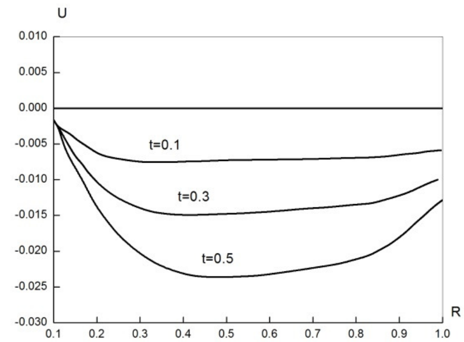

Fig.6 Show radial displacement distribution ($m = 0.5$) for problem II (Case A)

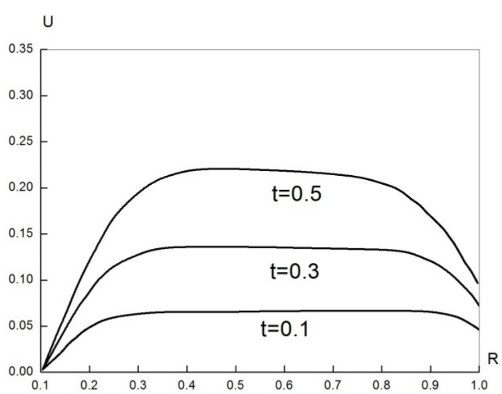

Fig.7 Show radial displacement distribution ($m = 0.0$) for problem II (Case A)

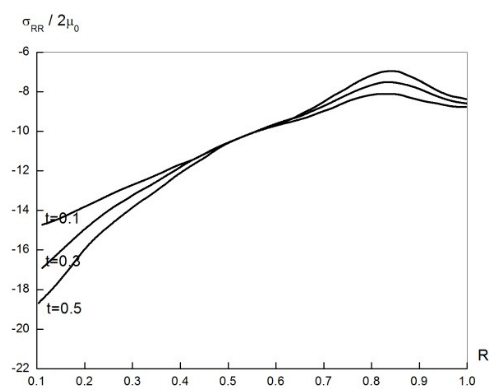

Fig.8 Show radial stress distribution ($m = 0.5$) for problem II (Case A)

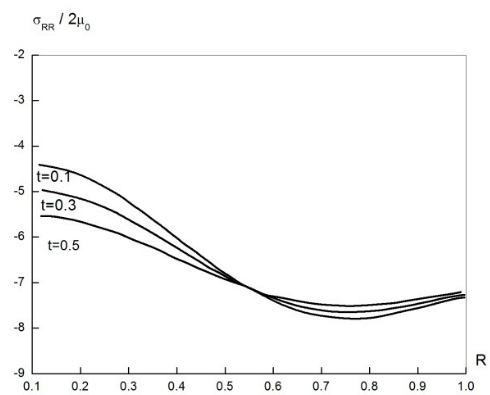

Fig. 9 Show radial stress distribution ($m = 0.0$) for problem II (Case A)

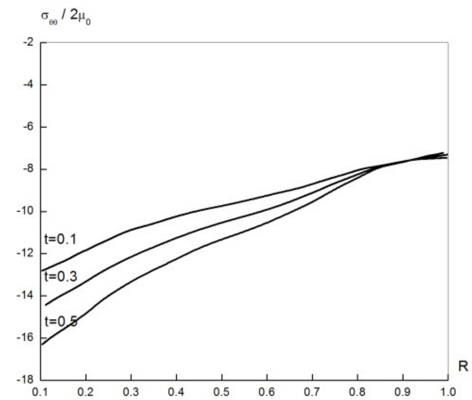

Fig. 10 Show tangential stress distribution ($m = 0.5$) for problem II (Case A)

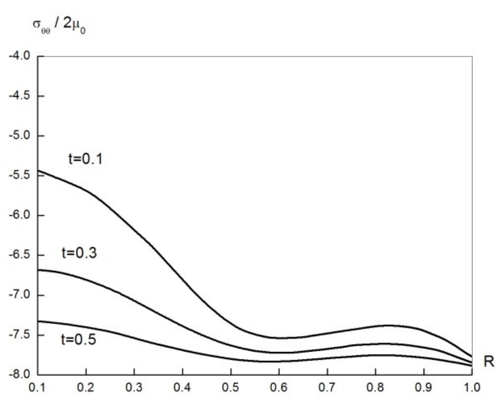

Fig. 11 Show tangential stress distribution ($m = 0.0$) for problem II (Case A)

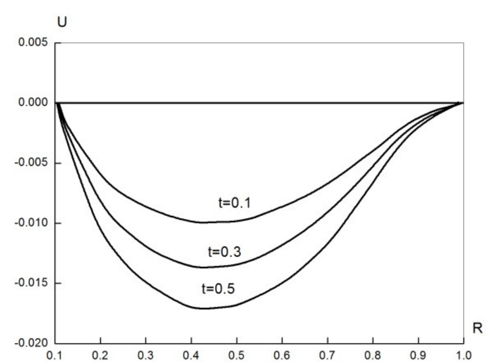

Fig. 12 Show radial displacement distribution ($m = 0.5$) for problem II (Case B)

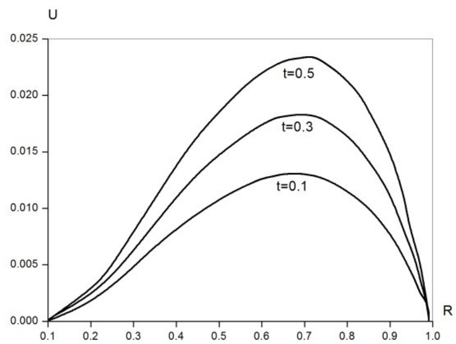

Fig. 13 Show radial displacement distribution ($m = 0.0$) for problem II (Case B)

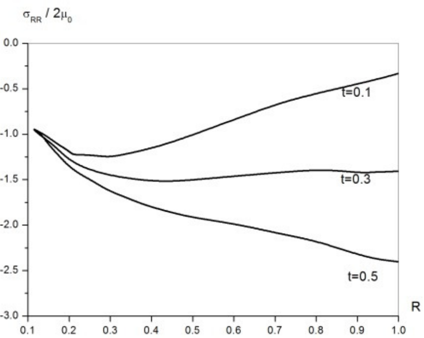

Fig. 14 Show radial stress distribution ($m = 0.5$) for problem II (Case B)

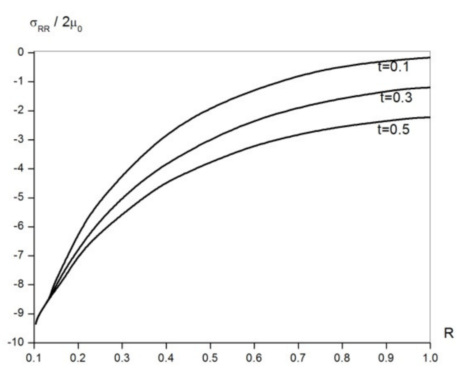

Fig. 15 Show radial stress distribution ($m = 0.0$) for problem II (Case B)

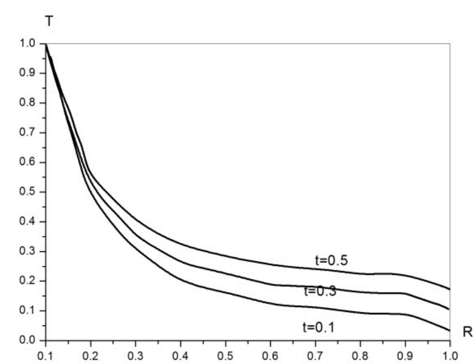

Fig. 16 Show temperature distribution for problem III

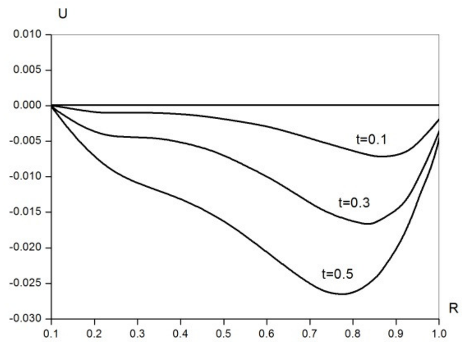

Fig. 17 Show radial displacement distribution ($m = 0.5$) for problem III

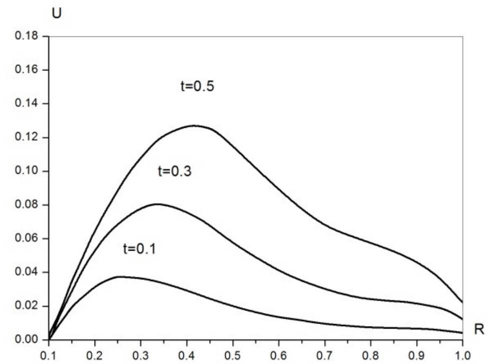

Fig. 18 Show radial displacement distribution ($m = 0.0$) for problem III

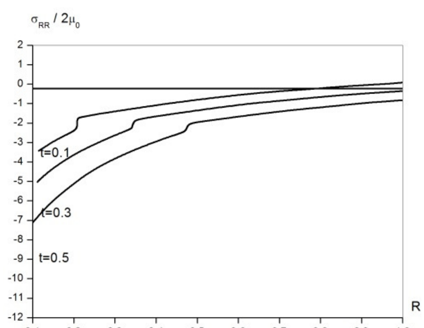

Fig. 19 Show radial stress distribution ($m = 0.5$) for problem III

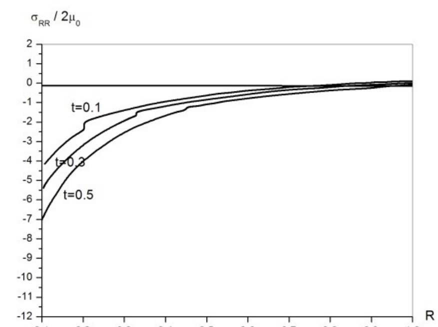

Fig. 20 Show radial stress distribution ($m = 0.0$) for problem III

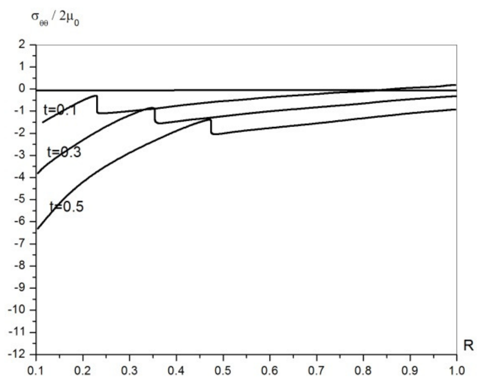

Fig. 21 Show tangential stress distribution ($m = 0.5$) for problem III

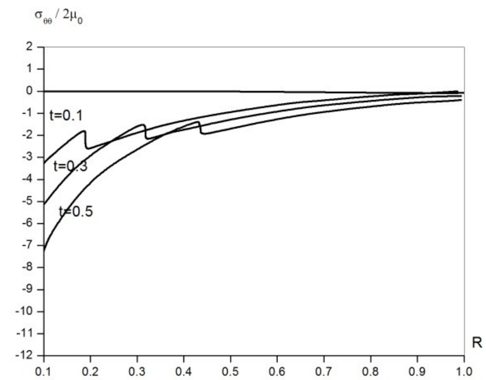

Fig. 22 Show tangential stress distribution ($m = 0.0$) for problem III

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