Some Results on IF Generalized Minimal Closed Set

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Abstract

The purpose of this paper is to introduce the new concept of IF generalized minimal closed set which implies IF generalized closed set. Then another concept of IF generalized * minimal closed set is introduced which is not coinciding with IF dense set, while other IF generalized * closed set concept are coinciding with the corresponding dense set. It can also be shown that this new structure is a weaker form of intuitionistic fuzzy minimal open set. The various properties of this newly formed set are studied and the corresponding topological structure is discussed in this paper. It is to be noticed that the collection of IF generalized minimal closed set forms an Alexandroff space if X is included but the collection of IF generalized*minimal closed set doesn’t forms a supra topological space. Also the connection of this set with some other sets is studied in this paper.

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1: Introduction & Preliminaries

The concept of fuzzy sets was introduced by Zadeh [28] and later Atanassov [2] generalized this idea to intuitionistic fuzzy sets. But a contradiction arose with the concept of Intuitionistic Logic. The concept of Intuitionistic Logic is not similar with Intuitionistic fuzzy logic and hence to avoid this contradiction various Researchers
suggested various nomenclatures. Following some of their suggestion we are using the nomenclature as IF Set in place of Intuitionistic fuzzy set.

Coker [13] introduced the notions of IF topological space and other related concepts. The initiation of the study of generalized closed sets was done by N. Levine in 1970[17] as he considered sets whose closure belongs to every open super set. He called them generalized closed sets and studied their most fundamental properties. Later on the concept of fuzzy generalized closed set has been introduced by S.S.Thakur and R.Malviya in 1995[26].Lots of researchers [4], [5], [6], [12], [14], [15], [17],[20],[22],[23],[24],[25] has studied various concepts of generalized closed set in ordinary topology and in fuzzy topological space.

The concept of minimal open set has been introduced by F Nakaoka and N Oda [21] in 2001 The concept of IF generalized minimal open set has been introduced by the author [9]. The concept of generalized * closed set has been introduced by the author in 2008[7] and the concept of dense mX set has been introduced by the author [8]. The aim of this paper is to study IF generalized* minimal closed set. It can be shown that IF generalized * minimal closed set is not coinciding with any IF dense set. It is an independent concept. It can also be shown that if a set is not a rare set then it is not an independent concept but it coincides with the concept of IF minimal open set.

In section 2 some preliminaries related to the topic is given.
In section 3 the concept of IF generalized * minimal open set is studied .Some properties of this set is studied and also connection of this set with some other set is introduced in this section of the paper. The topological space obtained by the collection of this set is also studied in this section of the paper
Lastly some of the applications of this space are shown.

Now let us recall some of the definitions and theorems related to intuitionistic fuzzy topology and ordinary topological space.

**Definition 1.1**[1] Let X be a nonempty set and I the unit interval [0,1]. An IF set U is an object having the form U={<x, \(\mu_u(x)\), \(\gamma_u(x)\)>:x \(\in\) X}where the functions \(\mu_u:X \rightarrow I\) and \(\gamma_u:X \rightarrow I\) denote respectively the degree of membership and degree of non-membership of each element x \(\in\) X to the set U, and \(0 \leq \mu_u(x) + \gamma_u(x) \leq 1\) for each x \(\in\) X.

**Lemma 1.2**[1] Let X be a nonempty set and let IF Set’s U and V be in the form U = \{<x,\(\mu_u(x)\),\(\gamma_u(x)\)>:x \(\in\) X\} and
V = \{<x,\(\mu_v(x)\),\(\gamma_v(x)\)>:x \(\in\) X\}.Then
1. U^c = \{<x,\(\gamma_u(x)\), \(\mu_u(x)\)>: x \(\in\) X\}
2. U \cap V = \{<x, \Lambda (\mu_u(x), \mu_v(x)), \vee (\gamma_u(x), \gamma_v(x))>: x \(\in\) X\}
3. U \cup V = \{<x, \vee (\mu_u(x), \mu_v(x)), \Lambda (\gamma_u(x), \gamma_v(x))>: x \(\in\) X\}
4. 1_\epsilon = \{<x, 1,0>: x \(\in\) X\}, 0_\epsilon = \{<x, 0,1>: x \(\in\) X\}
5. (U^c)^c = U, 0_\epsilon^c = 1_\epsilon, 1_\epsilon^c = 0_\epsilon.

**Definition 1.3**[3] An IF topology on a nonempty set X is a family \(\tau\) of IF Sets in X containing 0_\epsilon,1_\epsilon and closed under arbitrary infimum and finite supremum.
In this case the pair \((X, \tau)\) is called an IF Topological Space and each IF Set in \(\tau\) is known as an IF Open Set. The complement \(U^C\) of an IF Open Set \(U\) in an IF Topological Space \((X, \tau)\) is called an IF Closed Set in \(X\).

**Definition 1.4**\[^{15}\] Let \((X, \tau)\) is a topological space. A family \(\tau\) of IF sets on \(X\) is called an IF supra-topological space on \(X\) if \(0 \in \tau, 1 \in \tau\) and \(\tau\) is closed under arbitrary supremum. Each member of \(\tau\) is called an IF supra-open set and compliment of an IF supra-open set is an IF supra-closed set.

**Definition 1.5**\[^{3}\] Let \((X, \tau)\) is an IF Topological Space and \(A\) an IF Set in \(X\). Then closure of \(A\) is defined by \(\text{Cl}(A) = \bigcap\{F: A \subseteq F, F \in \tau\}\).

and the fuzzy interior of \(A\) is defined by \(\text{Int}(A) = \bigcup\{G: A \supseteq G, G \in \tau\}\).

**Definition 1.6**\[^{12}\] A fuzzy subset \(A\) of \(X\) is a fuzzy generalized closed set if \(\text{Cl}(A) \subseteq H\) whenever \(A \subseteq H, H\) being a fuzzy open subset of \(X\).

**Definition 1.7**\[^{12}\] A fuzzy subset \(A\) of \(X\) is a fuzzy dense set if \(\text{Cl}(A) = 1\).

**Definition 1.8**\[^{13}\] A subset \(A\) of a family \(\tau\) of IF sets on \(X\) is called an IF minimal open set in \(X\) if an IF open set which is contained in \(A\) is either \(0\) or \(A\).

**Definition 1.9**\[^{13}\] An IF set is said to be an IF Maximal open set of \(\text{IFTS} (X, \tau)\) iff it is not contained in any other open set of \(\tau\).

**Definition 1.10**\[^{12}\] A fuzzy subset \(A\) of \(X\) is a fuzzy generalized closed set if \(\text{Cl}(A) \subseteq H\) whenever \(A \subseteq H, H\) being a fuzzy open subset of \(X\).

**Definition 1.11** \[^{14}\] Let \(m_X\) an IF \(m_X\)-structure on \(X\). An IF \(m_X\) open set is said to be an open \(m_X\) if \(m_X - \text{Int}(A) = A\).

**Definition 1.12**\[^{5}\] An IF set is said to be an IF semi open set of IF Topological Space \((X, \tau)\) iff \(A \subseteq \text{Int}(\text{Cl}(\text{Int}A))\).

**Definition 1.13**\[^{3}\] Let \(f\) be a map from set \(X\) to set \(Y\). Let \(A = \{<x, \mu_A(x), \gamma_A(x)>: x \in X\}\) be an IF Open Set in \(X\) and \(B = \{<y, \mu_B(y), \gamma_B(y)>: y \in Y\}\) be an IF Open Set in \(Y\). Then \(f^{-1}(B)\) is an IF Open Set in \(X\) defined by \(f^{-1}(B) = \{<x, f^{-1}(\mu_B(x)), f^{-1}(\gamma_B(x))>: x \in X\}\) and \(f(A)\) is an IFOS in \(Y\) defined by \(f(A) = \{<y, f(\mu_A(y)), 1-f(1-\gamma_A(y))>: y \in Y\}\).

**Definition 1.14**\[^{3}\] A map \(f: (X, \tau) \rightarrow (Y, \sigma)\) is said to be an IF continuous function from IF topological space \((X, \tau)\) to IF topological space \((Y, \sigma)\) iff \(f^{-1}(V)\) is an IF open set in \(X\) for every open set \(V\) of \(Y\).

Throughout this paper IF Topological Space’s are denoted by \((X, \tau)\) and \((Y, \sigma)\) and complement of a set \(A\) is denoted by \(A^C\).

### 2. Some results on IF generalized minimal open set

In this section the concept of IF generalized minimal open set is introduced and some of its properties are discussed. Lastly the IF topological structure obtained by the collection of this set is studied.
**Definition 2.1:** An IF set $A$ is said to be an IF generalized minimal closed set, if there exist at least one IF Minimal Open Set $U$ containing $A$ such that $\text{Cl}A \subseteq U$.

**Example 2.2:** Let $A = \{<x, 0.3, 0.2>, x \in X\}$ and $B = \{<x, 0.5, 0.3>, x \in X\}$ be two IF subsets of $X$.

Let the corresponding topological space be $\tau = \{\emptyset, 1, A, B, A \cup B, A \cap B\}$. Here $A \cap B$ is an IF Minimal Open Set of $\tau$.

Consider a set $C = \{<x, 0.2, 0.4>: x \in X\}$, then $C \subseteq A \cap B$ and $\text{Cl}(C) = \{<x, 0.2, 0.3>: x \in X\} \subseteq A \cap B$.

Hence $C$ is an IF generalized minimal closed set.

**Theorem 2.3:**
(1) Let $A \subseteq B \subseteq U$, where $U$ is an IF minimal open set. If $B$ is an IF generalized minimal closed set, then $A$ is also so.

(2) If $A \subseteq B \subseteq \text{Cl}A$ and $B$ is an IF generalized minimal closed set then $A$ is also so.

**Proof:**
(1) Let $B \subseteq U$, where $U$ is an IF minimal open set i.e. $A \subseteq B \subseteq U$. From definition as $B$ is an IF generalized minimal closed set $\text{Cl}B \subseteq U$ implies $\text{Cl}A \subseteq \text{Cl}B \subseteq U$ i.e. $A$ is also an IF generalized minimal closed set.

(2) Since $B$ is an IF generalized minimal closed set i.e. $B \subseteq U$ where $U$ is an IF minimal open set and from definition as $B$ is an IF generalized minimal closed set $\text{Cl}B \subseteq U$ implies $\text{Cl}A = \text{Cl}B \subseteq U$ i.e. $A$ is also an IF generalized minimal closed set.

**Theorem 2.4:** An IF set $A$ is IF generalized minimal closed and IF minimal open set then $A$ is an IF closed set. Conversely if $A$ be an IF closed set and an IF minimal open set then $A$ is an IF generalized minimal closed set.

**Proof:** Let if possible $A$ be an IF generalized minimal closed set i.e. there exist an IF minimal open set $U$ containing $A$ such that $\text{Cl}A \subseteq U$. Since $A$ itself is IF minimal open set $\text{Cl}A \subseteq A$. But we know that $\text{Cl}A \supseteq A$. Hence $\text{Cl}A = A$ i.e. $A$ is an IF closed set

Conversely, Let $A$ be an IF closed set and an IF minimal open set then from definition it is an IF generalized minimal closed set.

**Theorem 2.5:** Every IF generalized minimal closed set is either IF rare set or an IF minimal open set i.e. $A$ is an IF rare set or the IF minimal open set containing $A$ is an IF closed set i.e. $A$ is an IF closed set.

**Proof:** Let if possible $A$ be an IF generalized minimal closed set then there exist an IF minimal open set $U$ containing $A$ such that $\text{Cl}A$ is contained in $U$. From theorem 2.3, Let $A \subseteq B \subseteq U$, where $U$ is an IF minimal open set. If $B$ is an IF generalized minimal closed set, then $A$ is also so. We know that $\text{Int}A \subseteq A \subseteq U$, Since $A$ is IF generalized minimal closed set $\text{Int}A$ is also so, but $\text{Int}A$ is an IF open set and no non-null IF open set can be a proper subset of an IF minimal open set. So $\text{Int}A = 0$ or $\text{Int}A = A = U$ i.e. $A$ is either an IF rare set or an IF minimal open set.
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Now if \( \text{Int}A = A = U, \ \text{Cl} U = \text{Cl} \text{Int}A \subseteq \text{Cl}A \subseteq U \). But we know that \( U \subseteq \text{Cl}U \) i.e. \( U \) is an IF minimal closed set.

**Remark 2.6:** Converse of the above theorem need not be true which follows from the following Example:

Let \( A = \{<x, 0.3, 0.5>: x \in X\}, B = \{<x, 0.4, 0.6>: x \in X\} \), and the IF topological space is \( T = \{0, 1, A, B, A \cap B, A \cup B\} \). Here \( A \cap B \) is the IF minimal open set but not the IF generalized minimal closed set. Let \( C = \{<x, 0.5, 0.4>: x \in X\} \) be another IF set. \( C \) is an IF rare set but not IF generalized minimal closed set.

But if the IF minimal open set containing \( A \) is an IF closed set then obviously \( A \) is an IF generalized minimal closed set.

**Theorem 2.7:** Every IF generalized minimal closed set is an IF generalized closed set

**Proof:** Let \( A \) be an IF generalized minimal closed set then there exist an IF minimal open set \( U \) such that \( A \subseteq U \) implies \( \text{Cl} A \subseteq U \). Since \( U \) is an IF minimal open set \( U \subseteq O \) where \( O \) is an IF open set. Hence \( \text{Cl}A \subseteq U \subseteq O \) i.e. \( A \) is an IF generalized closed set.

**Remark 2.8:** Converse of the above theorem need not be true which follows from the following example:

Let \( A = \{<x, 0.4, 0.2>: x \in X\}, B = \{<x, 0.5, 0.4>: x \in X\} \), and the IF topological space is \( T = \{0, 1, A, B, A \cap B, A \cup B\} \). Here \( A \cap B \) is the IF minimal open set. Let \( C = \{<x, 0.4, 0.3>: x \in X\} \) be another IF set. \( C \) is not IF generalized minimal closed set but IF generalized closed set.

**Theorem 2.9:** Let \( A \) be any IF generalized minimal closed set then \( \Lambda(A) \subseteq U \), for any IF minimal open set \( U \) and hence either \( \Lambda(A) \) is not an IF open set or \( \Lambda(A) = U \).

**Proof:** Let \( A \) be an IF generalized minimal closed set then \( A \subseteq U \) implies \( \text{Cl} A \subseteq U \) for any IF minimal open set \( U \). Therefore \( \Lambda(A) \subseteq \text{Cl}(A) \subseteq \Lambda(U) = U \), since \( U \) is an IF minimal open set. Hence \( \Lambda(A) \subseteq U \) i.e. infimum of all IF open set containing \( A \) is less than the IF minimal open set but it is possible iff either \( \Lambda(A) \) is not an open set or \( \Lambda(A) = U \). Since \( \Lambda(A) \) cannot be null IF set and any IF open set cannot be less than the IF minimal open set. Also we know that arbitrary infimum of IF open set need not be IF open set. So \( \Lambda(A) \) may not be an IF open set if \( X \) is an arbitrary set. But if \( X \) is a collection of IF finite set then \( \Lambda(A) = U \).

**Theorem 2.10:** Let \( A \) be any IF generalized minimal closed set then \( \text{Cl}A \subseteq \Lambda(A) \) if the set \( X \) is finite

**Proof:** Since \( A \) is an IF generalized minimal closed set, \( A \subseteq U \) where \( U \) is an IF minimal open set then \( \text{Cl}A \subseteq U \). Therefore \( \text{Cl}A \subseteq U \). Since \( \Lambda(A) \) is the infimum of all IF open set containing \( A \) and the set \( X \) being finite \( \Lambda(A) \) is an IF open set.
Theorem 2.11:
(1) $0$ is an IF generalized minimal closed set but $1$ is not an IF generalized minimal closed set.
(2) Arbitrary union of IF generalized minimal closed set is an IF generalized minimal closed set.
(3) Arbitrary intersection of IF generalized minimal closed set is an IF generalized minimal closed set.
Proof: (1) is obvious.
To prove (2)
Let $(A_i : i \in I)$ be an arbitrary collection of IF generalized minimal closed set. Since in an IF topological space there exist a unique IF minimal open set. Let $U$ be the corresponding IF minimal open set. i.e. $A_i \subseteq U$, where $U$ is an IF minimal closed set, implies $\text{Cl}(A_i) \subseteq U$.
Therefore $\bigcup(A_i : i \in I) \subseteq U$, implies $\bigcup \text{Cl}(A_i : i \in I) \subseteq U$. But we know that $\text{Cl} \bigcup(A_i : i \in I) = \bigcup \text{Cl}(A_i : i \in I) \subseteq U$.
Thus arbitrary union of IF generalized minimal closed set is an IF generalized minimal closed set.
To Prove (3)
Let $(A_i : i \in I)$ be an arbitrary collection of IF generalized minimal closed set. Since in an IF topological space there exist a unique IF minimal open set. Let $U$ be the corresponding IF minimal open set. i.e. $A_i \subseteq U$, where $U$ is an IF minimal closed set, implies $\text{Cl}(A_i) \subseteq U$.
Obviously $\bigcap(A_i : i \in I) \subseteq U$, and thus $\text{Cl} \bigcap(A_i : i \in I) \subseteq \bigcap \text{Cl}(A_i : i \in I) \subseteq U$. Therefore arbitrary intersection of IF generalized minimal closed set is an IF generalized minimal closed set.
Remark 2.12: The arbitrary collection of IF generalized minimal closed set forms an IF alexandroff space if $X$ is included in the set. Let us denote the space as $(X, G^{T_{MC}})$ where $T$ is the IF topological space.
Theorem 2.13: In an IF $T_{1/2}$ space every generalized minimal closed set is also a closed set.
Definition 2.14: Let $f : (X,T_1) \to (Y,T_2)$ be a mapping such that inverse image of IF generalized minimal closed set in $T_2$ is an IF generalized minimal closed set in $T_1$. Then this mapping is called an IF generalized minimal closed continuous mapping.
Example 2.15: Let $A=\{<x, 0.3, 0.2>: x \in X\}$ and $B=\{<x, 0.5, 0.3>: x \in X\}$, $C=\{<x, 0.5, 0.3>: x \in X\}$, $D=\{<x, 0.3, 0.2>: x \in X\}$ be some IF subsets of $X$.
Let the corresponding topological space be $T_1 = \{0, 1, A, B, A \cup B, A \cap B\}$. Here $A \cap B$ is an IF Minimal Open Set of $T_1$. $T_2 = \{0, 1, C, D, C \cup D, C \cap D\}$ The IF minimal open set is itself IF generalized minimal closed set. Therefore $\{U: U \subseteq \text{IF minimal open set in } T_1\}$ is the collection of IF generalized minimal closed set in $T_1$. And $\{U: U \subseteq \text{IF minimal open set in } T_2\}$ is the collection of IF generalized minimal closed set.
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in T2. Let \( f : (X, T_1) \rightarrow (Y, T_2) \) be a mapping such that \( f(x) = x \), then \( f \) is an IF generalized minimal closed continuous function.

**Theorem 2.16:** A mapping \( f : (X, G^{T_1}_{\text{MC}}) \rightarrow (X, G^{T_2}_{\text{MC}}) \) be an IF continuous function iff \( f : (X, T_1) \rightarrow (Y, T_2) \) be an IF generalized minimal closed continuous function.

**Proof:** Since every IF generalized minimal closed set in \( T_1 \) or \( T_2 \) is an IF open set in \((X, G^{T_1}_{\text{MC}})\) or \((X, G^{T_2}_{\text{MC}})\). Thus the theorem.

**Definition 2.17:** Let \( f : (X, T_1) \rightarrow (Y, T_2) \) be a mapping such that inverse image of IF closed set in \( T_2 \) is an IF generalized minimal closed set in \( T_1 \). Then this mapping is called an IF generalized minimal continuous mapping.

**Example 2.18:** Let \( A = \{<x, 0.5, 0.2>, x \in X\} \) and \( B = \{<x, 0.6, 0.3>, x \in X\} \), be two IF subsets of \( X \).

Let the corresponding topological space be \( T_1 = \{0 \sim, 1 \sim, A, B, A \cup B, A \cap B\} \). Here \( A \cap B \) is an IF Minimal Open Set of \( T_1 \).

Here \( C = \{<x, 0.3, 0.5>, x \in X\} \) is the maximum IF generalized minimal closed set. Therefore \( \{U: U \subseteq C\} \) is the collection of IF generalized minimal closed set in \( T_1 \). Let \( f : (X, T_1) \rightarrow (X, T_1) \) be a mapping such that \( f(x) = x \), then \( f \) is an IF generalized minimal continuous function.

**Theorem 2.19:** Let \( f : (X, T_1) \rightarrow (Y, T_2) \) be an IF generalized minimal continuous function then it is an IF generalized continuous function

**Proof:** It is obvious from theorem 2.7.

**Remark 2.20:** Converse of the above theorem need not be true which follows from the following example:

Let \( A = \{<x, 0.4, 0.2>, x \in X\} \), \( B = \{<x, 0.5, 0.4>, x \in X\} \), and the IF topological space is \( T = \{0 \sim, 1 \sim, A, B, A \cap B, A \cup B\} \). Here \( A \cap B \) is the IF minimal open set. Let \( C = \{<x, 0.4, 0.3>, x \in X\} \) be another IF set. \( C \) is not IF generalized minimal closed set but IF generalized closed set. Let us consider a mapping \( f : (X, T_1) \rightarrow (Y, T_2) \) such that \( f^{-1}(x) = C \) for all \( x \) in \( T_2 \). Here \( f \) is IF generalized continuous but not IF generalized minimal continuous function.

**Theorem 2.21:** Let \( f : (X, T_1) \rightarrow (Y, T_2) \) be an IF generalized minimal continuous function then it is also an IF continuous function if \( X \) is an IF \( T_{1/2} \) space.

**Proof:** It is obvious from theorem 2.13.

**Theorem 2.22:** Let \( f : (X, T_1) \rightarrow (Y, T_2) \) be an IF generalized minimal continuous function then it is also an IF generalized minimal closed continuous function if \( Y \) is an IF \( T_{1/2} \) space

**Proof:** It is obvious from theorem 2.13.

**Theorem 2.23:** Let \( f : (X, T_1) \rightarrow (Y, T_2) \) be an IF generalized minimal continuous function and \( g : (Y, T_2) \rightarrow (Z, T_3) \) be an IF continuous function then \( g \circ f : (X, T_1) \rightarrow (Z, T_3) \) is an IF generalized minimal continuous function.

**Proof:** Here \((g \circ f)^{-1} = f^{-1}g^{-1}\). Now \( g \) is IF continuous function then \( g^{-1}(z) \) is an IF closed set whenever \( z \) is an IF closed set in \( Z \) and hence \( f^{-1}g^{-1}(x) \) is an IF generalized
minimal closed, since \( f \) is an IF generalized minimal continuous function. Hence inverse image of a IF closed set in \( Z \) is an IF generalized minimal closed set in \( X \). Thus \( (gof) \) is an IF generalized minimal continuous function.

3: Some results on IF generalized* minimal open set

In this section the concept of IF generalized* minimal open set is introduced and some theorems related to this newly constructed set are studied and also related properties are discussed.

**Definition 3.1:** An IF set \( B \) is said to be an IF generalized* minimal closed set, if there exist at least one IF Minimal Open Set \( A \) containing \( B \) such that \( \text{Cl}(B) \supseteq A \).

**Example 3.2:** Let \( A = \{<x, 0.2, 0.6>: x \in X\} \) and \( B = \{<x, 0.3, 0.7>: x \in X\} \) be two IF subsets of \( X \). Let the corresponding topological space be \( \tau = \{0\~, 1\~, A, B, A \cup B, A \cap B\} \). Here \( A \cap B \) is an IF Minimal Open Set of \( \tau \).

Consider a set \( C = \{<x, 0.1, 0.8>: x \in X\} \), then \( C \subseteq A \cap B \) and \( \text{Cl}(C) = \{<x, 0.6, 0.3>: x \in X\} \supseteq A \cap B \). Hence \( C \) is an IF generalized* minimal closed set.

**Theorem 3.3:**

1. Let \( A \subseteq B \subseteq U \), where \( U \) is an IF minimal open set. If \( A \) is an IF generalized* minimal closed set, then \( B \) is also so.
2. If \( A \subseteq B \subseteq \text{Cl}(A) \) and \( B \) is an IF generalized* minimal closed set then \( A \) is also so.

**Proof:** (1) Let \( B \subseteq U \), where \( U \) is an IF minimal open set

i.e. \( A \subseteq B \subseteq U \). From definition as \( A \) is an IF generalized* minimal open set \( \text{Cl}(A) \supseteq U \) implies \( \text{Cl}(B) \supseteq U \) (as \( \text{Cl}(B) \supseteq \text{Cl}(A) \))

i.e. \( B \) is also an IF generalized* minimal open set.

(2) Since \( B \) is an IF generalized* minimal open set

i.e. \( B \subseteq U \) where \( U \) is an IF minimal open set and from definition as \( B \) is an IF generalized* minimal open set \( \text{Cl}(B) \supseteq U \) implies \( \text{Cl}(A) \supseteq \text{Cl}(B) \supseteq U \)

i.e. \( A \) be an IF generalized* minimal open set.

**Remark 3.4:** There does not exist any IF Minimal Open Set between \( A \) and \( B \) such that \( A \subset B \) and \( A \) is an IF generalized* minimal closed set.

**Theorem 3.5:** If \( A \) is an IF generalized* minimal open set then \( \text{Int}(A) = 0\~, \text{Int}(A) = A \)

i.e. \( A \) is an IF rare set or an IF minimal open set.

**Proof:** As \( \text{Int}(A) \) is an IF Open Set and \( \text{Int}(A) \subseteq A \subseteq B \) (for some IF Minimal Open Set) \( \Rightarrow \text{Int}(A) = 0\~, \) or \( A \) as IF Minimal Open Set does not contain any IF Open Set other than itself or 0.

**Remark 3.6:** The converse of the above theorem may not be true and it can be shown with the help of an example:
Let $A = \{<x, 0.2, 0.4>: x \in X\}$ be an IF of $X$ and the corresponding topological space be $\tau = \{0~, 1~, A\}$. Here $A$ is an IF Minimal Open Set of $\tau$. Consider a set $C = \{<x, 0.1, 0.3>: x \in X\}, \text{Int}C = 0~$, but $C$ is not an IF generalized* minimal open set.

**Theorem 3.7:** Every IF Minimal Open Set is an IF generalized* minimal open set in itself.

**Proof:** Let $A$ is an IF Minimal Open Set. We know that $\text{Cl}A \supseteq A$. Since $A$ is a minimal open set, so from definition $A$ is an IF generalized* minimal open set.

**Remark 3.8:** The converse of the above theorem need not be true, as IF Set $C$ in example 3.2 is IF generalized* minimal open set but it is not an IF Minimal Open Set. According to the theorem 3.5 the converse is true if the set is not a rare set. i.e. for a set which is not rare, IF minimal open set and IF generalized* minimal open set are similar concepts.

**Theorem 3.9:** If $B (\neq 0~)$ is an IF Open Set then $B$ will be IF generalized* minimal open set iff $B$ is an IF Minimal Open Set.

**Proof:** Let $B$ is an IF Open Set which is IF generalized* minimal open set. From definition there exist a IF minimal open set $A$ containing $B$ such that $\text{Cl}B \supseteq A$. But an IF Minimal Open Set does not contain any other IF Open Set except itself. i.e. $B = A$ implies $B$ is an IF Minimal Open Set.

Conversely, let $B$ is an IF Minimal Open Set, then as proved in theorem 3.7, $B$ is an IF generalized* minimal open set.

**Theorem 3.10:** Every IF-dense set is an intuitionistic fuzzy generalized* minimal open set if it is a subset of some IF Minimal Open Set but the converse is not true.

**Proof:** Let $A$ is an IF $\tau$ dense set $\Rightarrow \text{Cl}A = 1~$.

If $A \subseteq B$ (B is an IF generalized* minimal open set), then $\text{Cl}A = 1~ \supseteq B$.

$\Rightarrow A$ is an IF generalized* minimal open set.

The converse is not true as shown in example 3.6 C is an intuitionistic fuzzy generalized* minimal open set, but $C$ is not an IF $\tau$ dense set as $\text{Cl}C \neq 1~$.

**Theorem 3.11:** An IF fuzzy generalized* minimal open set $A$ is IF Generalized Closed Set iff $\text{Cl}A = B$, where $B$ is an IF Minimal Open Set.

**Proof:** Since $A$ is an IF generalized* minimal open set, $A \subseteq B$ where $B$ is an IF minimal open set. And $\text{Cl}A \supseteq B$.............(1)

But $A$ is IF Generalized Closed Set which implies $\text{Cl}A \subseteq B$.............(2)

So from (1) and (2) $\text{Cl}A = B$.

Conversely let $\text{Cl}A = B$ and $A$ is an IF generalized* minimal open set.

From definition $A \subseteq B$ implies $\text{Cl}A \supseteq B$, but $\text{Cl}A = B$ i.e. $\text{Cl}A \subseteq B$ implies $A$ is IF Generalized Closed Set.

**Theorem 3.12:** If the IF minimal open set containing a generalized* minimal closed set is IF closed set then the generalized* minimal closed set is a IF Pre-open set.

**Proof:** Let $U$ be a IF minimal open set containing $A$. Since $A$ is a generalized* minimal closed set $\text{Cl}A \supseteq U$. Since $U \supseteq A$, $\text{Cl}A \supseteq \text{Cl}U = U$. Since $U$ is an IF
closed set. Therefore \(U = \text{Cl}A\). Hence \(A\) is an IF Pre–Open Set.

**Theorem 3.13:** Let \(A\) be an closed set and an IF generalized * minimal closed set then \(A\) is the minimal open set.

**Proof:** Let \(U\) be a IF minimal open set containing \(A\). Since \(A\) is an IF generalized * minimal closed set, \(\text{Cl}A \supseteq U\) i.e. \(A \supseteq U\) i.e. \(A= U\). Hence \(A\) is an If minimal open set.

**Theorem 3.14:** A IF Set \(A\) contained in an IF minimal open set is an IF generalized * minimal closed set if it is an IF generalized *closed set.

**Proof:** Let \(A \subseteq U\), an IF minimal open set. Since an IF minimal open set is a subset of any IF open set, \(A \subseteq U \subseteq O\), an IF open set. Here \(A\) is an IF generalized *closed set, implies \(\text{Cl}A \supseteq O \supseteq U\) i.e. \(A\) is an IF generalized * minimal closed set.

**Remark 3.15:** The converse of the above theorem need not be true. Let us consider the following example:

**Remark 3.16:** \(0\sim\) and \(1\sim\) are not an IF generalized* minimal open set.

**Theorem 3.17:** Arbitrary union of IF generalized* minimal open set is an IF generalized* minimal open set if it is contained in an IF minimal open set.

**Proof:** Let \(\bigcup \{B_i : i \in I\} \subseteq A\) (where \(A\) is an IF Minimal Open Set and \(i \in I\)). \(\Rightarrow \{B_i : i \in I\} \subseteq A \Rightarrow \text{Cl} \{B_i : i \in I\} \supseteq A\) \(\Rightarrow \bigcup \text{Cl} \{B_i : i \in I\} \supseteq \bigcup \text{Cl} \{B_i : i \in I\}\). \(\Rightarrow \bigcup \{B_i : i \in I\}\) is also an IF generalized* minimal open set.

**Remark 3.18:** The intersection of two IF generalized* minimal open sets need not be an IF generalized* minimal open set. This can be shown with the help of an example:

**Remark 3.19:** The collection of all IF generalized * minimal open set forms an IF supra topological space if \(0\sim\) and \(1\sim\) are included in the collection. This supra topological space may be denoted as \((X, g*M)\) and is named as IF generalized * minimal supra topological space.

**Theorem 3.20:** An IF set \(A\) of \(X\) is both IF generalized minimal closed set and IF generalized*minimal closed set iff \(\text{Cl}A = U\).
**Theorem 3.21:** The union of an IF generalized minimal closed set and an IF generalized * minimal closed set is an IF generalized * minimal closed set.

**Proof:** Let $A$ be an IF generalized minimal closed set and $B$ be an IF generalized * minimal closed set in the same IF topological space. Let $P = A \cup B$. Here $\text{Cl}A \subseteq U \subseteq \text{Cl}B$. Therefore $P = A \cup B \subseteq U$ and $\text{Cl}(A \cup B) = \text{Cl}A \cup \text{Cl}B = \text{Cl}B \supseteq U$. Hence $P$ is an IF generalized* minimal closed set.

**Remark 3.22:** Intersection of an IF generalized minimal closed set and an IF generalized*minimal closed set is an IF generalized minimal closed set.

Let us now form the relational structure between the newly introduced sets and other sets introduced earlier by various researchers.

\[ \text{IF Generalized minimal closed set} \]

\[ \text{IF Generalized closed set} \quad \text{IF Minimal open set} \]

\[ \text{IF Rare set} \quad \text{IF Generalized minimal *} \]

**Conclusion:** Through out this paper the concept of IF generalized minimal closed set and IF generalized * minimal closed set has been introduced. It is seen that the collection of IF generalized minimal closed set forms an IF Alexandroff space if $X$ is included but the collection of IF generalized* minimal closed set doesn’t forms even a Supra topological space. Also it is seen that the IF generalized * minimal closed set is similar as IF minimal open set if it is not an IF rare set. Though the other concepts of IF generalized* closed set coincide with IF dense set. These two spaces may have lots of applications which may appear in our coming papers.

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