Int. Journal of Math. Analysis, Vol. 4, 2010, no. 33, 1599 - 1606

On $\alpha\psi$-Closed Sets in Bi-$\check{C}$ech Closure Spaces

R. Devi

Department of Mathematics
Kongunadu Arts and Science College
Coimbatore-641029, India
rdevicbe@yahoo.com

M. Parimala

Department of Mathematics
Bannari Amman Institute of Technology
Sathyamangalam-638401
rishwanthpari@gmail.com

Abstract

In this paper, we introduce the concept of $\alpha\psi$-closed (resp.$\alpha\psi$-open) sets in bi-$\check{C}$ech closure space and some characterization and properties are investigated. Further, the concept of $_{\alpha\psi}C_0$ bi-$\check{C}$ech spaces and $_{\alpha\psi}C_1$ bi-$\check{C}$ech spaces are introduced and their basic properties are studied.

Mathematics Subject Classification: 54A05, 54D10, 54F65, 54G05

Keywords: bi-$\check{C}$ech closure operator, bi-$\check{C}$ech closure spaces, bi-$\check{C}$ech-$\alpha\psi$ closed sets, bi-$\check{C}$ech-$\alpha\psi$ open sets, $_{\alpha\psi}C_0$ bi-$\check{C}$ech spaces and $_{\alpha\psi}C_1$ bi-$\check{C}$ech spaces

1. Introduction

$\check{C}$ech closure spaces were introduced by $\check{C}$ech [1] and then studied by many authors, see e.g. [4,5,7,8]. In $\check{C}$ech's approach operator satisfies idempotent condition among kuratowski axioms. This condition need not hold for every set A of x when this condition is also true, The operator becomes topological closure operator. Thus the concept of closure space is the generalization of a topological space. R.Devi [6] et al. introduced the concept of $\alpha\psi$- closed set to investigate some topological properties. This paper deals with the concept of $\check{C}$ech - $\alpha\psi$ closed sets, ${}_{\alpha\psi}C_0$ bi-$\check{C}$ech spaces, ${}_{\alpha\psi}C_1$ bi-$\check{C}$ech spaces and some of their properties.

2. Preliminaries

Definition 2.1. [2] Two functions k_1 and k_2 from power set X to itself are called bi-$\check{C}$ech closure operators (simply biclosure operator) for X if they satisfies the following properties.

(i) $k_1(\phi) = \phi$ and $k_2(\phi) = \phi$

(ii) $A \subset k_1(A)$ and $A \subset k_2(A)$ for any set $A \subset X$

(iii) $k_1(A \cup B) = k_1(A) \cup k_1(B)$ and $k_2(A \cup B) = k_2(A) \cup k_2(B)$ for any $A, B \subset X$

(X, k_1, k_2) is called bi-$\check{C}$ech closure space.

Example 2.2. Let $X = \{a, b, c\}$ and define a closure operator k_1 on X by $k_1(\{a\}) = \{a\}$, $k_1(\{b\}) = \{b, c\}$, $k_1(\{c\}) = k_1(\{a, c\})=\{\text{a, c}\}$, $k_1(\{a, b\}) = k_1(\{b, c\}) = k_1(\{X\}) = X$, $k_1(\phi) = \phi$. Define a closure operator k_2 on X by $k_2(\{a\}) = \{a\}$, $k_2(\{b\}) = k_2(\{c\}) = k_2(\{b, c\}) = \{b, c\}$, $k_2(\{a, b\}) = k_2(\{a, c\}) = k_2(\{X\}) = X$, $k_2(\phi) = \phi$. Now, (X, k_1, k_2) is a bi-$\check{C}$ech closure space.

DEFINITION 2.3 [3] A subset A in a bi-$\check{C}$ech closure space (X, k_1, k_2) is said to be

1. k_i-*regular open* if $A = int_{ki}(k_i(A))$, $i = 1, 2$
2. k_i-*regular closed* if $A = k_i(int_{ki}(A))$, $i = 1, 2$
3. k_i-*semi open* if $A \subseteq k_i(int_{ki}(A))$, $i = 1, 2$
4. k_i-*semi closed* if $int_{ki}(k_i(A)) \subseteq A$, $i = 1, 2$
5. k_i-*pre open* if $A \subseteq int_{ki}(k_i(A))$, $i = 1, 2$
6. k_i-*pre closed* if $k_i(int_{ki}(A)) \subseteq A$, $i = 1, 2$
7. k_i-α *open* if $A \subseteq int_{ki}(k_i(int_{ki}(A)))$, $i = 1, 2$ and
8. k_i-α *closed* if $k_i(int_{ki}(k_i(A))) \subseteq A$, $i = 1, 2$.

DEFINITION 2.4 A subset A in a bi-$\check{C}$ech closure space (X, k_1, k_2) is said to be

1. a (k_1, k_2)-*semi generalized closed* set if $k_{scl2}(A) \subseteq U$ whenever $A \subseteq U$ and U is k_1-*semi open* in (X, τ).

2. a (k_1, k_2)-ψ *closed* set if $k_{scl2}(A) \subseteq U$ whenever $A \subseteq U$ and U is k_1-*sg open* in (X, τ).

3. (k_1, k_2) - $\alpha\psi$ closed sets

Definition 3.1. A subset A is a bi-Čech closure space (X, k_1, k_2) is said to be (k_1, k_2)-$\alpha\psi$ closed if $k_{\psi 2}(A) \subseteq U$ whenever $A \subseteq U$ and U is k_1 - α open set in X.

Example 3.2. In example 2.2 when U=$\{b, c\}$, A=$\{b\}$ is (k_1, k_2)-$\alpha\psi$ closed.

Theorem 3.3. If A and B are (k_1, k_2)-$\alpha\psi$ closed sets and so is $A \cup B$.
Proof. Let A and B be two (k_1, k_2)-$\alpha\psi$ closed sets. Let U be k_1 - α open set in X. Let $(A \cup B) \subseteq U$, we have $A \subseteq U$ and $B \subseteq U$. Then $k_{\psi 2}(A) \subseteq U$ and $k_{\psi 2}(B) \subseteq U$ implies $(k_{\psi 2}(A) \cup k_{\psi 2}(B)) \subseteq U$. Hence $k_{\psi 2}(A \cup B) \subseteq U$. Thus $A \cup B$ is (k_1, k_2)-$\alpha\psi$ closed set.

Theorem 3.4. If A is (k_1, k_2)-$\alpha\psi$ closed set,Then $k_{\psi 2}(A)$-A contains no non-empty k_1-α closed sets.
Proof. Let A be (k_1, k_2)-$\alpha\psi$ closed. Let U be k_1-α closed contained in $k_{\psi 2}(A)$-A. Now,

$$U \subseteq k_{\psi 2}(A) \text{ and } U \subseteq A^c. \tag{1}$$

Now, $U \subseteq A^c$ then $A \subseteq U^c$. Since U is k_1-α closed, U^c is k_1-α open. Thus we have, $k_{\psi 2}(A) \subseteq U^c$. Consequently,

$$U \subseteq [k_{\psi 2}(A)]^c. \tag{2}$$

From (1) and (2), $U \subseteq k_{\psi 2}(A) \cap [k_{\psi 2}(A)]^c$=$\phi$. Therefore, $U = \phi$. Hence $k_{\psi 2}(A)$-A contains no non-empty k_1-α closed sets.

Theorem 3.5. If A is (k_1, k_2)-$\alpha\psi$ closed set, then $k_{\psi 1}(x) \cap A$=ϕ holds for each $x \in k_{\psi 2}(A)$.
Proof. Let A be (k_1, k_2)-$\alpha\psi$ closed set. Suppose $k_{\psi 1}(x) \cap A$=ϕ, for some $x \in k_{\psi 2}(A)$, we have $A \subseteq [k_{\psi 1}(x)]^c$. Now, $k_{\psi 1}(x)$ is k_1-ψ closed. Therefore $[k_{\psi 1}(x)]^c$ is k_1-ψ open. Thus $[k_{\psi 1}(x)]^c$ is k_1-α open. Since A is (k_1, k_2)-$\alpha\psi$ closed set, we have $k_{\psi 2}(A) \subseteq [k_{\psi 1}(x)]^c$ implies $k_{\psi 2}(A) \cap k_{\psi 1}(x)$=$\phi$. Then $x \notin k_{\psi 2}(A)$ is a contradiction. Hence $k_{\psi 1}(x) \cap A$=ϕ holds for each $x \in k_{\psi 2}(A)$.

Theorem 3.6. Let (X, k_1, k_2) be bi-$\check{C}$ech closure space. For each x in X, $\{x\}$ is k_1-α closed or $\{x\}^c$ is (k_1, k_2)-$\alpha\psi$ closed set.
Proof. Let (X, k_1, k_2) be bi-$\check{C}$ech closure space. Suppose that $\{x\}$ is not k_1-α closed, $\{x\}^c$ is not k_1-α open. Therefore, the only k_1-α open set containing $\{x\}^c$ is X. Thus, $\{x\}^c \subset X$. Now, $k_{\psi 2}[\{x\}^c] \subseteq k_{\psi 2}(X)$=$X$. Hence $\{x\}^c$ is (k_1, k_2)-$\alpha\psi$ closed set.

Theorem 3.7. Let A be (k_1, k_2)-$\alpha\psi$ closed set and if A is k_1-α open then $A = k_{\psi 2}(A)$.
Proof. Let A be (k_1, k_2)-$\alpha\psi$ closed subset of a bicech closure spaces (X, k_1, k_2) and let A be k_1-α open set. Then $k_{\psi 2}(A) \subseteq U$ whenever $A \subseteq U$ and U is k_1-α open set in X. Since A is k_1-α open and $A \subseteq A$, we have $k_{\psi 2}(A) \subseteq A$. But always, $A \subseteq k_{\psi 2}(A)$. Thus, $A = k_{\psi 2}(A)$.

Theorem 3.8. Let $A \subseteq Y \subseteq X$ and suppose that A is (k_1, k_2)-$\alpha\psi$ closed in (X, k_1, k_2). Then A is (k_1, k_2)-$\alpha\psi$ closed relative to Y.
Proof. Let S be any k_1-α open set in Y such that $A \subseteq S$. Then $S = U \cap Y$ for some U is k_1-α open in X. Therefore $A \subset U \cap Y$ implies $A \subseteq U$. Since A is (k_1, k_2)-$\alpha\psi$ closed set in X, we have $k_{\psi 2}(A) \subseteq U$. Hence $Y \cap k_{\psi 2}(A) \subseteq Y \cap U = S$. Thus A is (k_1, k_2)-$\alpha\psi$ closed set relative to Y.

4. (k_1, k_2)-$\alpha\psi$ open sets

Definition 4.1. A subset A in bi-$\check{C}$ech closure space (X, k_1, k_2) is called (k_1, k_2)-$\alpha\psi$ open set if A^c is (k_1, k_2)-$\alpha\psi$ closed in (X, k_1, k_2).

Example 4.2. In example 2.2, when $U = \{b, c\}$, $A = \{a, c\}$ is (k_1, k_2)-$\alpha\psi$ open set.
Theorem 4.3. A subset A of (X, k_1, k_2) is (k_1, k_2)-$\alpha\psi$ open set if and only if $F \subset (int_{k\psi 2}(A))$ whenever F is k_1-α closed set and $F \subseteq A$.
Proof. Suppose A is (k_1, k_2)-$\alpha\psi$ open in (X, k_1, k_2). Let F be k_1-α closed set and $F \subseteq A$. Then F^c is k_1-α open set and $A^c \subseteq F^c$. Since A^c is (k_1, k_2)- $\alpha\psi$ closed set, we have $k_{\psi 2}(A^c) \subseteq F^c$. Implies $F \subseteq [k_{\psi 2}(A^c)]^c = int_{k_{\psi 2}}(A)$. That is $F \subseteq int_{k_{\psi 2}}(A)$ whenever F is k_1-α closed set and $F \subseteq A$. Let V be any k_1-α open set in X such that $A^c \subseteq V$. Thus $V^c \subseteq A$ and V^c is k_1-α closed. Therefore $V^c \subseteq int_{k_{\psi 2}}(A)$. Then $[int_{k_{\psi 2}}(A)]^c \subseteq V$. Implies $k_{\psi 2}(A^c) \subseteq V$ gives A^c is (k_1, k_2)-$\alpha\psi$ closed set. Thus A is (k_1, k_2)-$\alpha\psi$ open set.

Corollary 4.4. A subset A of (X, k_1, k_2) is (k_1, k_2)-$\alpha\psi$ closed set, then $k_{\psi 2}(A) - A$ is (k_1, k_2)-$\alpha\psi$ open set.

Proof. Let F be k_1-α closed set such that $F \subseteq k_{\psi 2}(A) - A$. $F = \phi$ (by Theorem 3.7). Therefore $F \subseteq int_{\psi 2}(k_{\psi 2}(A) - A)$. Hence $k_{\psi 2}(A) - A$ is (k_1, k_2)-$\alpha\psi$ open set.

Theorem 4.5. If A and B and (k_1, k_2)-$\alpha\psi$ open sets, then so is $A \cap B$.
proof. Let $(A^c \cup B^c) \subseteq U$ where U is k_1-α open. This implies $A^c \subseteq U$ and $B^c \subseteq U$, gives $k_{\psi 2}(A^c) \subseteq U$ and $k_{\psi 2}(B^c) \subseteq U$. Thus $(k_{\psi 2}(A^c) \cup k_{\psi 2}(B^c)) \subseteq U$. Thus $k_{\psi 2}(A^c \cup B^c) \subseteq U$. Therefore $A \cap B$ is (k_1, k_2)-$\alpha\psi$ open set.

Theorem 4.6. Let $A \subseteq Y \subseteq X$ and suppose that Y is k_2-ψ closed in X and A is (k_1, k_2)-$\alpha\psi$ open in X, then A is (k_1, k_2)-$\alpha\psi$ open set relative to Y.
Proof. Let S be any k_1-α closed set in Y such that $S \subseteq A$. Then $S = U \cap Y$ for some U is k_1-α closed in X. Therefore $U \cap Y \subset A$ implies $U \subseteq A$. Since A is (k_1, k_2)-$\alpha\psi$ open set in X, we have $U \subseteq int_{k\psi 2}(A)$. Hence $S = Y \cap U \subseteq Y \cap\ int_{k\psi 2}(A)$. Thus A is (k_1, k_2)-$\alpha\psi$ open set relative to Y.

5. ${}_{\alpha\psi}C_0$ bi-$\check{C}$ech spaces and ${}_{\alpha\psi}C_1$ bi-$\check{C}$ech spaces

Definition 5.1. A bi-$\check{C}$ech closure space (X, k_1, k_2) is said to be a ${}_{\alpha\psi}C_0$ bi-$\check{C}$ech space if for every $\alpha\psi$-open subset U of (X, k_1), $x \in U$ implies $k_2(\{x\}) \subseteq U$.

Example 5.2. Let $X = \{a, b, c\}$ and define a closure operator k_1 on X by $k_1(\{\phi\}) = \phi$, $k_1(\{a\}) = \{a\}$, $k_1(\{b\}) = k_1(\{c\}) = k_1(\{b, c\}) = \{b, c\}$ and $k_1(\{a, b\}) = k_1(\{a, c\}) = k_1(X) = X$. Define a closure operator k_2 on X by $k_2(\{\phi\}) = \phi$, $k_2(\{a\}) = \{a\}$, $k_2(\{b\}) = \{b, c\}$, $k_2(\{c\}) = k_2(\{a, c\}) = \{a, c\}$ and $k_2(\{a, b\}) = k_2(\{b, c\}) = k_2(X) = X$. Then (X, k_1, k_2) is a ${}_{\alpha\psi}C_0$ bi-$\check{C}$ech space.

Theorem 5.3. *A bi-$\check{C}$ech closure space (X, k_1, k_2) is a ${}_{\alpha\psi}C_0$ bi-$\check{C}$ech space if and only if for every $\alpha\psi$-closed subset F of (X, k_1) such that $x \notin F$, $k_2(\{x\}) \cap F = \phi$.*
Proof. Let F be a $\alpha\psi$-closed subset of (X, k_1) and let $x \notin F$. Since $x \in X - F$ and $X - F$ is a $\alpha\psi$-open subset of (X, k_1), $k_2(\{x\}) \subseteq X - F$. Consequently $k_2(\{x\}) \cap F = \phi$.

Conversely, let U be a $\alpha\psi$-open subset of (X, k_1) and let $x \in U$. Since $X - U$ is a $\alpha\psi$-closed subset of (X, k_1) and $x \notin X - U$, $k_2(\{x\}) \cap (X - U) = \phi$. Consequently $k_2(\{x\}) \subseteq U$. Hence, (X, k_1, k_2) is a ${}_{\alpha\psi}C_0$ bi-$\check{C}$ech space.

Definition 5.4. A bi-$\check{C}$ech closure space (X, k_1, k_2) is said to be ${}_{\alpha\psi}C_1$ bi-$\check{C}$ech space if for each $x, y \in X$ such that $k_1(\{x\}) \neq k_2(\{y\})$, there exist a disjoint

$\alpha\psi$-open subset U of (X, k_2) and a $\alpha\psi$-open subset V of (X, k_1) such that $k_1(\{x\}) \subseteq U$ and $k_2(\{y\}) \subseteq V$.

Example 5.5. Let $X = \{a, b\}$ and define a closure operator k_1 on X by $k_1(\{\phi\}) = \phi$ and $k_1(\{a\}) = k_1(X) = X$. Define a closure operator k_2 on X by $k_2(\{\phi\}) = \phi$ and $k_2(\{b\}) = k_2(X) = X$. Then (X, k_1, k_2) is a ${}_{\alpha\psi}C_1$ bi-Čech space.

Theorem 5.6. *Every ${}_{\alpha\psi}C_1$ bi-Čech space is a ${}_{\alpha\psi}C_0$ bi-Čech space.*
Proof. Let (X, k_1, k_2) be a ${}_{\alpha\psi}C_1$ bi-Čech space. Let U be a $\alpha\psi$-open subset of (X, k_1) and let $x \in U$. If $y \notin U$, then $k_2(\{x\}) \neq k_1(\{y\})$ because $x \notin k_1(\{y\})$. Then there exists a $\alpha\psi$-open subset V_y of (X, k_2) such that $k_1(\{y\}) \subseteq V_y$ and $x \notin V_y$, which implies $y \notin k_2(\{x\})$. Consequently, $k_2(\{y\}) \subseteq U$. Hence, (X, k_1, k_2) is a ${}_{\alpha\psi}C_0$ bi-Čech space.

The converse of the above theorem need not be true by the following example.

Example 5.7. Let $X = \{a, b\}$ and define a closure operator k_1 on X by $k_1(\{\phi\}) = \phi$ and $k_1(\{a\}) = k_1(X) = X$. Define a closure operator k_2 on X by $k_2(\{\phi\}) = \phi$, $k_2(\{a\}) = \{a\}$ and $k_2(\{b\}) = k_2(X) = X$. Then (X, k_1, k_2) is a ${}_{\alpha\psi}C_0$ bi-Čech space but it is not a ${}_{\alpha\psi}C_1$ bi-Čech space.

Theorem 5.8. *A bi-Čech closure space (X, k_1, k_2) is a ${}_{\alpha\psi}C_1$ bi-Čech space if and only if every pair of points x, y of (X, k_1, k_2) such that $k_1(\{x\}) \neq k_2(\{y\})$, there exists a $\alpha\psi$-open subset U of (X, k_1) and $\alpha\psi$-open subset V of (X, k_2) such that $x \in V$, $y \in U$ and $U \cap V = \phi$.*
Proof. Suppose that (X, k_1, k_2) is a ${}_{\alpha\psi}C_1$ bi-Čech space. Let x, y be points of (X, k_1, k_2) such that $k_1(\{x\}) \neq k_2(\{y\})$. There exists a $\alpha\psi$-open subset U of (X, k_1) and $\alpha\psi$-open subset V of (X, k_2) such that $x \in k_1(\{x\}) \subseteq V$ and $y \in k_2(\{y\}) \subseteq U$.

Conversely, suppose that there exist a $\alpha\psi$-open subset U of (X, k_1) and $\alpha\psi$-open subset V of (X, k_2) such that $x \in V$, $y \in U$ and $U \cap V = \phi$. Since every ${}_{\alpha\psi}C_1$ bi-Čech space is a ${}_{\alpha\psi}C_0$ bi-Čech space, $k_1(\{x\}) \subseteq V$ and $k_2(\{y\}) \subseteq U$.

Theorem 5.9. Let $\{(X_i, k_i^1, k_i^2) : i \in I\}$ be a family of bi-Čech closure spaces. If $\prod_{i\in I}(X_i, k_i^1, k_i^2)$ is an ${}_{\alpha\psi}C_0$ bi-Čech space, then (X_i, k_i^1, k_i^2) is an ${}_{\alpha\psi}C_0$ bi-Čech space for each $i \in I$.
Proof. Suppose that $\prod_{i\in I}(X_i, k_i^1, k_i^2)$ is an ${}_{\alpha\psi}C_0$ bi-Čech space. Let $j \in I$ and

let G be an $\alpha\psi$-open subset of (X_j, k_j^1) such that $x_j \in G$. Then $G \times \prod_{\substack{i \neq j \\ i \in I}} X_i$ is an $\alpha\psi$-open subset of $\prod_{i \in I}(X_i, k_i^1)$ such that $(x_i)_{i \in I} \in G \times \prod_{\substack{i \neq j \\ i \in I}} X_i$. Since $\prod_{i \in I}(X_i, k_i^1, k_i^2)$ is an ${}_{\alpha\psi}C_0$ bi-$\check{C}$ech space, $\prod_{i \in I} k_i^2 \pi_i(\{(x_i)_{i \in I}\}) \subseteq G \times \prod_{\substack{i \neq j \\ i \in I}} X_i$. Consequently, $k_j^2\{x_j\} \subseteq G$. Hence, (X_i, k_i^1, k_i^2) is an ${}_{\alpha\psi}C_0$ bi-$\check{C}$ech space.

Theorem 5.10. Let $\{(X_i, k_i^1, k_i^2) : i \in I\}$ be a family of bi-$\check{C}$ech closure spaces. If (X_i, k_i^1, k_i^2) is an ${}_{\alpha\psi}C_1$ bi-$\check{C}$ech space for each $i \in I$, then $\prod_{i \in I}(X_i, k_i^1, k_i^2)$ is an ${}_{\alpha\psi}C_1$ bi-$\check{C}$ech space .

Proof. Suppose that (X_i, k_i^1, k_i^2) is an ${}_{\alpha\psi}C_1$ bi-$\check{C}$ech space for each $i \in I$. Let $(x_i)_{i \in I}$ and $(y_i)_{i \in I}$ be points of $\prod_{i \in I} X_i$ such that $\prod_{i \in I} k_i^1 \pi_i(\{(x_i)_{i \in I}\}) \neq \prod_{i \in I} k_i^2 \pi_i(\{(y_i)_{i \in I}\})$. There exists $j \in I$ such that $k_j^1\{x_j\} \neq k_j^2\{y_j\}$. Since (X_j, k_j^1, k_j^2) is a ${}_{\alpha\psi}C_1$ bi-$\check{C}$ech space, there exist an $\alpha\psi$-open subset U of (X_j, k_j^1) and an $\alpha\psi$-open subset V of (X_j, k_j^2) such that $U \cap V = \phi$, $k_j^2\{y_j\} \subseteq U$ and $k_j^1\{x_j\} \subseteq V$. Consequently, $\prod_{i \in I} k_i^2 \pi_i(\{(y_i)_{i \in I}\}) \subseteq U \times \prod_{\substack{i \neq j \\ i \in I}} X_i$ and $\prod_{i \in I} k_i^1 \pi_i(\{(x_i)_{i \in I}\}) \subseteq V \times \prod_{\substack{i \neq j \\ i \in I}} X_i$ such that $U \times \prod_{\substack{i \neq j \\ i \in I}} X_i$ is an $\alpha\psi$-open subset of $\prod_{i \in I}(X_i, k_i^1)$, $V \times \prod_{\substack{i \neq j \\ i \in I}} X_i$ is an $\alpha\psi$-open subset of $\prod_{i \in I}(X_i, k_i^2)$ and $(U \times \prod_{\substack{i \neq j \\ i \in I}} X_i) \cap (V \times \prod_{\substack{i \neq j \\ i \in I}} X_i) = \phi$. Hence, $\prod_{i \in I}(X_i, k_i^1, k_i^2)$ is an ${}_{\alpha\psi}C_1$ bi-$\check{C}$ech space .

References

[1] E. Cech, Topological Spaces, *Inter Science Publishers, john Wiley nd Sons*, NewYork (1966).

[2] K. Chandrasekhara Rao and R. Gowri, On biclosure spaces, *Bulletine of pure and applied sciences*, 25E (2006), 171-175.

[3] K. Chandrasekhara Rao and R. Gowri, Regular generalised closed sets in biclosure spaces, *Jr. of institute of mathematics and computer science*,(Math. Ser.), 19(3) (2006), 283-286.

[4] J. Chvalina, On homeomorphic topologies and equivalent set-systems, *Arch. Math. (Brno)*, 12(2) (1976), 107-115.

[5] J. Chvalina, Stackbases in power sets of neighbourhood spaces preserving the continuity of mappings, *Arch. Math. (Brno)*, 17(2) (1981), 81-86.

[6] R. Devi, A. Selvakumar and M.Parimala, $\alpha\psi$-closed sets in topological spaces, (Submitted).

[7] L. Skula, System von stetigen Abbildungen, *Czechoslovak Math. J*, 17(92) (1967),45-52.

[8] J. Slapal, Closure operations for digital topology, *Theoret. Comput. Sci*, 305(1-3) (2003), 457-471.

Received: January, 2010